NUMERICAL AND EXPERIMENTAL INVESTIGATION OF THE BEHAVIOR OF CYLINDRICAL SHELLS

O. Temami\(^1\), A. Ashraf\(^2\), D. Hamadi\(^1\) and I. Bennoui\(^3\)

\(^1\)Laboratory of Civil Engineering, Hydraulics, Development and Durability, Department of Civil Engineering and Hydraulics, Biskra University, Algeria
\(^2\)School of Mathematics, Computer Science and Engineering, City University London, Northampton Square London EC1V OHB, U.K
\(^3\)Laboratory of Public Works Engineering and Environment (LTPiTE), High National School of Public Works (ENSTP) 1, Sidi Garidi Street, B.P.32 Vieux Kouba, 16051, Algiers, Algeria

Received: 27 March 2017; Accepted: 30 July 2017

ABSTRACT

The analysis of thin shell structures has generally been purely carried out based on a theoretical basis, usually by superposing the membrane and bending behavior. It is of great importance to carry out experimental tests to validate the numerical models. In this paper, a series of tests have been conducted on cylindrical shell models with different types of boundary conditions and wall thicknesses under the effect of concentrated progressive loads. The experimental results obtained are compared to those derived numerically. A new flat shell element called ACM-Q4SBE1 composed of strain-based quadrilateral membrane element Q4SBE1 and the standard plate bending element ACM is used for the numerical analysis, in addition to the S4R ABAQUS element. The results obtained confirm the effectiveness and superiority of the flat shell approach for the linear analysis of shell structures.

Keywords: Experimental test; cylindrical shell model; flat shell element; experimental test; rigid diaphragm (boundary conditions).

1. INTRODUCTION

The numerical analysis of shell structures is based on essentially two types of elements: flat shell and curved shell elements. We can approach the shell geometry by polygonization using flat elements. This approach ensures the representation of rigid modes. The flat shell element is obtained through combining a plane membrane element and a plate bending element. This technique is frequently used to analyze the behavior of thin shell structures.
and it can be easily formulated. In addition, it provides convenience in managing complex loading and boundary conditions as discussed by Ashwell et al. [1]. The analysis of shell structures as an assembly of flat shell elements was exposed in detail earlier by Zienkiewics [2]. The stiffness matrix is obtained by combining membrane and plate bending elements; the convergence of such element was analyzed and the results obtained for several examples are quite good [3-5]. A general formulation for a curved arbitrary-shape shell element as well as a simplified form suitable for an axisymmetric problem was developed by Ahmed et al. [6]. A simplified form of Cantin and Clough’s cylindrical shell element, reducing the size of the element stiffness matrix from $24 \times 24$ to $20 \times 20$ is presented in [3] and the two elements are compared through different examples. The new Strain-Based Approach (S.B.A.) developed at Cardiff University (UK) has also been used for the analysis of shell structures [1]. Sabir et al. developed an element based on the strain approach that was tested through a series of examples for the analysis of shell structures [4-5]. Dhatt et al. [7] formulated a new 6-node triangular bending element called DKTP based on a discrete Kirchhoff model and a new flat-shell element DLTP having quadratic variation of in-plane displacement. Dawe[8] formulated a simple triangular flat element for the analysis of shells using the displacement method. Batoz and Dhatt [9] developed many elements, from those elements there is the DKT12 obtained from the superposition of the CST membrane element and DKT6 bending element. Sabir and Ramadhani [10] developed an even simpler curved element for general shell analysis. The element is rectangular and has only the essential five external nodal degrees of freedom at each of the four corner nodes; the formulated element was tested by applying it to the analysis of cylindrical as well as spherical shells [11], and the results show a high degree of accuracy to the reference solution which can be obtained with relatively fine meshes. Some other elements for shells have been also developed by Djoudi et al. [12] and Sabir et al. [13]. From the validation tests, these elements have been shown to produce results of an acceptable degree of accuracy without the use of large number of elements. Another shell element with 24 DOF was formulated by Poulsen and Damkilde [14]. Also, Cook [15] developed a quadrilateral flat shell element having 24 degrees of freedom and good results have been obtained. Further, Djoudi and Bahai [16] have developed a new strain-based shell element for the linear and nonlinear analysis of cylindrical shells; the effectiveness of this element was demonstrated and good convergence was also observed. Recently, a new simple four-node quadrilateral shell element with 24-dof which can be used to analyze thick and thin shell problems have been developed by Irwan et al. [17] called DKMQ24 taking into account membrane, bending and shear problems. The development of the element is based on 3-nodal (triangular) and 4-nodal (quadrilateral) elements. The associated formulations allow taking into account a wide variety of thicknesses, irregular geometry, different boundary conditions and rough modeling.

In this paper, we will use a quadrilateral flat shell element with four nodes and 6 degrees of freedom per node baptized as ACM-Q4SBE1 to analyze thin shell structures. The formulated element is obtained by superposition of a new rectangular membrane element Q4SBE1 based on the strain approach and the well-known plate bending element ACM. The stiffness matrix of the shell element is obtained by combining the two independent membrane and bending stiffness matrices. Tests on standard problems have been examined. The convergence of the new formulated element is also compared to other types of
quadrilateral shell elements. Since the analysis of thin shell structures has generally been purely carried out on a theoretical basis such as the superposition of the membrane and bending behavior, it is of importance to carry out additional experimental tests. In this paper, a series of tests has been conducted for a cylindrical shell model under the effect of a concentrated static load, with different thickness and boundary conditions. The results obtained by the numerical analysis are compared to those given by the experimental tests.

2. EXPERIMENTAL PROGRAM

This experimental investigation was conducted at the civil engineering laboratory at City University London; we will detail all the test’s procedures in the next paragraphs.

2.1 System set-up

2.1.1. Specimens

The present study was performed on 3 cylindrical shell models; each model is made of stainless steel 304 and with different thicknesses: for the first model the thickness \( t = 2 \text{mm} \), while for the second and the third models \( t = 1.2 \text{ mm} \). The three models have the same shape and dimensions, and a shape of a semi cylinder with a radius \( R = 160 \text{ mm} \) and a length \( L = 900 \text{ mm} \) (see Fig. 1). The material properties were measured to be: The elasticity modulus \( E = 190000 \text{ N/mm}^2 \), and the Poisson ratio = 0.265. A concentrated load is applied at the top for all the models. The boundary conditions are described separately for each model as follows:

- For tests 1 and 2: The model is free along the four edges, and fixed at the 4 corner points (Fig. 1(a)).
- For test 3: The model is free along the two lateral long sides and fixed with two rigid diaphragms at the curved sides (Fig. 1(b)).

(a) Cylindrical shell model without Rigid Diaphragms       (b) Specimen with Rigid Diaphragms

Figure 1. Cylindrical shell model
Fig. 2 shows the shell model and the dial gauges positions.

Figure 2. The shell model and the dial gauges positions

2.1.2 Apparatus
The machine used is 50-C1601 UNIFLEX 300 (see Fig. 2). It has a high precision load unit for the load measurement and a large testing space for a wide range of tests under both load and displacement control. This machine is connected to a suitable control consoles 50-C9842 ADVANTEST 9. The ADVANTEST 9 can be easily used for a specific load, displacement and strain control. The displacements at the top of the cylinder where the load is applied can be measured by the ADVANTEST 9 and it gives readings with 0.001mm of resolution, while at the other points it is measured by the dial gauge ABSOLUTE Digimatic Indicator ID-U Series 575 with 0.01 mm resolution and 25.4 mm range.

2.1.3 Measurements
For the first test, we choose three points to record displacements; point 1 and 2 that are given by the dial gauges and the displacement at point 3 is given by the apparatus. For the 2nd and the 3rd tests we recorded the displacements just for one point at the top of the cylinder. The points 1, 2 and 3 are illustrated in Fig. 3.

Figure 3. Dial gauge positions; (distance in mm)
2.1.4 Applied loads
The applied load is a concentrated static load for all the tests. The structure is controlled by a vertical rod hydraulically clamped and controlled assuring high rigidity, fitted with high precision load cells for accurate and reliable test results. The tests under control of displacement and strain rate can only be performed with the ADVANTEST 9 (Servo-hydraulic control console).

3. NUMERICAL ANALYSIS

In this work we performed the numerical analysis using 2 elements: the newly-developed ACM-Q4SBE1 [18] element, and the S4R element of the commercial package ABAQUS.

3.1 ACM-Q4SBE1
The quadrilateral shell element used is obtained by the superposition of the Q4SBE1 membrane strain-based finite element with the ACM standard plate bending element [19-20]. The resulting flat shell element is called ACM-Q4SBE1. Fig. 4 shows the geometric properties of the Q4SBE1 element, and the corresponding nodal displacements. At each node (i) the degrees of freedom are \(U_i\) and \(V_i\).

![Figure 4. Co-ordinates and nodal points for the quadrilateral element "Q4SBE1"](image)

The displacement fields of the ACM element are shown in Fig. 5.

![Figure 5. Coordinates and nodal points for the rectangular plate bending element "ACM"](image)
The shell element obtained ACM_Q4SBE1 is composed by assembling the two elements Q4SBE1 and ACM with an effective rotation $z$ (see Fig. 6).

The stiffness matrix of the shell element ACM-Q4SBE1 is obtained by using the analytical integration of the membrane and bending stiffness matrix. The calculation of the element stiffness matrix is summarized with the following well known expressions:

$$ [K_e] = [A^{-1}]^T \int_s [\bar{Q}]^T [D] [\bar{Q}] \, dx \, dy \, [A^{-1}] $$  \hspace{1cm} (1)

$$ [K_s] = [A^{-1}]^T [K_s] [A^{-1}] $$  \hspace{1cm} (2)

$$ [K_s] = \int_s [\bar{Q}]^T [D] [\bar{Q}] \, dx \, dy $$  \hspace{1cm} (3)

Figure 6. The shell element ACM-Q4SBE1

3.2 S4R (ABAQUS/CAE) element
The S4R is a 4-node doubly curved element used for thin and thick shells. It has 6 DOF at each node, and its stiffness matrix is calculated using a reduced integration and hourglass control [21].

4. RESULTS

In this section we present and discuss the results obtained for each test. For each one; we compare the experimental and numerical results of the displacements at points 1, 2 and 3 for the first test, and at point 3 for the second and third tests.

4.1 Test1 (cylindrical shell with no end diaphragms, $t=2\text{mm}$)
The following loads were applied (2750N, 3000N, 3250N and 3500N) and the vertical displacements for points 1, 2 and 3 for each load were recorded. The diagrams presented below show the convergence of displacements. In each diagram the number of elements used is presented along the X-axis and the normalized results ($W_{\text{num}}/W_{\text{exp}}$) along the Y-
axis. The results obtained for the vertical displacements are given in Figs. 7 to 9 (a, b, c and d loading cases) and compared with those obtained by elements ACM-Q4SBE1 and S4R.

![Figure 7. Convergence curve for the deflections $W_1$ at point 1 under different loads](image)

Figure 7. Convergence curve for the deflections $W_1$ at point 1 under different loads.
For the first loading case (a), we observe that the ACM-Q4SBE1 element shows a very good convergence resulting in up to 99% accuracy for a10x10 mesh (Figs. 7 to 9). Also, the ACM-Q4SBE1 element converges much better than the S4R ABAQUS element. Other load cases show excellent convergence as well.
Table 1 summarizes the vertical displacements $W$ obtained from the experimental tests and compared to the numerical analysis using a 10x10 mesh for points 1, 2 and 3 under different loads.

<table>
<thead>
<tr>
<th>Points</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case a</strong>&lt;br&gt;Load =2750 N</td>
<td>ACM-Q4SBE1</td>
<td>3.418</td>
<td>3.562</td>
</tr>
<tr>
<td></td>
<td>S4R Element</td>
<td>3.459</td>
<td>3.603</td>
</tr>
<tr>
<td></td>
<td>Exp.Work</td>
<td><strong>3.495</strong></td>
<td><strong>3.631</strong></td>
</tr>
<tr>
<td><strong>Case b</strong>&lt;br&gt;Load =3000 N</td>
<td>ACM-Q4SBE1</td>
<td>3.728</td>
<td>3.885</td>
</tr>
<tr>
<td></td>
<td>S4R Element</td>
<td>3.770</td>
<td>3.928</td>
</tr>
<tr>
<td></td>
<td>Exp.Work</td>
<td>3.95</td>
<td><strong>4.07</strong></td>
</tr>
<tr>
<td><strong>Case c</strong>&lt;br&gt;Load =3250 N</td>
<td>ACM-Q4SBE1</td>
<td>4.039</td>
<td>4.209</td>
</tr>
<tr>
<td></td>
<td>S4R Element</td>
<td>4.082</td>
<td>4.253</td>
</tr>
<tr>
<td></td>
<td>Exp.Work</td>
<td><strong>4.298</strong></td>
<td><strong>4.458</strong></td>
</tr>
<tr>
<td><strong>Case d</strong>&lt;br&gt;Load =3500 N</td>
<td>ACM-Q4SBE1</td>
<td>4.350</td>
<td>4.533</td>
</tr>
<tr>
<td></td>
<td>S4R Element</td>
<td>4.393</td>
<td>4.578</td>
</tr>
<tr>
<td></td>
<td>Exp.Work</td>
<td><strong>4.716</strong></td>
<td><strong>4.824</strong></td>
</tr>
</tbody>
</table>

Figs. 10 to 12 shows that the results obtained with the ACM-Q4SBE1 element and the S4R element are very close to the experimental results. Fig. 13 show the deflected shape at a load of 3500 N.
Fig. 11. Vertical displacement at point 2

Fig. 12. Vertical displacement at point 3

Fig. 13 shows the shell model undergoing loading and the deformed shape of the shell.

Fig. 13. Cylindrical shell model undergoing the loading
4.2 Test2 (cylindrical shell with no end diaphragms, $t=1.2\, \text{mm}$)

4.2.1 Vertical Displacements at point 3

In this test, several loads are applied (800N, 825N, 850N and 875N) and the Vertical displacements are recorded just at point 3. The same procedure is used here as in the first test.

For the second test using a different thickness $t = 1.2 \, \text{mm}$, the ACM-Q4SBE1 element gives very good results as shown in Fig. 14. The element shows an excellent convergence with an accuracy of up to 98% for a 10 x10 mesh, and it converges faster than the S4R ABAQUS element.

![Convergence curve for the deflection $W_3$ at point 3 under different loads](image)

4.2.2 Vertical displacements $W$ (mm) under different applied loadings with numerical analysis (10x10 meshes), deflection for point 3

Table 2 recapitulates the vertical deflections under different applied loads at point 3. Fig. 15 shows that the results of the ACM-Q4SBE1 and S4R elements are very close to the experimental values.
Table 2: Vertical deflection $W$ (mm) under different applied loads at point 3

<table>
<thead>
<tr>
<th>Loads (N)</th>
<th>800</th>
<th>825</th>
<th>850</th>
<th>875</th>
<th>900</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACM-Q4SBE1</td>
<td>5.224</td>
<td>5.387</td>
<td>5.551</td>
<td>5.714</td>
<td>5.877</td>
</tr>
<tr>
<td>S4R Element</td>
<td>5.241</td>
<td>5.403</td>
<td>5.564</td>
<td>5.726</td>
<td>5.888</td>
</tr>
<tr>
<td>Exp.Work</td>
<td><strong>5.098</strong></td>
<td><strong>5.606</strong></td>
<td><strong>6.060</strong></td>
<td><strong>6.422</strong></td>
<td><strong>6.822</strong></td>
</tr>
</tbody>
</table>

Figure 15. Vertical displacement at point 3

4.3 Test3 (cylindrical shell with end rigid diaphragms, $t=1.2\text{mm}$)

4.3.1 Vertical displacements at point 3

The same procedure as the previous one is used for this test. The applied loads are (575N, 600N, 625N, 650N, 675N and 700N) and the vertical displacements for point 3 are recorded.
The deflections obtained with the ACM-Q4SBE1 element converges rapidly towards the experimental results with an accuracy of up to 99% using a mesh of 10x10 elements as shown in Fig. 16. Also, the ACM-Q4SBE1 element converges better than the S4R ABAQUS element.

4.3.2 Vertical displacements $W$ (mm) under different applied loads with numerical analysis (10x10) meshes, deflections for point 3

<table>
<thead>
<tr>
<th>Loads (N)</th>
<th>ACM-Q4SBE1</th>
<th>S4R element</th>
<th>Exp.Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>0.8238</td>
<td>0.7585</td>
<td>0.858</td>
</tr>
<tr>
<td>625</td>
<td>0.8581</td>
<td>0.7900</td>
<td>0.864</td>
</tr>
<tr>
<td>650</td>
<td>0.8925</td>
<td>0.8216</td>
<td>0.910</td>
</tr>
<tr>
<td>675</td>
<td>0.9268</td>
<td>0.8532</td>
<td>0.958</td>
</tr>
<tr>
<td>700</td>
<td>0.9611</td>
<td>0.8848</td>
<td>1.231</td>
</tr>
</tbody>
</table>

Table 3: Vertical deflection $W$ (mm) under different applied loadings at point 3

![Figure 16. Convergence curve for the deflection $W_3$ at point 3 under different loads](image1)

![Figure 17. Vertical displacement at point 3](image2)
Table 3 recapitulates the vertical deflections under different applied loads at point 3. Figs. 16 and 17 show that the results obtained with both elements ACM-Q4SBE1 and S4R ABAQUS element are close to the experimental results; but the ACM-Q4SBE1 gives much closer values to the experiment. Also, the same comments can be noticed when the applied load attains 675 N.

5. CONCLUSIONS AND RECOMMENDATION

In this study, an experimental work was performed to examine the bending behaviour of cylindrical shells. Three cylindrical shell models with different conditions were tested under concentrated loads. From the results obtained in this study, the following conclusions can be summarized:

- Excellent agreement was obtained between the ACM_Q4SBE1 element results and those from the experimental tests. The newly-developed flat shell element ‘ACM_Q4SBE1’ proved to be robust, effective and useful in analyzing thin shell structures. It also exhibits strong convergence, as can be seen from the numerical results presented. It did perform in general better than widely-used elements such as the S4R element currently adopted by the commercial software package ABAQUS.

- The ACM_Q4SBE1 element can be used to analyze shell structures in the linear stage with great accuracy. However, it is of importance to extend the presented element ACM_Q4SBE1 to take into account nonlinear effects, which will be addressed in future research studies.

Acknowledgments: The authors would like to thank all persons involved in this work at the Structural Laboratory at City University London. In particular, they would like to thank Dr. Brett McKinley, the laboratory manager, for his invaluable advices regarding the test rig.
instrumentation and control. The effort of Mr. JN Hooker is to be also acknowledged. The authors would like to also thank Mr. S Gendy, Mr. R. Mohamed, and Mr. D. Das for their help to construct the shell models.

REFERENCES

17. Irwan K, Batoz JL, Imam JM, Hamdouni A, Millet O. The development of DKMQ plate bending element for thick to thin shell analysis based on the Naghdi/Reissner/Mindlin