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SHRINKAGE BEHAVIOUR MODELLING OF STEEL-CONCRETE COMPOSITE BEAMS WITH VARYING DEGREE OF CONNECTION

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ABSTRACT

The work introduced in this article is a theoretical study of the behaviour of composite beams with respect to the shrinkage of concrete. In fact, our contribution resides essentially in consideration of the degree of connection (N/N_f) between the slab and steel beam. While using the theory of the linear viscoelasticity of the concrete, and on the basis of the Rate of Creep Method of concrete, in proposing an analytical model, made up by a system of two linear differential equations, emphasizing the effects caused by shrinkage on the resistance of a steel-concrete composite beams regardless degree of connection employed.

Keywords: Composite beams; shrinkage; time; degree of connection (N/N_f); rate of creep method; differential equations.

1. INTRODUCTION

Steel-concrete composite beams are a popular and economical form of construction in both buildings and bridges. A reinforced concrete slab is mechanically connected to the top flange of a rolled or fabricated steel beam, thereby forming a composite member that is considerably stronger and stiffer than the steel beam acting on its own.

The problem of investigating the statically determinate composite plate beam in the time t has for 60 years drawn the attention of engineers who were dealing with the problems of their design. This problem has, however, received a certain currency in the past few years, due to the new facts gathered about the rheological qualities of concrete [1].

It is known that while in the steel beam, under the effect of the serviceability loads, we see

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only elastic deformations, in the concrete plate during the time significant plastic deformation takes place as a consequence of shrinkage of concrete [1,2].

Shrinkage designates the time-dependent strains which concrete suffers at constant temperature without any external strains. Since it is essentially a consequence of the drying process, the dimensions of the structural element have a great on the shrinkage strains [3].

As a result of these deformations and because of the stiff connection between the two elements of the composite plate beam, in every cross-section subjected to the effect of shrinkage in the time t arises a new additional group of forces and moments $N_c(t)$, $M_c(t)$, $N_s(t)$, $M_s(t)$ (Fig. 1) [1]. The influence of this group of forces and moments over the general stress conditions of the statically determinate composite plate beam is expressed by the increase of the stresses in the concrete plate and in the steel beam [4,5,6,7].

The papers dealing with the solution of the problem of finding the unknown normal forces $N_c(t)$, $N_s(t)$ and the bending moments $M_c(t)$, $M_s(t)$ are numerous and diverse. Their chronological analysis shows from the first aspect the aspirations of various authors to penetrate further into the actual behavior of the structures, which will eventually lead to the creation of more accurate calculation methods; and from the second aspect it shows the aspirations to replace the complicated methods by simple ones for practical usage.

The effect of shrinkage of concrete on the behavior of steel-concrete composite beams has been studied analytically by [8]. The proposed method is to determine, at any time t, the solicitations and the additional stresses provided by the shrinkage of concrete in the steel-concrete mixed beams having perfect connection only.

By introducing the influence of the degree of connection (N/N_f) on the steel-concrete interface (case of composite beams in which the degree of connection $N/N_f < 1$), our present contribution is therefore to develop and enrich the model developed by Rahal *et al* [8] which was designed only for composite beams with full connection (in which the degree of connection $N/N_f = 1$). Hence, the present methodology is more general and applicable to composite beams regardless of the degree of connection (N/N_f) employed.

The work introduced in this article is a theoretical study of the behaviour of composite beams with respect to the phenomenon of concrete shrinkage. In fact, our contribution through this work resides essentially in consideration of the degree of connection (N/N_f) between the slab and steel beam. While using the theory of the linear viscoelasticity of the concrete, and on the basis of the Rate of Creep Method (RCM) of concrete, in proposing an analytical model, made up by a system of two linear differential equations, emphasizing the effects caused by shrinkage on the resistance of a steel-concrete composite section.

The well stocked results, by the use of formulation proposed, are satisfactory while confronting them to those given by Rahal *et al* [8] part and the results obtained by Hendy *et al* [9]. They also show that the variation of the efforts, in normal efforts and in bending moments, soliciting the slab made of concrete and the metallic beam is in the beginning important and so that it decreases, continually, during the time until the stabilization. They also show that new local constraint redistribution will be developed in the slab made of concrete and in the metallic beam.

2. TİME-DEPENDENT DEFORMATİON

2.1 Concrete strain components

At any time t, the total concrete strain ε(t) in an uncracked, uniaxially-loaded specimen consists of a number of components that include the instantaneous strain $\varepsilon_e(t)$, creep strain $\varepsilon_{cr}(t)$, shrinkage strain $\varepsilon_{sh}(t)$ and temperature strain $\varepsilon_T(t)$ [10,11,12] (Fig. 1). Although not strictly correct, it is usual to assume that all four components are independent and may be calculated separately and combined to obtain the total strain. When calculating the in service behaviour of a concrete structure at constant temperature [11,13,14,15], it is usual to express the concrete strain at a point as the sum of the instantaneous, creep and shrinkage components. The strain components in a drying specimen held at constant temperature and subjected to a constant sustained compressive stress:

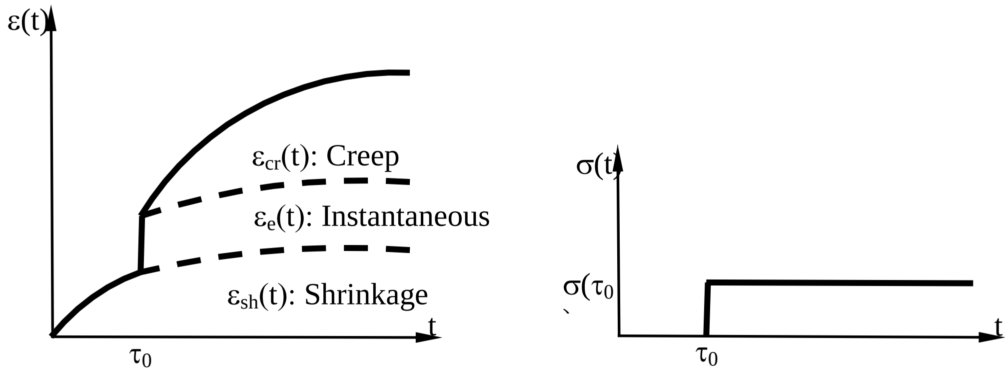

Figure 1. Concrete strain components under sustained load

$$\varepsilon(t)=\varepsilon_{sh}(t)+\varepsilon_e(t)+\varepsilon_{cr}(t) \tag{1}$$

2.2 The creep coefficient φ(t, τ), and the creep function J(t, τ)

The capacity of concrete to creep is usually measured in terms of the creep coefficient, φ(t,τ). In a concrete specimen subjected to a constant sustained compressive stress, $\sigma_c(\tau)$, first applied at age τ , the creep coefficient at time t is the ratio of the creep strain to the instantaneous strain and is given by [2,6,10,11,14]:

$$\varphi(t,\tau)=\frac{\varepsilon_{cr}(t,\tau)}{\varepsilon_e(\tau)} \tag{2}$$

Therefore, the creep strain at time t caused by a constant sustained stress $\sigma_c(\tau)$ first applied at age τ is:

$$\varepsilon_{cr}(t,\tau)=\varphi(t,\tau)\varepsilon_e(\tau)=\varphi(t,\tau)\frac{\sigma_c(\tau)}{E_c(\tau)} \tag{3}$$

where $E_c(\tau)$ is the elastic modulus at time τ . For concrete subjected to a constant sustained stress, knowledge of the creep coefficient allows the rapid determination of the creep strain at any time.

Another frequently used time function is known as specific creep, C(t,τ), and is the proportionality factor relating stress to linear creep [10,11,12,13], i.e.:

$$\varepsilon_{cr}(t,\tau)=C(t,\tau)\sigma_c(\tau) \text{ or } C(t,\tau)=\frac{\varepsilon_{cr}(t,\tau)}{\sigma_c(\tau)} \tag{4}$$

C(t,τ) is the creep strain at time t produced by a sustained unit stress first applied at age τ.

The relationship between the creep coefficient and specific creep can be obtained from equations (Eqs.1 and 3) [10,11,12,13]:

$$\varphi(t,\tau)=C(t,\tau)E_c(\tau) \tag{5}$$

In many models relatives to structures analysis, the creep function J(t, τ) is composed of an instantaneous part and a differed component (observed during time) [10,11,14]. The sum of the instantaneous and creep strains at time t produced by a sustained unit stress applied at τ is defined as the creep function, J(t,τ), and is given by:

$$J(t,\tau)=\frac{1}{E_c(\tau)}+C(t,\tau)=\frac{1}{E_c(\tau)}[1+\varphi(t,\tau)] \tag{6}$$

2.3 Principle of superposition of the loads and its integral representation

Because the load-dependent strains in concrete at service loads are proportional to stress, the principle of superposition is frequently used to estimate the deformation caused by a time-varying stress history [10,11,12,13,14]. The principle of superposition was first applied to concrete by McHenry [15] (Henry 1943) who stated that the strain produced by a stress increment applied at any time τ_i is not affected by any stress applied either earlier or later. This principle is illustrated in Fig. 2.

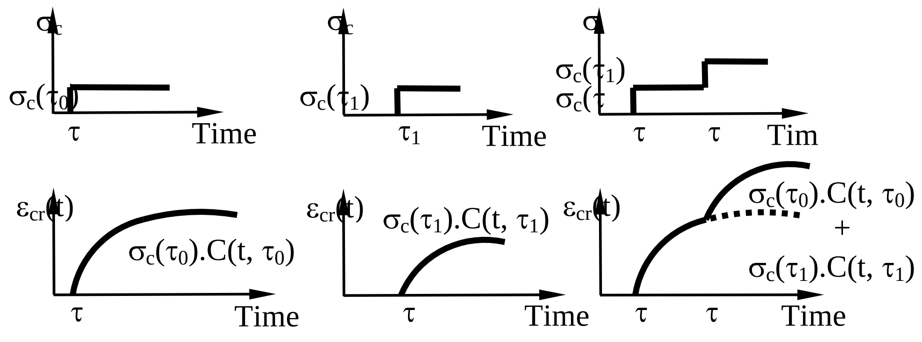

Figure 2. The principle of superposition of creep

According to the principle of superposition, the total stress-dependent strain in concrete at time *t* (elastic plus creep strains) can be written as [11,12]:

$$\varepsilon_e(t)+\varepsilon_{cr}(t)+\varepsilon_{sh}(t)=\frac{\Delta\sigma_c(\tau_0)}{E_c(\tau_0)}\left[1+\varphi(t,\tau_0)\right]+\frac{\Delta\sigma_c(\tau_1)}{E_c(\tau_1)}\left[1+\varphi(t,\tau_1)\right]+\varepsilon_{sh}(t) \tag{7}$$

$$\varepsilon_e(t)+\varepsilon_{cr}(t)+\varepsilon_{sh}(t)=\sum_{i=0}^{1}\frac{\Delta\sigma_c(\tau_i)}{E_c(\tau_i)}\left[1+\varphi(t,\tau_i)\right]+\varepsilon_{sh}(t) \tag{8}$$

where the argument τ has been eliminated from the elastic strain $\varepsilon_e(t)$ and creep strain $\varepsilon_{cr}(t)$ as the stress history varies with time. This notation is more appropriate for realistic time-varying stress histories and will be used throughout the remainder of the paper. Equations (Eqs: 7 and 8) can also be written in terms of creep functions as [11,12]:

$$\varepsilon_e(t)+\varepsilon_{cr}(t)+\varepsilon_{sh}(t)=J(t,\tau_0)\Delta\sigma_c(\tau_0)+J(t,\tau_1)\Delta\sigma_c(\tau_1)=\sum_{i=0}^{1}J(t,\tau_i)\Delta\sigma_c(\tau_i)+\varepsilon_{sh}(t) \tag{9}$$

In the case of increasing loading gradually in time from τ to t (Fig.2), the deformation can be computed using equation (Eq.10) [10,11,12,14]:

$$\varepsilon(t)=J(t,\tau_0)\sigma_c(\tau_0)+\int_{\tau_1}^{t}\frac{d\sigma(\tau)}{d\tau}J(t,\tau)d\tau+\varepsilon_{sh}(t) \tag{10}$$

Equation (Eq.10) is often referred to as the hereditary of Volterra integral equation [10,11,12]. It shows that at time t the total strain ε(t) not only depends on the stress σ(τ), but rather on the whole stress history.

This deformation with respect to the time t can be expressed by the function of the stress σ(τ) [10,12]. Thus, a simple integration by part of the formula (Eq.10) should be carried out to obtain the relationship which expresses the following creep, which can be called the ``equation of creep":

$$\varepsilon(t)=\frac{\sigma_c(t)}{E_c(t)}-\int_{\tau_0}^{t}\sigma_c(\tau)\frac{\partial}{\partial\tau}\left[\frac{1}{E_c(\tau)}+J(t,\tau)\right]d\tau+\varepsilon_{sh}(t) \tag{11}$$

Substituting the coefficient of creep φ given by equation (Eq.6) and neglecting the variation of modulus of elasticity of concrete in time [10,11,12,14], we find the form alternative creep equation given by the relation (Eq.12).

$$\varepsilon(t)=\frac{\sigma_c(t)}{E_c(\tau_1)}-\frac{1}{E_c(\tau_1)}\int_{\tau_1}^{t}\sigma_c(\tau)\frac{\partial}{\partial\tau}\left[1+\varphi(t,\tau)\right]d\tau+\varepsilon_{sh}(t) \tag{12}$$

2.4 The rate of creep method for time-dependent deformations

According to the theory [10,11], the value of the rate of creep C depends only on the time τ_1 due to the first application of load. After the removal of the load, the deformation that has developed until time t=τ remains preserved. The value of C after unloading becomes:

$$C(t,\tau)=C(t,\tau_1)-C(t,\tau)=\frac{1}{E_c(t)}\left[\varphi(t,\tau_1)-\varphi(t,\tau)\right] \tag{13}$$

φ : Creep coefficient defined as the ratio of the creep deformation at time t over the initial deformation relative to a stress applied at 28 days.

The equation of creep (Eq. 8), according to this theory, becomes:

$$\varepsilon(t)=\frac{\sigma_c(t)}{E_c(\tau_1)}-\frac{1}{E_c(\tau_1)}\int_{\tau_1}^{t}\sigma_c(\tau)\frac{\partial}{\partial\tau}\left[1+\varphi(t,\tau_1)-\varphi(t,\tau)\right]d\tau+\varepsilon_{sh} \tag{14}$$

The relative deformation at time t, is calculated by carrying out an integration by parts of the formula (Eq. 14). After integration by parts, we obtain [10,11,14]:

$$\varepsilon(t)=\frac{\sigma_c(t)}{E_c(\tau_1)}+\frac{1}{E_c(\tau_1)}\int_{\tau_1}^{t}\sigma_c(\tau)\frac{\partial\varphi(t,\tau)}{\partial\tau}d\tau+\varepsilon_{sh} \tag{15}$$

Taking into account of the homothety that exists between the shrinkage deformations and those of creep (shrinkage develops at the same rate as creep i.e. the creep and shrinkage curves are affine) [10,12,14], and that is represented by equation (Eq. 16):

$$\varepsilon_{sh}(t)=\frac{\varepsilon_{sh}(\infty)}{\varphi(\infty)}\varphi(t,\tau_0)\Rightarrow\frac{d\varepsilon_{sh}(t)}{d\varphi}=\frac{\varepsilon_{sh}(\infty)}{\varphi(\infty)} \tag{16}$$

After the derivation with respect to the creep coefficient φ, the final equation expressing the total deformation will be given by the relationship we obtain [10,12,14]:

$$\frac{d\,\varepsilon(t,\tau)}{d\,\varphi}=\frac{1}{E_c(\tau_1)}\frac{d\,\sigma_c(t)}{d\,\varphi}+\frac{\sigma_c(t)}{E_c(\tau_1)}+\frac{\varepsilon_{sh}(\infty)}{\varphi(\infty)} \tag{17}$$

3. FORMULATİON OF THE PROPOSED METHOD

Let's consider that a cross section of a steel-concrete mixed beam is subjected to a normal force N_0 and a bending moment M_0, applied at the center of gravity of its cross section made homogeneous (Fig. 3).

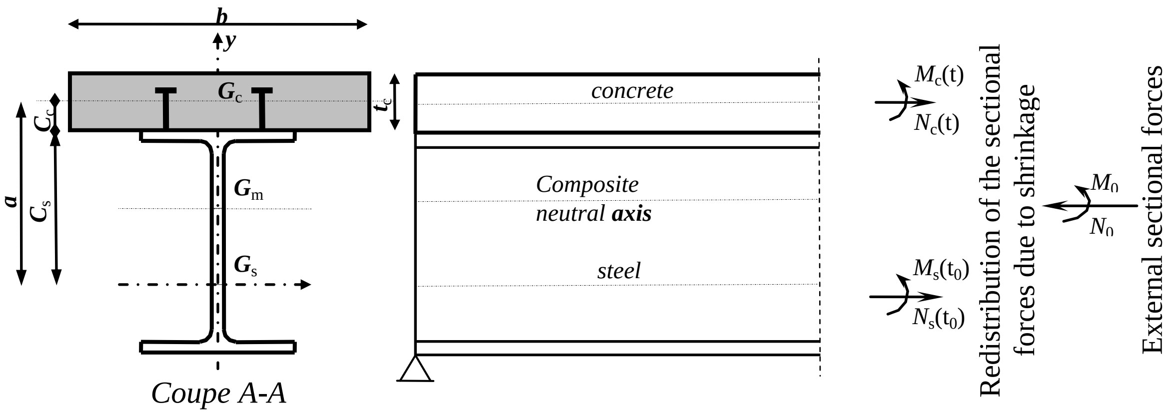

Figure 3. Internal efforts in steel-concrete composite beams subjected to shrinkage

3.1 Basic equations of equilibrium

In a steel-concrete composite cross section with perfect or partial connection, the shrinkage will affect in time, the diagram of stress distribution but the equilibrium conditions must be preserved [15,16,17]. By introducing the effect of the degree of connection (N/N_f), we can write the static equilibrium of the composite cross section, as follows:

$$\begin{cases} N_s(t) - N_c(t)\left[\left(\frac{A_s}{A_m} - \frac{A_c.C_c^2}{I_m}\right)\left(1 - \frac{N}{N_f}\right) - 1\right] = N_0 \\ M_s(t) - N_c(t)\ a\left[1 - \left(\frac{A_s}{A_m} - \frac{A_c.C_c^2}{I_m}\right)\left(1 - \frac{N}{N_f}\right)\right] + M_c(t) = M_0 \end{cases} \tag{18}$$

So that we can replace the forces $N_s(t)$ and $M_s(t)$ in the system of equations (Eqs.18) by expressions in term of $N_c(t)$ and $M_c(t)$ given by the static equilibrium equations of the cross section (Eqs.19).

$$\begin{cases} N_s(t) = N_0 + N_c(t)\left[\left(\frac{A_s}{A_m} - \frac{A_c.C_c^2}{I_m}\right)\left(1 - \frac{N}{N_f}\right) - 1\right] \\ M_s(t) = M_0 + N_c(t)a\left[1 - \left(\frac{A_s}{A_m} - \frac{A_c.C_c^2}{I_m}\right)\left(1 - \frac{N}{N_f}\right)\right] - M_c(t) \end{cases} \tag{19}$$

A : distance between the neutral axis of the steel section and that of the concrete slab.
b_{eff} : effective width of the reinforced concrete slab.
$M_c(t)$: bending moment in the concrete slab due to creep.
$M_s(t)$: bending moment in the steel beam.

M_0 : bending moment due to the external loads.
N : number of partial connection.
$N_c(t)$: normal effort in the concrete slab due to creep.
N_f : number of total connection.
$N_s(t)$: normal effort in the steel section.
N_0 : normal effort applied outside.
N/N_f : rate of connection in (%).
G_c : center of gravity of the concrete slab.
G_m : center of gravity of the mixed section made homogeneous.
G_s : center of gravity of the metallic beam.
t_c : thickness of the slab.

3.2 Compatibility equations

Due to the fact that the problem is a twice internally statically indeterminate system, the equilibrium equations (Eqs.19) are not sufficient to solve it. It is necessary to produce two additional equations in the sense of compatibility and deformations of both steel girder and concrete slab in time t.

The behaviour of the mixed beams is characterized by the interaction between steel and concrete [15,16,17]. The use of the connexion that prevents slip and lifting of the two materials allows us writing the interface condition at which the curvatures and deformations of the concrete slab and the steel beam are equal and one can write:

3.2.1 Compatibility of curvatures on the contact surfaces concrete/steel

$$\chi_c(t) = \frac{M_c(t)}{E_c\, I_c} = \frac{M_s(t)}{E_s\, I_s} \tag{20a}$$

3.2.2 Compatibility of deformations on the contact surfaces concrete/steel

$$\varepsilon_c(t) = \frac{N_c(t)}{E_c\, A_c} + \frac{M_c(t)}{E_c\, I_c} C_c = \frac{N_s(t)}{E_s\, A_s} + \frac{M_s(t)}{E_s\, I_s} C_s \tag{20b}$$

C_s : distance from the steel beam centre of gravity to the neutral fibre of the mixed section.
I_c : moment of inertia of the concrete slab.
I_s : moment of inertia of the metallic beam.

In accordance to equation (Eq.14), the two conditions (Eqs.21a and 21b) can be reduced to equations giving the variation, in time, of the internal forces and which will take the underneath form:

$$\chi_c(t) = \frac{1}{E_c I_c}\left[M_c(t) - \int_{\tau}^{t} M_c(\tau) \frac{\partial}{\partial \tau}\left[1 + \varphi(t,\tau)\right] \right] = \frac{M_s(t)}{E_s I_s} \tag{21a}$$

$$\varepsilon_c(t)=\frac{1}{E_cA_c}\left[N_c(t)-\int_{\tau}^{t}N_c(\tau)\frac{\partial}{\partial\tau}[1+\varphi(t,\tau)]\right]$$

$$+\frac{C_c}{E_cI_c}\left[M_c(t)-\int_{\tau}^{t}M_c(\tau)\frac{\partial}{\partial\tau}[1+\varphi(t,\tau)]\right] \qquad \text{21b}$$

$$-\varepsilon_r(t)=\frac{N_s}{E_sA_s}-\frac{M_s}{E_sI_s}C_s$$

In accordance with the irreversible law of concrete, the two conditions (Eqs.21a and 21b) can be transformed into differential equations of the type of relation (Eq.17), one can write:

$$\chi_c(t)=\frac{1}{E_cI_c}\left[dM_c(t)+M_c(t).d\varphi\right]=\frac{M_s(t)}{E_sI_s} \qquad (22a)$$

$$\varepsilon_c(t)=\frac{1}{E_cA_c}\left[dN_c(t)+N_c(t).d\varphi\right]+\frac{C_c}{E_cI_c}\left[dM_c(t)+M_c(t).d\varphi\right]$$

$$-d\varepsilon_r(t)=\frac{N_s}{E_sA_s}-\frac{M_s}{E_sI_s}C_s \qquad (22b)$$

Knowing that the shrinking of the concrete is independent of loading, it balances then a system externally nil. So that we can replace the forces $N_s(t)$ and $M_s(t)$ in the system of equations (Eqs. 22a and 22b) by expressions in term of $N_c(t)$ and $M_c(t)$ given by the static equilibrium equations of the cross section (Eqs. 19). It is then obtained the following system of differential equations:

$$A_1\frac{dM_c(t)}{d\varphi}+A_2\ \frac{dN_c(t)}{d\varphi}+M_c(t)=0 \qquad (23a)$$

$$A_3\frac{dM_c(t)}{d\varphi}+A_4\frac{dN_c(t)}{d\varphi}+A_5\,M_c(t)+A_6\,N_c(t)=0 \qquad (23b)$$

$$A_1=\left(1+\frac{I_c}{n\,I_s}\right)\left[1-\left(\frac{A_s}{A_m}-\frac{A_c\,.C_c{}^2}{I_m}\right)\left(1-\frac{N}{N_f}\right)\right]\quad A_2=-\frac{a.I_c}{n.I_s}\left[1-\left(\frac{A_s}{A_m}-\frac{A_c\,.C_c{}^2}{I_m}\right)\left(1-\frac{N}{N_f}\right)\right]$$

$$A_3=\left(\frac{C_c}{I_c}-\frac{C_s}{n\,I_s}\right)\left[1-\left(\frac{A_s}{A_m}-\frac{A_c\,.C_c{}^2}{I_m}\right)\left(1-\frac{N}{N_f}\right)\right]\quad A_5=\frac{C_c}{I_c}\left[1-\left(\frac{A_s}{A_m}-\frac{A_c\,.C_c{}^2}{I_m}\right)\left(1-\frac{N}{N_f}\right)\right]$$

$$A_4=\left(\frac{1}{A_c}+\frac{1}{nA_s}+\frac{C_s\,.a}{nI_s}\right)\left[\left(\frac{A_s}{A_m}-\frac{A_c\,.C_c{}^2}{I_m}\right)\left(1-\frac{N}{N_f}\right)-1\right]\quad A_6=-\frac{1}{A_c}\left[1-\left(\frac{A_s}{A_m}-\frac{A_c\,.C_c{}^2}{I_m}\right)\left(1-\frac{N}{N_f}\right)\right]$$

If we consider in the equations (Eqs.23a and 23b), that $N=N_f$ (case of beams with full connection), we obtain the system of differential equations developed by Rahal *et al* [8]:

The general solution of the system of differential equations (Eqs.23a and 23b), and which constitutes the relations giving the variation, in time, of the loadings, i.e: bending moments *$M_c(t)$* and normal efforts *$N_c(t)$*, are given respectively by the following matrix expression (Eqs.24):

$$\begin{Bmatrix} M_c(t) \\ N_c(t) \end{Bmatrix} = \begin{bmatrix} C_1 & C_2.a_1 \\ C_1.a_2 & C_2 \end{bmatrix} \begin{Bmatrix} e^{\lambda_1 \varphi(t)} \\ e^{\lambda_2 \varphi(t)} \end{Bmatrix} + \begin{Bmatrix} 0 \\ E_c \dfrac{\varepsilon_{sh}(\infty)}{\varphi(\infty)} \end{Bmatrix} \tag{24}$$

C_1 and C_2: are constants computed from the condition that at time $t = 0$ one have $M_c(t) = N_c(t) = 0$ (boundary conditions). Their values are:

$$C_1 = -C_2.a_1 ; \; C_2 = \frac{\varepsilon_{sh}(\infty)}{\varphi(\infty)} \frac{1}{A_6} \frac{1}{1 - a_1.a_2} ; \; a_1 = -\frac{A_4\lambda_2 + A_6}{A_3\lambda_2 + A_5} ; \; a_2 = -\frac{(A_1\lambda_1 + 1)}{A_2\lambda_1}$$

4. NUMERİCAL COMPUTATİON

To validate this model, we examined composite beam treated in Eurocode 4 [9], using our proposed method. In this example, the parameters of shrinkage were calculated according to Eurocode 2 [6]. The geometric and physical characteristics of the example discussed are:

b_{eff} =3100 mm, t_c = 250+25 mm, b_{ft} = 400 mm, t_{ft} = 20 mm, b_{fb} = 400 mm, t_{fb} = 30 mm. h_w = 1175 mm, t_w = 12.5 mm, A_c = 0.785 m^2, A_s = 0.0346875 m^2, I_c = 0.004223633272 m^4 , I_s = 0.0346875 m^4, C_c = 0.375 m, C_s = 0.451 m, a= 0.826 m, E_c = 33×10^4 MPa, E_s =2.1×10^5 MPa, n = 6.56. The deck concrete is grade C30/37 and the relative humidity is 70%.

5. DİSCUSSİON OF RESULTS

In this work, the different degrees of connection selected are: N/N_f = (100%, 80% and 60%). Calculation of the bending moments $M_c(t)$ and the efforts normal $N_c(t)$ in the concrete slab, were carried out until the stabilization of these loadings was observed. The results, showing the evolution of loadings in the time, are presented graphically in diagrams (Figs.4, 5, 6, 7, 8, 9, 10 and 11).

The obtained results by the present formulation (Figs. 4 to 11) reflect sufficiently the connection degree effect (N/N_f) on the redistribution of local stresses due to shrinkage of concrete. Fig. 4 show the comparison of obtained results by this formulation to those Eurocode 4 (Hendy and Johnson 2006) [9], and those obtained by Rahal *et al* [8]. Figs. 9, 10 and 11 show clearly that by reducing the connection degree N/N_f, the constraints will be reduced as a result. Hence in order to reduce the effects of shrinkage on the behavior of steel-concrete composite beams, it is very useful to use a connection degree of $N/N_f < 1$ instead of a connection degree of $N/N_f = 1$.

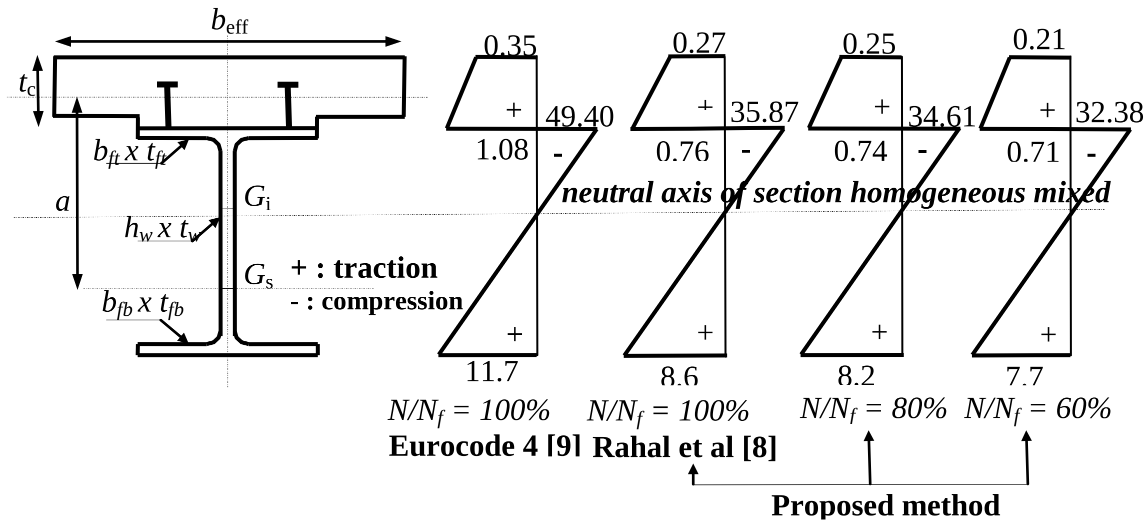

Figure 4. Diagrams comparing the final stress (MPa) due to shrinkage of concrete

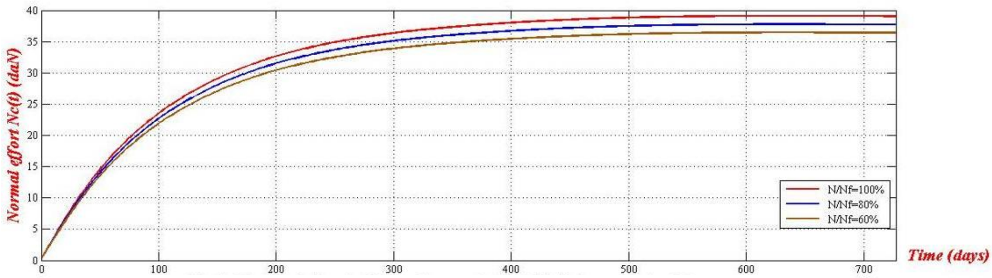

Figure 5. The evolution in time of normal effort $N_c(t)$ in concrete slab

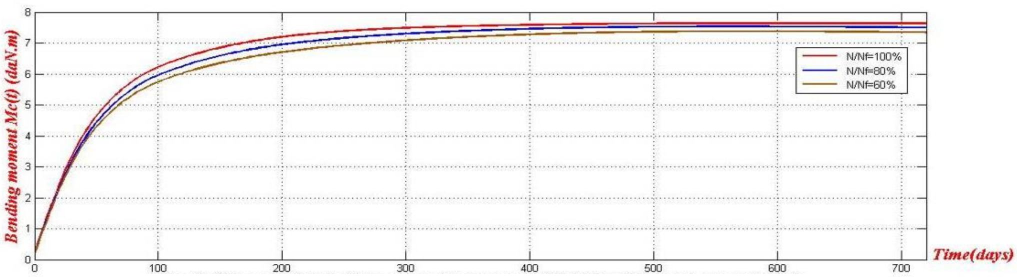

Figure 6. The evolution in time of bending moment $M_c(t)$ in concrete slab

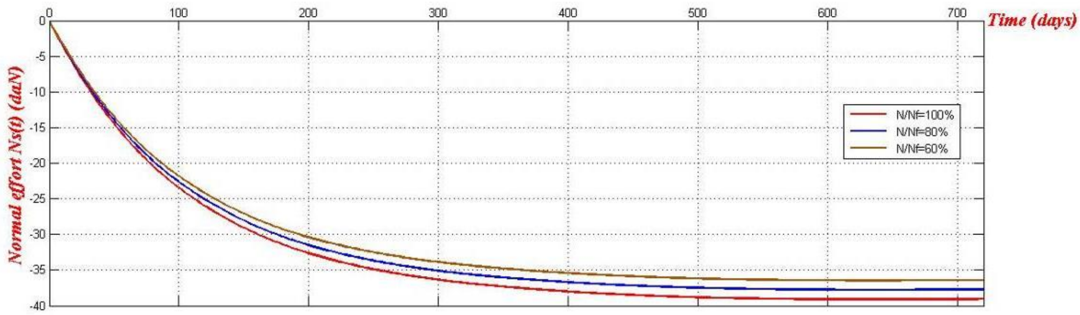

Figure 7. The evolution in time of normal effort $N_s(t)$ in steel beam

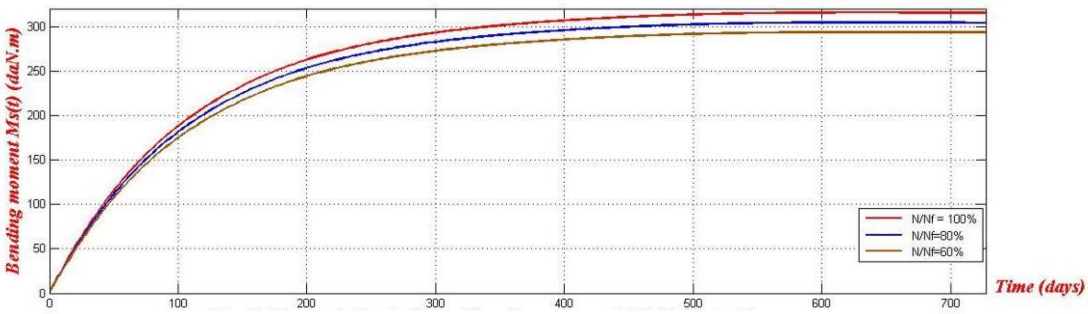

Figure 8. The evolution in time of bending moment $M_s(t)$ in steel beam

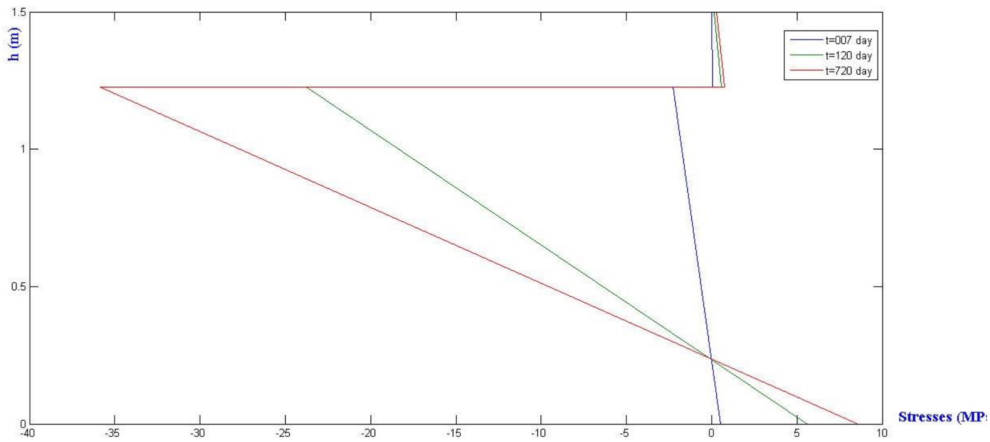

Figure 9. The redistribution of stresses for N/N_f = 100%

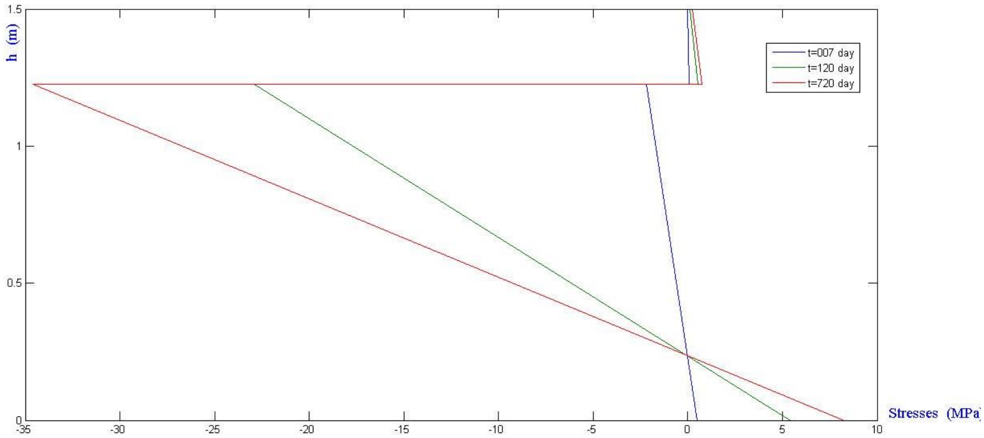

Figure 10. The redistribution of stresses for N/N_f = 80%

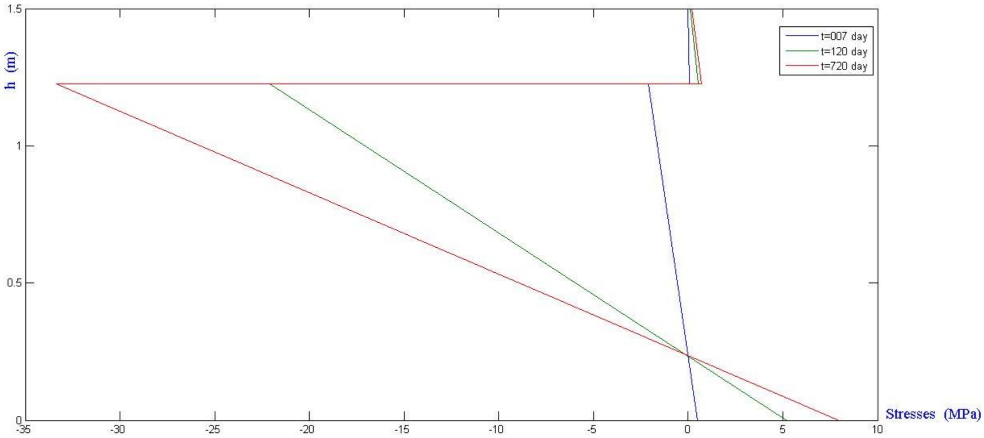

Figure 11. The redistribution of stresses for N/N_f = 60%

6. CONCLUSION

During time, the equilibrium state of a steel-concrete mixed beam progresses under the concrete shrinkage. In this context, a framework for analytical formulation was developed in this paper. It then allows us to treat properly the influence of the time-dependant behaviour of mixed beams, including the shrinkage phenomenon of the concrete slab.

This model leads writing equilibrium and compatibility equations and the constitutive laws for the steel part and a Volterra integral equation for the concrete part. The problem is governed by a system of two simultaneous differential equations (Eqs. 24). As known in this differential equations it exists a group of normal forces $N_c(t)$, $N_s(t)$ and bending moments $M_c(t)$, $M_s(t)$ which influence the general stress conditions of the statically determinate composite beam is expressed by the increase of the stresses in the concrete slab and in the steel beam with respect to time t.

The results provided by the application of our method, show that the redistribution of efforts, resulting from shrinkage of concrete, is not negligible. In the steel-concrete mixed beams, the action of concrete shrinkage has the effect of causing a stress self-balanced state characterized by:

1- Positive bending moment and a normal force of traction in the slab.

2- Positive bending moment and a normal force of compression in the steel beam.

For the service load analysis, this analytical method makes it possible to follow with great precisions the migration of the stresses from the concrete slab to the steel beam, which occurs gradually during the time as a result of shrinkage of the concrete.

The model thus developed in this paper is to extend and generalize the one proposed by Rahal and al [8] (designed only for composite beams with full connection: $N/N_f = 1$), while introducing a new variable that is the degree of connection (N/N_f). It will then find the new stress redistribution due to shrinkage (Fig. 4) and correspond to the case of complete connection and the partial connection. Hence, our present methodology is more general and applicable to composite beams regardless of the degree of connection (N/N_f) employed.

The results (Figs. 4 to 11) obtained by this analytical method, are completely comparable with the results derived by Rahal et al [8] part and the results obtained by Hendy and al [9]. They show also that evolution over time of efforts, acting respectively on the reinforced concrete slab and the metal profile is important in the short term. Because of the ageing of concrete, then this variation decreases with time until stabilization.

Following the results found from the application of our model, we recommend using a partial connection. This type of connection has the advantage of minimizing the size of the joint cross section, and thus reduce the production cost.

Depending on the degree of connection (N/N_f) and the direct application of our formulas (Eqs. 24), this model will be a working tool and a simple way to predict in terms of the local redistribution of stresses due to flow regardless of the type of connection used.

The proposed model has a major advantage which is the simplification of expressions for calculating the additional stresses due to shrinkage of concrete and therefore saves computing time by the direct application of our formulas (Eqs. 24).

For future work, the proposed model can be enriched and developed for applications related to creep of concrete.

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