



## A NEW APPROACH TO IDENTIFY ACTIVE FREQUENCIES OF DYNAMIC RATCHETING- INDUCING GROUND MOTION

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### ABSTRACT

Dynamic ratcheting (DR) is a phenomenon that occurs in nonlinear systems and one aspect of complicated nonlinear dynamic behaviors. DR behavior have a strong effect on the magnitude of plastic deformation in single-degree of freedom (SDOF) hysteretic damping systems, therefore there is need to precisely examine this phenomenon caused by earthquake excitations. DR-inducing frequencies of ground excitation are called “active frequencies”. A new method is proposed in this study to identify active frequencies of DR-inducing ground excitation using wavelet transform. The identification method is based on a reduced dimensionality and filtered frequency content of original record according to closeness of displacement time history of SDOF system under both original and modified record. The objective of this study is divided into two main parts: In the first part of this study, applying the proposed method for a DR- inducing ground motion and it was an example that how the method could be utilized to identify active frequencies of DR- inducing ground motion. The second part of this study is devoted to identify the DR- inducing ground motions between a set of ground motions records, containing 504 records up to the year 2012, in Iran. The reduced representation of ground motion is a useful technique for extracting active frequencies from ground motion.

**Keywords:** Damping system; dynamic ratcheting; active frequencies; wavelet transform.

### 1. INTRODUCTION

Variation in response of structures with nonlinear behavior subjected to strong ground motion records is significant. The effect of earthquake excitation randomness on the variation of structural response is defined as Record-to-Record (RTR) variability. In a hysteretic damping system under earthquake excitation when inelastic displacement response continuous increments asymmetrically in successive cycles, a phenomenon called Dynamic Ratcheting (DR) occurs which is likely a contributor to variability. Therefore,

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there is need to further understand the sources of variability due to DR. Ahn I-S et al. [1] used different hysteretic behavior (i.e., linear kinematic hardening, elastic-perfectly-plastic and two surface model) for study DR behavior of a hysteretic damping SDOF system under dual-frequency sinusoidal excitations. The results show that elastic-perfectly-plastic (EPP) model develops more significant DR behavior than does the other hysteretic models. The results show that the frequency ratios are the most significant parameters and also DR can occur when the excitation frequencies hold integer ratios and the product of terms comprising the ratio is an even number. Various SDOF systems under different combinations of sinusoidal excitations were analyzed in another studies and their Results indicate that the persistence of frequency ratio depend on DR behavior [2, 3]. The effect of DR behavior on magnitude of displacement response of nonlinear SDOF systems having different values of damping ratio  $\xi$  and yield displacement were examined by Ahn I-S et al. [4].

The earthquake ground motions are known to have random and non-stationary nature, both in their amplitude and frequency unlike simple sinusoidal excitations. So, it is difficult to achieve the identification and extraction of the DR- inducing frequencies of ground motions. DR-inducing frequencies of ground excitation are called “active frequencies”. Recently, Ahn I-S et al [3] employed Fourier Transform and spectrogram to identify the active frequencies of DR-inducing ground motion. They in their investigation, employing the SDOF systems with EPP hysteretic model, under 160 earthquakes excitation, they found that occurrences of DR can depend on frequency content, magnitude of ground motion, local site conditions and the hysteretic shape of the structural system, [2]. In the present study, wavelet transform is utilized to identify the active frequencies of DR- inducing ground motion. Wavelet transform is utilized as one of the new and useful tools in earthquake engineering for describing the time- varying frequency characteristic of the earthquake record. Several studies have proposed the application of wavelet analysis on seismic signals [5-10]. One of the most interesting properties of wavelet transform is that the signal can be decomposed into different signals such that each new signal covers specified frequency content of the main signal. In this way, it is more precise to study frequency content of earthquake. The present study was conducted to develop a method base of wavelet transform to identify the active frequencies of DR- inducing ground motions. The proposed method is based on a reduced dimensionality and filtered frequency content of original record according to closeness of displacement time history of SDOF system under both original and modified record. The ground motion active frequency is determined utilizing pseudo-frequency of the mother wavelet. For simplicity, a generalized SDOF structure with EPP hysteretic behavior was considered.

As the present work is based on wavelet decomposition and reconstruction of ground excitation, a brief review of wavelet transform used in the paper is given in section 2 and also the methodology of the study is presented in Section 3. In section 4 the applying proposed method for Chino Hills earthquake record at fire station in Mecca to demonstrate the process and in section 5, conclusion is presented.

## 2. OVERVIEW ON WAVELET TRANSFORM

Wavelet transform is used as one of the new and useful tools in engineering for studying the frequency content of earthquakes. The mathematical formula for wavelet transform is defined as follows:

$$\psi_{a,b}(t) = |a|^{-\frac{1}{2}} \psi\left(\frac{t-b}{a}\right) \quad (1)$$

Where,  $a$  is the scale parameters and  $b$  is the translation parameter. The continuous wavelet transform of the signal  $S(t)$  is defined as

$$W_{\psi}(a,b) = \int_{-\infty}^{+\infty} S(t) \psi^*\left(\frac{t-b}{a}\right) dt \quad (2)$$

Where,  $\psi^*$  is the complex conjugate of  $\psi$  and  $W_{\psi}(a,b)$  is called the wavelet coefficient for the wavelet  $\psi_{a,b}(t)$ . In signal processing, a discrete version of wavelet transform is often used by discretizing the scale parameter  $a$ , and the translation parameter  $b$ . In general, the procedure becomes much more efficient if dyadic values of  $a$  and  $b$  are used, i.e.:

$$a = 2^{-j}, \quad b = 2^{-j} k \quad (3)$$

Where  $j$  and  $k$  are integers: The corresponding discretized wavelet  $\psi_{j,k}(t)$  is as follows

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \quad (4)$$

The discrete wavelet transform can be efficiently realized as a multi-resolution decomposition of the time series as:

$$S(t) = \sum_{j=1}^n d_j(t) + a_n(t) \quad (5)$$

Where  $n$  is the total number of decomposition levels; and  $(d_j)$  the detail components,  $(a_n)$  Approximation components of the original signal  $(S)$ . We can extend of decomposition procedure up to a level where there is no significant information remaining in the approximation component. After truncating the decomposition at level  $n$ , the original signal can be reconstructed through the inverse discrete wavelet transform from the details only as follow

$$S(t) = \sum_{j=1}^n d_j(t) \quad (6)$$

In this paper, the discrete wavelet transform is applied to decompose of earthquake ground motion records at different level of frequencies. Daubechies of order10 (db10) wavelet has been chosen as mother wavelet in the analysis. The Daubechies mother wavelet is suitable for the seismic performance analyses and is orthogonal and there is relatively small frequency overlapping between adjacent levels [11-13]. The db10 wavelet function is shown in Fig. 1.

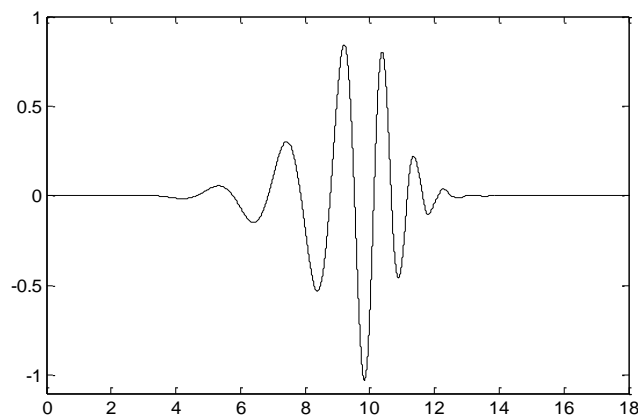


Figure 1. Daubechies basis function (db10)

### 3. PROPOSED METHODOLOGY

This section presents a step-by-step of the proposed method to identifying the active frequencies of ground motions that will cause DR behavior in SDOF systems. The procedure for identifying the active frequencies of DR-inducing ground motion is explained in the following steps:

- (1) The nonlinear displacement time history of SDOF EPP model experiencing DR behavior under ground motion is computed and recorded.
- (2) Breaking the ground motion acceleration into its constituent components with isolated frequency bands using Discrete Wavelet.
- (3) The nonlinear displacement response of the SDOF EPP model under consideration is calculated as subjected to each component of the ground motion. Hence, the contribution each component on occurs the DR behavior in SDOF EPP model is identified.
- (4) Select the smallest number of component for generation of modified record so that the displacement response of SDOF EPP model under modified record closely follows the trends of the displacement time history recorded in step 1. Actually, chosen Components have maximum effect on the displacement response of SDOF system.

(5) Active frequency of ground motion can be given in a simple way by defining the pseudo-frequency corresponding to each scale of chosen components as follows:

$$f_a = \frac{f_c}{\alpha \times \Delta t} \quad (7)$$

The variables,  $\alpha$ ,  $\Delta t$ ,  $f_c$ ,  $f_a$  represent scale, sampling period, center frequency of the mother wavelet, and frequency corresponding to the given scale respectively. This study utilized the value of  $f_c = 0.6842$  Hz as the center frequency of the *db10* mother wavelet.

#### 4. STRUCTURAL MODEL

In previous studies [2-4], it was found that the elastic-perfectly-plastic (EPP) hysteretic damping model develops substantial DR than other hysteretic models. Therefore, a SDOF system with EPP hysteretic behavior and viscously damped force–deformation relationship is used to investigate the statistical structural response under DR-inducing ground excitation. A typical force–displacement response of SDOF EPP model under seismic loading is shown in Fig. 2. The initial damping ratio,  $\xi$ , was considered to be equal to 5%. The hysteretic SDOF system is generalized as a function of only two variables, initial fundamental period  $T$  and yield displacement,  $u_y$ . The inelastic displacement time histories are computed for a set of SDOF systems with normalized periods between 0.1 and 1.5 with an interval of 0.05.

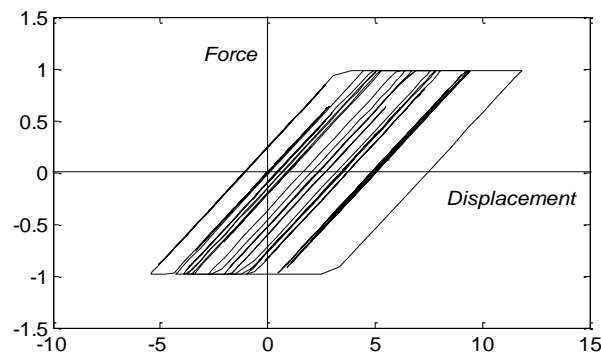


Figure 2. Force–displacement response of elastic-perfectly plastic SDOF

#### 5. DETAILED RESULTS FOR CHINO HILLS RECORD

The objective of this section is to how the proposed methodology can be utilized to identify active frequencies ground motion that lead to DR behavior. With this purpose, one ground motion was analyzed to demonstrate the process and verify the accuracy of the proposed method. The ground motion chosen is the 2008 Chino Hills (M=5.4) at fire station in Mecca, at epicentral distance of 160 km that active frequencies of this record was evaluated in Re

[3]. The results of nonlinear analysis show that a maximum effect of DR behavior is seen at the SDOF system with ( $T=1s$ ,  $u_y = 10$  cm), where the displacement ductility is increased by 4 times. Fig. 3 illustrates the displacement time history of the SDOF system with ( $T=1s$ ,  $u_y = 10$  cm). It is obvious that the occurrence of DR is confirmed.

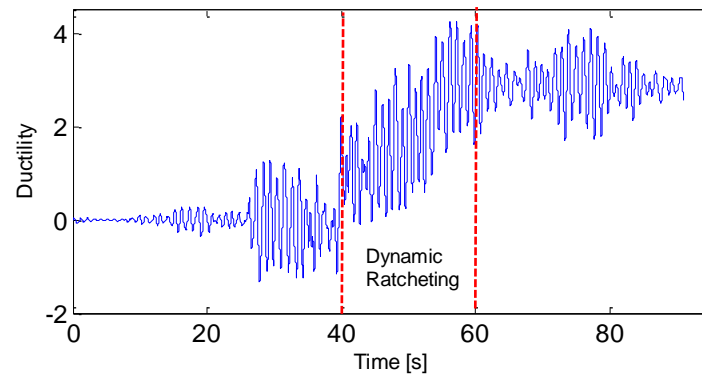


Figure 3. Dynamic ratcheting in SDOF system with ( $T=1s$ ,  $u_y = 10$  cm) under the Chino Hills earthquake record

In the first step, the ground motion is decomposed into component signals with limited frequency band utilizing discrete wavelet analysis. Frequency and period corresponding to each component of the Chino Hills at fire station are presented in Table 1. The nonlinear structural response to each ground motion component is computed utilizing response history analyses. Contribution of each component in occurs the DR behavior were identified thorough the procedure.

Table 1: Frequency and period ranges of Chino Hills record

Component	Scale	Period range (s)	Frequency range (Hz)
1	2	0.01 – 0.02	100 – 50
2	4	0.02 – 0.04	50 – 25
3	8	0.04 – 0.08	25 – 12.5
4	16	0.08 - 0.16	12.5 - 6.25
5	32	0.16 – 0.32	6.25 – 3.12
6	64	0.32 – 0.64	3.12 – 1.56
7	128	0.64 – 1.28	1.56 – 0.80
8	256	1.28 – 2.56	0.80 – 0.40
9	512	2.56 – 5.12	0.40 – 0.20

The results of nonlinear analysis show that among all the components, the 6th component (corresponding to periods between 0.32 and 0.64 s) and the 7th component (corresponding to periods between 0.64 and 1.28 s) of the original ground motion record have considerable energy and heavily affects the structural response . The four wavelet components of Chino Hills ground motion and their responses are shown in Fig. 4. Next step this work is representation of original ground motion record by a relatively small number of wavelet

components, which would be powerful technique for identify active frequencies. The main step in this method the number of wavelet components needed to represent original record that exactly is proportional to closeness of displacement time history of SDOF system under both original and reduced dimensionality representation of original (modified) records. According to this principle, modified record was constructed by only these two (6th and 7th) components that have a strong effect on structural response of SDOF system.

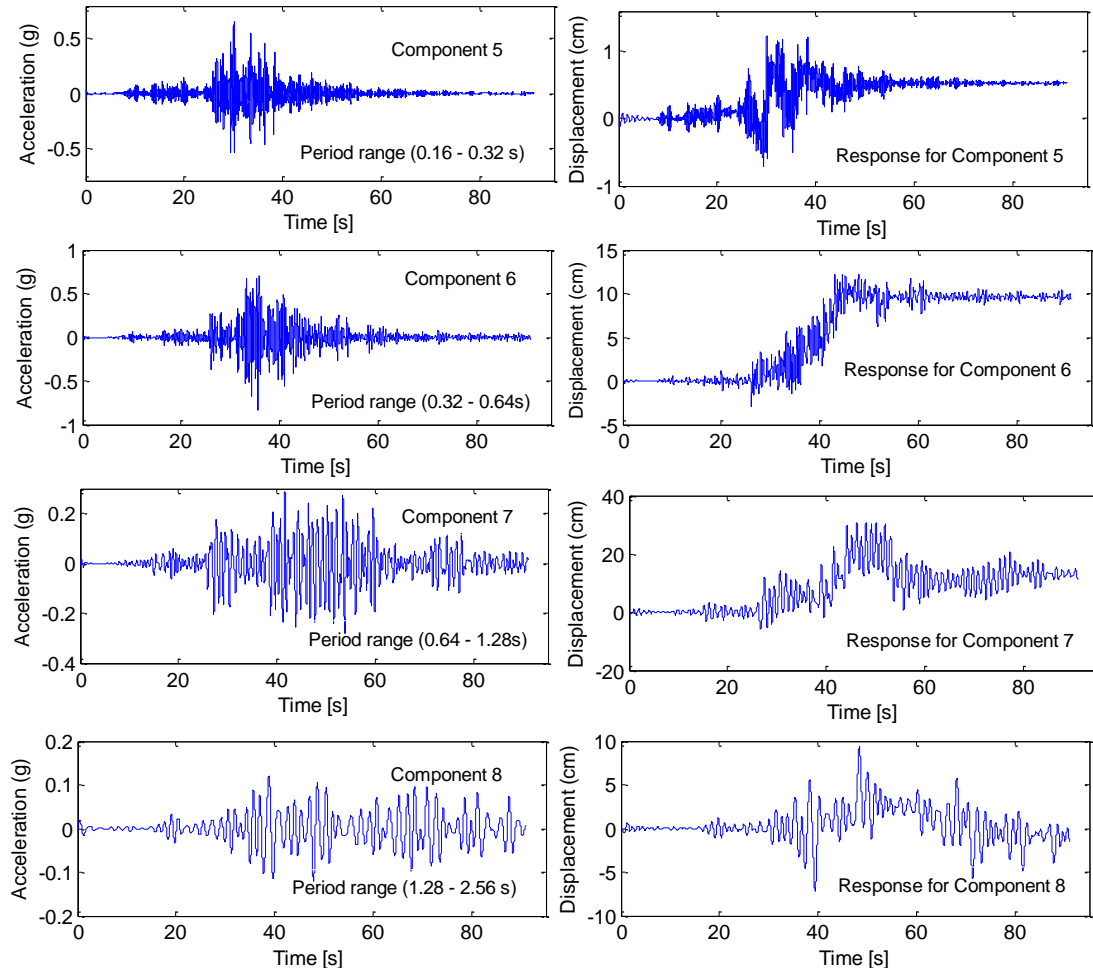


Figure 4. Example of four wavelet components of the Chino Hills ground motion and their responses

The acceleration time history of original and modified ground motions are illustrated in Fig. 5a. Fig. 5b shows a comparison between the displacement response history due to the original ground motion and the displacement response history due to the modified ground motion. It can be seen that the displacement time history response of the modified motion closely follows the trends of the displacement time history response of the original motion. The displacement time history of SDOF EPP model under modified motion show a signature of DR. Figs. 5c also show the Fourier Transform of original and modified ground motion. It

can be observed that despite great discrepancies in frequency content of original and modified ground motions (frequency content of the modified ground motion is concentrated and covers limited frequency band of the original ground motion), the displacement response of the modified motion closely follow the trends of the displacement response of the original motion, particularly in the duration of DR behavior.

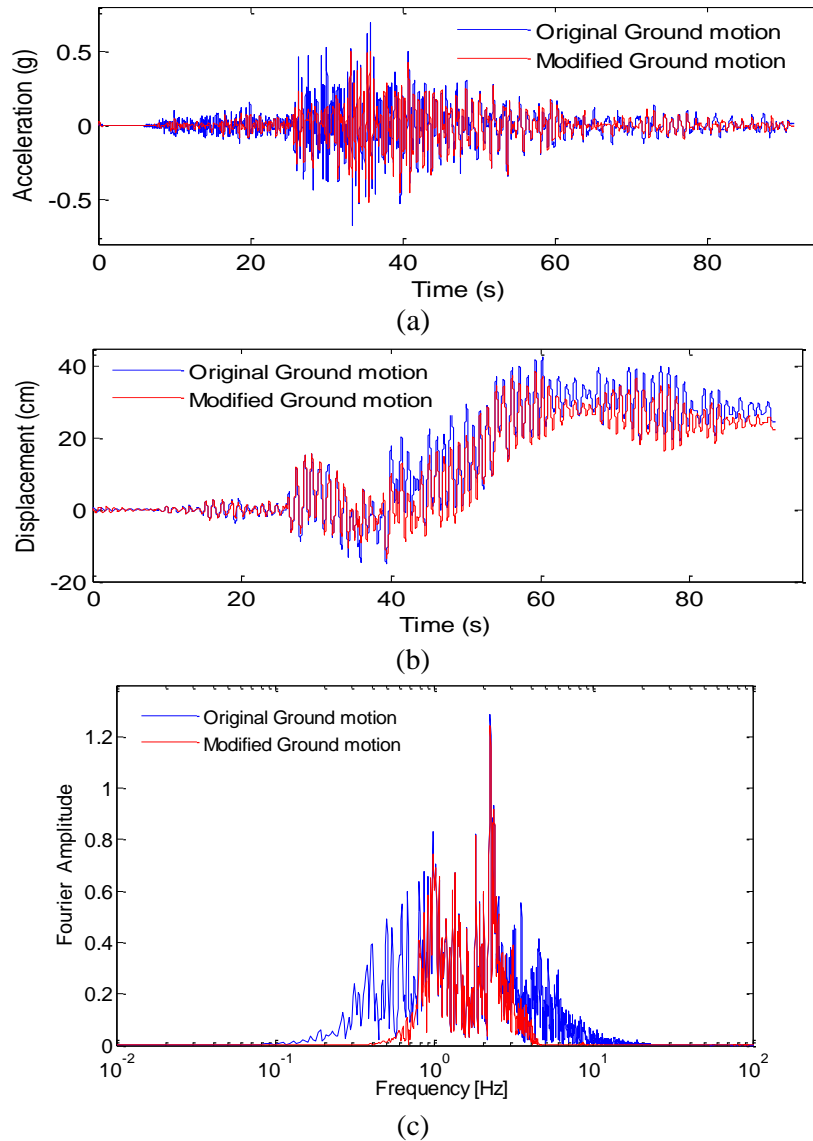


Figure 5. Comparison of the (a) acceleration time history, (b) displacement time history, and (c) Fourier spectra, of original and modified ground motion in the case of Chino Hills Recorded at the fire station

In the final step, active frequencies of ground motion are calculated by Pseudo-frequency concept. Pseudo-frequency concept has been applied by Yaghmaei-Sabegh [14, 15] for



time–frequency analysis of earthquake ground-motion in the past. The specified wavelet scales corresponding to the level of chosen components are converted to frequency utilizing the relationship described in Eqn. (7). Pseudo-frequency and period corresponding to each chosen component are presented in Table 2. It was observed that the frequency ratio of active frequencies is 1:2 which demonstrates that the concept of dynamic ratcheting induced often by active frequencies with smallest integer values.

Table 2: Active frequencies of the Chino Hills record.

Component	Scale	Pseudo-Period (s)	Pseudo-Frequency (Hz )
6	64	0.467	2.13
7	128	0.935	1.07

In previous study [3] results analysis of Chino Hills ground motion demonstrated that among the frequencies with high intensities, 0.9121, 1.8132, 2.2637, and 2.2747 Hz can lead to DR behavior. The Fourier Transform of the ground excitation in Fig. 6(a and b) indicates that the active frequency (0.9121 Hz) falls within frequency range of 6th component and active frequencies (1.8132, 2.2637 and 2.2747) falls within frequency range of 7th component. The effects of neighboring frequencies (1.81, 2.2637 and 2.2747 Hz) are mixed with together. It is now clear why the 6th and 7th components were chosen because the frequency ranges of the 6th and 7th components covering the active frequencies of Chino Hills ground motion.

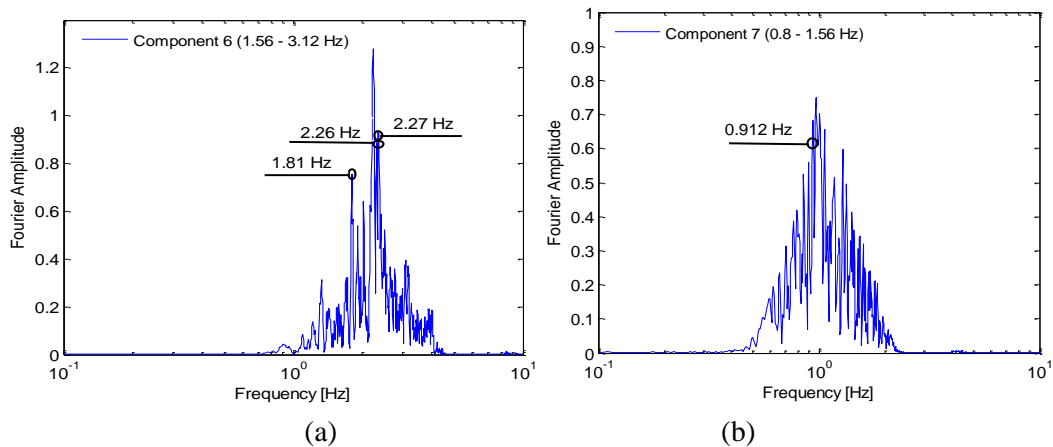


Figure 6. Fourier spectra of (a) 6th component, and (b) 7th component

It can be observed from wavelet map in Fig. 7 that the significant energy of the ground motion is starting at time  $t=36$  s hitting around active period  $T= 0.46$  and the intensity of excitation is increasing and then again decreasing as time increases.

Compare the active frequencies obtained by proposed method and the method in Ref [4] shows the effectiveness of the wavelet approach for extracting active frequencies DR-inducing ground motion, Table 3.

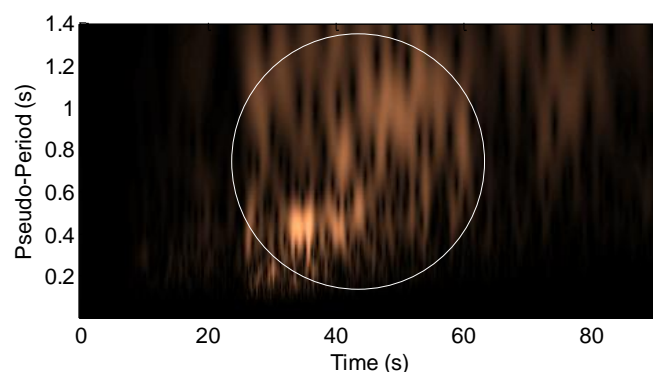


Figure 7. Time-period variation of Chino Hills record

Table 3: Comparison of the active frequencies

Active frequency (Hz)	f1	f2
This study	1.07	2.13
Ref [3]	0.912	[1.81 – 2.26- 2.27]

## 6. SEISMIC INPUT

The dataset used in this study contains 504 two-component horizontal accelerograms from 16 earthquakes occurred in Iran up to the year 2012 and have been collated from the Building and Housing Research Center database ([www.bhrc.com](http://www.bhrc.com))[16]. Table 4 lists the earthquakes event.

Table 4: Ground-motion data set used for the analysis

Earthquake Name	Event Date	Earthquake Magnitude	Style faulting
Tabas	1978/09/16	7.4	Reverse
Rudbar-Manjil	1990/06/21	7.7	Reverse
Sefidabeh	1990/06/20	4.0	Reverse
Garmkhan	1997/02/04	6.5	Strike-slip
Sarin	1997/02/28	6.1	Reverse
Ardakul	1997/05/10	7.1	Strike-slip
Golbaf	1998/03/14	6.6	Reverse
Karebas	1999/05/06	6.2	Strike-slip
Poleabghine	1999/10/31	5.2	Strike-slip
Salehabad	1999/11/08	5.5	Reverse
Avaj	2002/07/22	6.5	Reverse
Bam	2003/11/27	6.5	Strike-slip
Kojur-Firoozabad	2004/05/28	6.3	Strike-slip
Zarand	2005/02/22	6.5	Reverse
Silakhor	2006/03/31	6.1	Reverse
Varzaghan-Ahar	2012/08/11	6.3	Reverse

## 7. INVESTIGATION DETAILS

The objective of this section is to identify Iranian DR- inducing ground motions. In the first step the seismic response of SDOF system under each ground motion record is analyzed considering EPP behavior and displacement time history is obtained at every interval of period. The each ground motion was scaled with dividing by its PGA. For the identification of DR under Iranian ground excitations, nonlinear time history analysis is used with the Fourier Transform and spectrogram. The increase in ductility demand of SDOF systems is considered to be the effect of DR behavior (displacement ductility greater than 3). Therefore, the ones having ductility of 3 or greater were selected [3]. Based on the analyses results, 12 records of 604 records exhibit DR behavior, and Table 5 Shows the characteristics of ground motion records. Fig. 8 depicts four cases where DR occurs under different Earthquake excitation. The displacement time histories show a signature of DR.

Table 5: Dynamic ratcheting cases

Earthquake name	Magnitude	PGA (g)	Dist (km)	DR effect
Tabas	7.7	0.38	12	DR
Rudbar-Manjil	7.4	0.53	62	DR
Sarin	6.1	0.63	48	DR
Sarin	6.1	0.23	54	Potential-DR*
Ardekul	7.1	0.47	126	DR
Ardekul	7.1	0.34	118	DR
Poleabghine	5.2	0.39	31	Potential-DR*
Avaj	6.5	6.5	74	Potential-DR*
Golbaf	6.6	0.28	57	Potential-DR*
Bam	6.5	6.5	157	DR
Sefidabe	4	0.16	22	Potential-DR*
Kojur-Firoozabad	6.3	0.22	58	DR

\*The Potential-DR cases are the ones that reveal a signature of DR in the displacement time history, but whose ductility is less than 3.0.

A star (\*) next to one of the DR cases in Table 2 indicates that the Potential-DR cases are the ones that reveal a signature of DR in the displacement time history, but whose ductility is less than 3.0. Based on the analyses results, the effect of DR behavior was significant in some cases. For instance, a displacement ductility of 6 for SDOF system with ( $T=0.7s$ ,  $u_y = 10cm$ ) is observed for Ardakul earthquake at Birjand station. For the Ardakul record, active frequencies were identified by proposed method. after decomposing the earthquake record into a set of components, 6th, 7th and 8th components were used in generating modified ground motion record. The acceleration time history of original and modified ground motions are illustrated in Fig. 5a. Figure 9b shows a comparison between the displacement response history due to the original ground motion and the displacement response history due to the modified ground motion. It can be seen the degree of correlation of displacement tie history response under both original and modified records is very high. Also it can be seen that the structural response of modified ground motion is capable of experiencing DR

because the excitation only contains active frequencies. Active frequencies of Ardakul record are presented in Table 6. Figure 9c Fourier spectra, of original and modified ground motion in the case of Ardakul recorded at the fire station.

Table 6: Active frequencies of the Ardakul record

Component	Scale	Pseudo- Period (s)	Active frequency
6	64	0.467	2.13
7	128	0.935	1.07
8	256	1.88	0.53

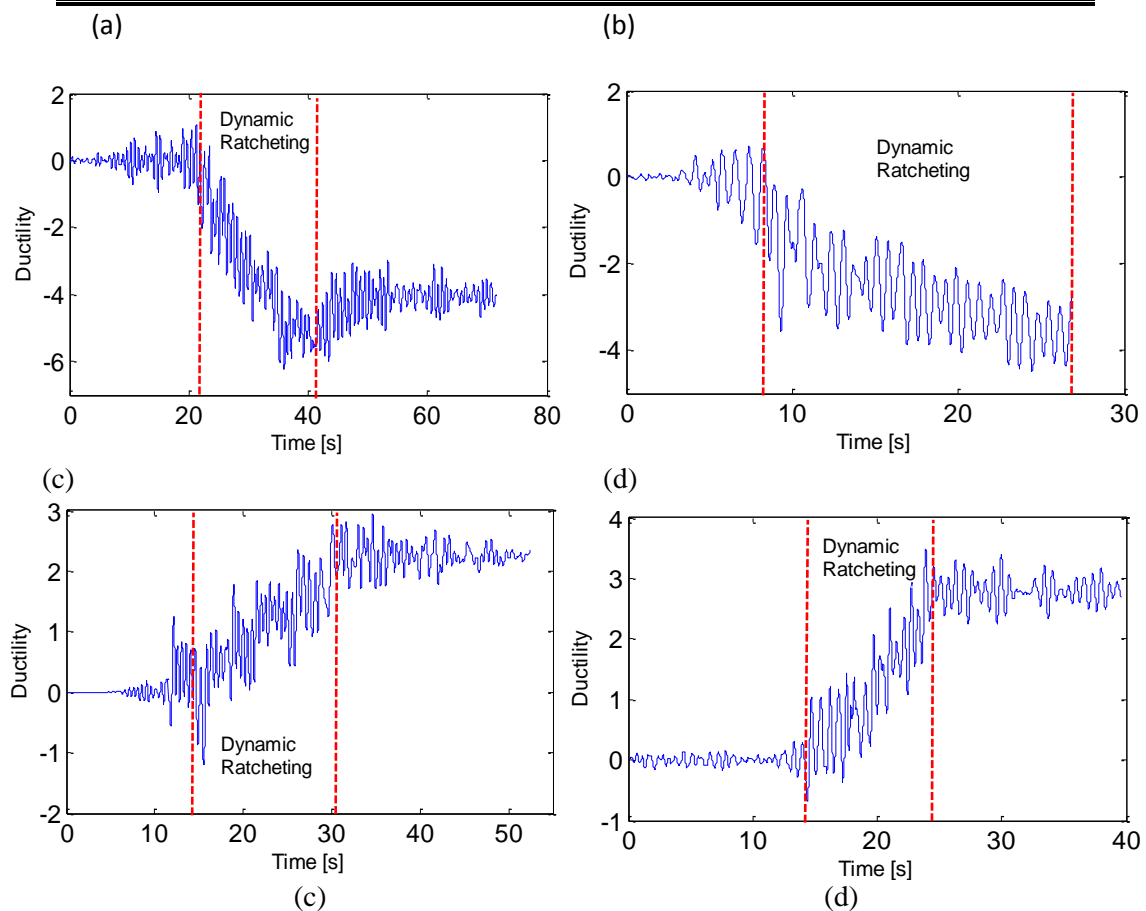


Figure 8. Dynamic Ratcheting under ground motion (PGA: 1.0 g): (a) Ardakul earthquake (5/10/1997, 7:57 UTC) recorded at Birjand station , Fundamental Period: 0.7 s(b) firuzabad Earthquake (7/29/2008, 18:42 UTC) recorded at Farashband station, Fundamental Period: 0.75 s (c) Sarin Earthquake (28/2/1997, 12:57 UTC) recorded at Sarab station, Fundamental Period: 0.55 s and (d) Bam Earthquake (12/26/2003, 1:56 UTC) recorded at the Laleh Zar station, Fundamental Period: 0.65 s

Fig. 10 shows a wavelet map of Ardakul record. Lighter colors in the spectrogram indicate greater energy in the ground motion. The spectrogram shows that there are two

dominant ridges of energy in this ground motion, the second has longer period than the first with major intensity. The two bulks of energy are at periods around 0.45 and 0.9 s at time approximately equal to 24 and 35 seconds respectively.

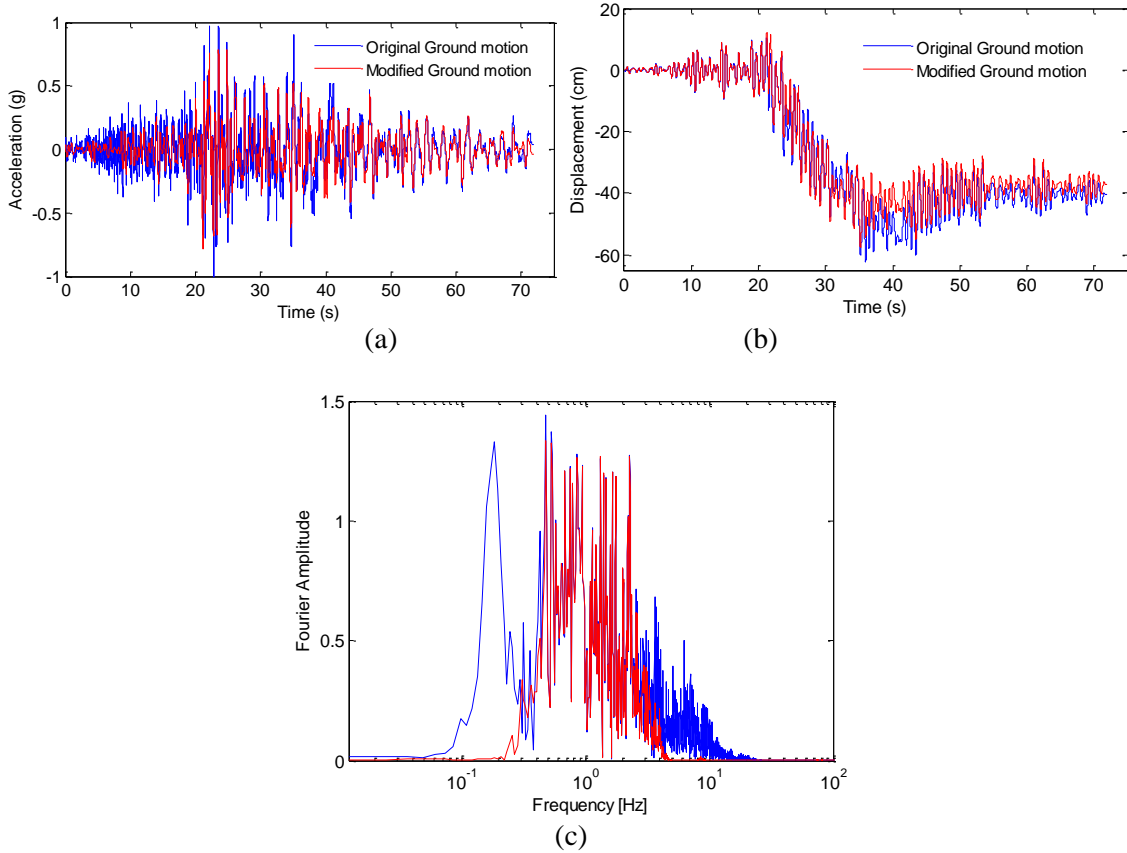


Figure 9. Comparison between the (a) acceleration time history, (b) displacement time history, and (c) Fourier spectra, of original and modified ground motion in the case of Ardakul recorded at the fire station

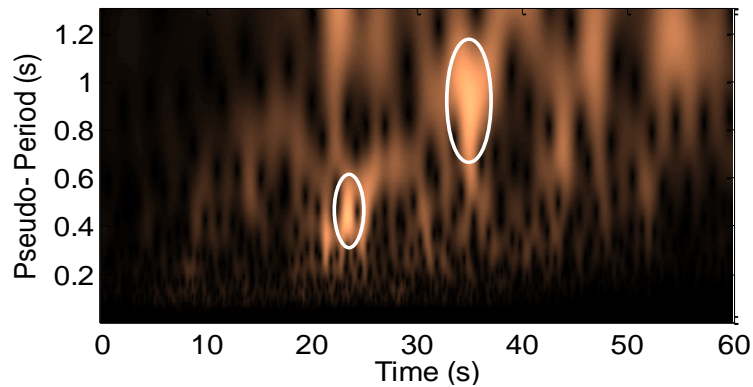


Figure 10. Time-period variation of Ardakul record

On May-10-1997, an earthquake with the magnitude of  $M_s 7.1$  occurred in the eastern stretch of Ghaenat (Zirkouh) area and caused major casualties and damages. Haji-Abad, Kalateh, Ardekul, Esfaragh, Fakhr-Abad and Bashiran villages were totally destroyed, and Abiz, Esfandan and Pishbar villages were seriously damaged. Severe structural damages during Ardakul earthquake event might be caused by DR behavior of structure. Figure 11 shows significant damage of RC frame buildings near to Birjand city during Ardakul earthquake event.



Figure 11. An example of damage to R/C frame buildings during the Ardakul earthquake

## 8. CONCLUSIONS

The new approach taken in the research proposed herein is representation of DR-inducing motion record by a relatively small number of wavelet components, which would be powerful technique for extracting active frequencies. From the results of this work, the following conclusions are derived:

The reduced representation of strong motion records as a sum of a relatively small number of components is an efficient technique for extracting active frequencies of DR-inducing ground motion.

A maximum effect of DR is seen for Ardekul earthquake at  $T=0.7$  sec where the displacement ductility is increased by 6 times and change of displacement ductility demand can reflect the damage accumulation of structures due to DR behavior.

It is expected that the only SDOF EPP models with short and moderate periods likely to experience DR behavior.

Safe structural design needs better understanding of this phenomenon as the consideration of DR can be included in ground motion selection for non-linear time history analysis or structural design.

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