DYNAMIC RESPONSE OF RECTANGULAR PLATE SUBJECTED TO MOVING LOADS USING SPECTRAL FINITE STRIP METHOD

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ABSTRACT

This paper presents development of spectral finite strip method for dynamic analysis of thin rectangular plates subjected to moving loads. Dynamic stiffness matrix of a plate strip in frequency domain is derived. For obtaining dynamic response of plate subjected to moving loads, both Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT) are utilized. Numerical examples are presented to validate accuracy of the presented method.

Keywords: Thin rectangular plate; moving load; dynamic of continues plates; spectral finite strip method.

1. INTRODUCTION

Many structural elements can be simulated effectively using beam or plate elements such as slabs, airplane wings and bridge deck. In bridge engineering field, moving load is one of the most important types of loading in analysing and designing bridge decks; therefore, dynamic analysis of plates subjected to the moving load has been considered as an important on-going subject by researchers for a long time. Methods for analysing dynamic response of plates subjected to moving loads can be categorized into two major groups. The first group includes the analytical methods, and the second one includes the numerical methods.

Fryba has analytically solved the dynamic response of flat plates under moving load [1]. Ichikawa et al. considered the general continuous Euler-Bernoulli beam subjected to forces applied by a moving mass and gave an analytical solution based on eigen function expansion [2]. A good review of moving force identification analytical methods on bridges was studied by Yu and Chan [3].

Finite element method (FEM) and finite strip method (FSM) are two of the well-used numerical methods in the field of bridge engineering. Yoshida and Weaver are the first researchers who applied these methods to study the moving load problem [4]. Wu et al. and
Taheri and Ting analyzed the dynamic response of flat plate subjected to varying moving loads using FEM [5, 6]. Smith investigated the dynamic problem of a simply supported slab bridge under the action of a moving load using FSM [7].

In order to increase computational efficiency and decrease the computational cost and time, many numerical methods have been presented in recent decades. These methods are all formulated in the frequency domain and the frequency-dependency simplifies the inclusion of frequency-dependent material characteristics and boundary condition. The spectral element method (SEM) is one of these methods. SEM can be considered as the combination of Spectral Analysis Method (SAM), Dynamic Stiffness Method (DSM) and FEM. In SEM, the governing partial differential equation is reduced to a set of ordinary differential equations using Fast Fourier Transformation (FFT) algorithm. The advantage of this method is that the inertial effects are exactly represented and as a result the wave behavior can be captured accurately. Because of using a few elements in SEM, volume of computational operations decreases, remarkably.

According to the literature, Doyle is one of the first researchers, applied the SEM to study the wave propagation problem in structural elements. In 1988, Doyle introduced FFT-based spectral analysis methodology to investigate the wave propagations in structural elements [8]. He proved the accuracy of the SEM in comparison with the FEM. He also showed the applicability of the SEM to the non-uniform structural elements such as multi-connected beams and the tapered beams. Later Gopalakrishnan analysed the multiple connected Timoshenko beam by using SEM [9]. Birgersson et al. are developed a spectral super element for modelling of plate vibration [10]. Lee and Lee developed the conventional SEM for structures with distributed dynamic loads [11]. The application of SEM to forced vibration of beam and plates [11], dynamic of smart structures [12] and the spectral transfer matrix [13] has been developed by Lee et al. For the first time, Shirmohammadi et al. and Bahrami et al. combined SAM and FSM entitled Spectral Finite Strip Method (SFSM) to develop the formulation of wave propagation in rectangular moderately thick plate and annular sector plate under impact loads [14, 15].

Most of the researches on the SEM application have been focused on dynamic of structures subject to impact concentrated loads. In the study presented here, the SFSM formulations are developed to analyze the dynamic response of a thin rectangular plate under moving loads. The accuracy of the developed formulations in calculating the natural frequencies of thin rectangular and continuous plates is compared with other numerical method in the literature.

2. ANALYTICAL ANALYSIS

A thin rectangular plate with a uniform thickness of $h$ and dimensions of $l \times a$ along x- and y-axis of established Cartesian coordinate system is considered (Figure 1). The plate is subjected to a moving load which moves in x-direction with a constant speed $v$. The plate is simply supported on the two opposite edges parallel to y-axis (at $x=0$ and $x=l$) and has arbitrary boundary condition along other edges (at $y=0$ and $y=a$). Based on small displacement theory, the governing equation of motion for a plate subjected to a dynamic
load \( p(x, y, t) \) is given as:

\[
D \times \nabla^2 \nabla^2 w + \rho h \frac{\partial^2 w}{\partial t^2} + \eta h \frac{\partial w}{\partial t} = p(x, y, t)
\]  
(1)

where:

\[
\nabla^2 \nabla^2 w = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \times \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w
\]  
(2)

and

\[
D = \frac{Eh^3}{12(1-v^2)}
\]  
(3)

In above equations, \( t \) is time; \( w(x, y, t) \) is transverse displacement of the plate at point \((x, y)\). \( D, E, \rho, \eta, \) and \( v \) are flexural stiffness, Young’s module of elasticity, mass density, damping coefficient per unit volume, and Poisson’s ratio, respectively. Considering \( t_d \) as the time that moving loads needs to pass the plate length in \( x \)-direction, the moving concentrated load function is written as:

\[
p(x, y, t) = \begin{cases} 
P \times \delta(x - v \times t) \times \delta(y - y_0) & 0 \leq t \leq t_d \\
0 & t > t_d 
\end{cases}
\]  
(4)

where \( \delta \) is Dirac delta function, \( P \) is constant magnitude of concentrated load, \( y_0 \) is \( y \)-coordinate where the force exerts, and \( v \) is constant speed of moving load \((l = v \times t_d)\).

Figure 1. Thin rectangular Levy-type plate subjected to moving load

In modal form, the transverse displacement of the plate and the applied dynamic load are expressed as:
\[
\begin{align*}
\text{F. Shirmohammadi, S. Bahrami and M. M. Saadatpour} \\
\text{706} \\
\end{align*}
\]

\[w(x, y, t) = \sum_{m=1}^{\infty} w_m(y, t) \times \sin \alpha_m x \quad (5)\]

\[p(x, y, t) = \sum_{m=1}^{\infty} f_m(y, t) \times \sin \alpha_m x \quad (6)\]

where,

\[\alpha_m = \frac{m \times \pi}{l} \quad (6)\]

And \(\alpha_m, w_m(y, t)\) and \(f_m(y, t)\) are wave number in x-direction, amplitude of displacement and force in \(m^{th}\) mode, respectively. The amplitude of force for the moving load with constant magnitude of \(P\) and speed of \(v\) is expressed as:

\[f_m(y, t) = \frac{2P}{l} \sin \alpha_m vt \quad (7)\]

Substituting Eqs. (5) and (6) into Eq. (1), the modal form of the dynamic equation of motion for the Levy-type plate is derived as:

\[
\begin{align*}
D \times \sum_{m=1}^{\infty} \left[ \left( \alpha_m^4 \times w_m - 2 \alpha_m^2 \times \frac{\partial^2 w_m}{\partial y^2} + \frac{\partial^4 w_m}{\partial y^4} \right) + \rho h \frac{\partial^2 w_m}{\partial t^2} + \eta h \frac{\partial w_m}{\partial t} \right] \times \sin \alpha_m x \right] = \\
\sum_{m=1}^{\infty} \left[ f_m(y, t) \times \sin \alpha_m x \right] \quad (m = 1, 2, \ldots) \\
\end{align*}
\]

For \(m^{th}\) mode, Eq. (8) becomes:

\[
\begin{align*}
D \times \left[ \left( \alpha_m^4 \times w_m - 2 \alpha_m^2 \times \frac{\partial^2 w_m}{\partial y^2} + \frac{\partial^4 w_m}{\partial y^4} \right) + \rho h \frac{\partial^2 w_m}{\partial t^2} + \eta h \frac{\partial w_m}{\partial t} \right] \times \sin \alpha_m x = f_m(y, t) \times \sin \alpha_m x \\
\end{align*}
\]

\[D \times \left[ \left( \alpha_m^4 \times w_m - 2 \alpha_m^2 \times \frac{\partial^2 w_m}{\partial y^2} + \frac{\partial^4 w_m}{\partial y^4} \right) + \rho h \frac{\partial^2 w_m}{\partial t^2} + \eta h \frac{\partial w_m}{\partial t} \right] \times \sin \alpha_m x = f_m(y, t) \\
\]  \quad (9)

hence

\[
\begin{align*}
D \times \left( \alpha_m^4 \times w_m - 2 \alpha_m^2 \times \frac{\partial^2 w_m}{\partial y^2} + \frac{\partial^4 w_m}{\partial y^4} \right) + \rho h \frac{\partial^2 w_m}{\partial t^2} + \eta h \frac{\partial w_m}{\partial t} = f_m(y, t) \\
\end{align*}
\]

\[D \times \left( \alpha_m^4 \times w_m - 2 \alpha_m^2 \times \frac{\partial^2 w_m}{\partial y^2} + \frac{\partial^4 w_m}{\partial y^4} \right) + \rho h \frac{\partial^2 w_m}{\partial t^2} + \eta h \frac{\partial w_m}{\partial t} = f_m(y, t) \quad (10)\]

The general solution of Eq. (10) for \(m^{th}\) mode is written in the spectral form as:
\[
\omega_n = \frac{2n\pi}{T}, \quad (n = 1, 2, \ldots, N)
\] (12)

And \(\bar{w}_{mn}(y; \omega_n)\) is Fourier component of \(w_{mn}(y, t)\), corresponding to discrete frequency \(\omega_n\), defined by \(\omega_n = \frac{2n\pi}{T}\), where \(T\) is time-window and \(N\) is number of sample. \(T\) is related to \(N\) using following equation:

\[
T = \frac{N}{2f_{Nyq}}
\] (13)

where, \(f_{Nyq}\) is Nyquest frequency, the maximum frequency which can be reached using \(N\) sample in the time domain window. Substituting Eq. (11) into Eq. (10), the equation of motion in time-domain, the equation of motion in frequency-domain is derived as:

\[
D \left( \frac{\partial^4 \bar{w}_{mn}}{\partial y^4} - 2\alpha_m^2 \frac{\partial^2 \bar{w}_{mn}}{\partial y^2} + \left( \alpha_m^4 - \lambda_{mn}^4 \right) \bar{Y}_{mn} \right) = \bar{f}_{mn}(y; \omega_n) \quad (n = 1, 2, \ldots, N)
\] (14)

where

\[
\lambda_{mn}^4 = \frac{\rho h \omega_n^2 - i n h \omega_n}{D}
\] (15)

and

\[
f_{mn}(y, t) = \sum_{n=1}^{N} \bar{f}_{mn}(y, \omega_n) \times e^{i\omega_n t}
\] (16)

In the above equations \(\bar{f}_{mn}(y; \omega_n)\) is the external force function in frequency domain. Using Eq. (16), the solution of homogenous equation of motion in frequency domain can be derived as:

\[
\bar{w}_{mn} = C_{mn} e^{-ik_{mn} y}
\] (17)
where $i = \sqrt{-1}$ and $C_{mn}$ is constant coefficient in each frequency and each mode and $k_{mn}$ is wavenumber in y-direction. Substituting this solution into homogenous version of Eq. (14) we obtain the relation between wave numbers ($k_{mn}$) and discrete frequency ($\omega_n$) known as dispersion relation or spectrum relation in the literature:

$$\left(\alpha_m^2 + k_{mn}^2\right)^2 = \lambda_{mn}^4$$

(18)

Solving the characteristic Eq. (18) leads to:

$$\left(k_{mn}\right)_{1,3} = \pm i\left(\alpha_m^2 + \lambda_{mn}^2\right)^{1/2} \left(k_{mn}\right)_{2,4} = \pm \left(-\alpha_m^2 + \lambda_{mn}^2\right)^{1/2}$$

(19)

Using Eqs. (19) in Eq. (14) gives:

$$\bar{w}_{mn} = C_{mn,1}e^{-\left(k_{mn}\right)_y} + C_{mn,2}e^{\left(k_{mn}\right)_y} + C_{mn,3}e^{-\left(k_{mn}\right)_y} + C_{mn,4}e^{\left(k_{mn}\right)_y}$$

(20)

Finally the general solution of Eq. (1) can be written as:

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{N} \left[\left(\alpha_m^2 + k_{mn}^2\right)^2 \sin \alpha_m x e^{i\omega_n t}\right]$$

(21)

Eq. (21) must satisfy boundary conditions in each specific mode ($m$) and discrete frequency ($\omega_n$)

3. SPECTRAL DYNAMIC STIFFNESS MATRIX FORMULATION

The exact dynamic stiffness matrix is called spectral element matrix in the literature and there are three major methods to formulate it; the force-displacement relation method, the state-vector equation method and the variational method. The detailed explanation of these methods can be found in Ref. [14]. In this paper, the force-displacement relation method is applied to derive spectral element matrix. In this method, the exact dynamic stiffness matrix is derived finding the direct relations between nodal forces and nodal displacements [14]. Figure 2 shows a spectral plate strip which has dimensions $l$ and $b$ in x- and y-directions, respectively. Using Eq. (21) the spectral nodal degree of freedoms (DOFs) (displacement and slope) on the boundary edges at $y = 0$ and $y = b$ can be expressed as:
(a) Figure 2. Sign Conventions defined for (a) the mechanics of materials, (b) the matrix analysis

\[
\begin{bmatrix}
\bar{w}_{mn,1} & \bar{\vartheta}_{mn,1} & \bar{w}_{mn,2} & \bar{\vartheta}_{mn,2}
\end{bmatrix}^T = \left[ A_{mn} \right] \times \begin{bmatrix}
C_{mn,1} & C_{mn,2} & C_{mn,3} & C_{mn,4}
\end{bmatrix}^T
\]

or

\[
[U]_{mn} = \left[ A_{mn} \right] \times [C]_{mn}
\]

where, the components of linking matrix \( \left[ A_{mn} \right] \) are presented as follows:

\[
\left[ A_{mn} \right] = \begin{bmatrix}
1 & 1 & 1 & 1 \\
-(k_{mn})_1 & (k_{mn})_1 & -(k_{mn})_2 & (k_{mn})_2 \\
\exp(-(k_{nn})y_b) & \exp((k_{nn})y_b) & \exp(-(k_{nn})y_b) & \exp((k_{nn})y_b) \\
-(k_{mn})_1\exp(-(k_{nn})y_b) & (k_{mn})_1\exp((k_{nn})y_b) & -(k_{mn})_2\exp(-(k_{nn})y_b) & (k_{mn})_2\exp((k_{nn})y_b)
\end{bmatrix}
\]

(24)

The spectral shear forces and flexural moments on the boundary edges at \( y = 0 \) and \( y = b \) of the plate strip are given by [15]:

\[
\begin{bmatrix}
\tilde{V}_x \\
\tilde{V}_y \\
\tilde{M}_x \\
\tilde{M}_y
\end{bmatrix}_{mn} = -D \times \begin{bmatrix}
\frac{\partial^3 \bar{w}_{mn}}{\partial y^3} + (2 - \nu) \frac{\partial^3 \bar{w}_{mn}}{\partial x^2 \partial y} \\
\frac{\partial^2 \bar{w}_{mn}}{\partial y^2} + \nu \frac{\partial^2 \bar{w}_{mn}}{\partial x^2}
\end{bmatrix}
\]

(25)

Using Eq. (21) and Eq. (25), the spectral moment and the shear force on the boundary edges of the strip are expressed as:
\[ \begin{bmatrix} (\vec{V}_y)_{mn,1} & (\vec{M}_y)_{mn,1} & (\vec{V}_y)_{mn,2} & (\vec{M}_y)_{mn,2} \end{bmatrix}^T = \begin{bmatrix} \vec{B}_{mn} \end{bmatrix} \times \begin{bmatrix} C_{mn,1} & C_{mn,2} & C_{mn,3} & C_{mn,4} \end{bmatrix}^T \] (26)

or

\[ [F]_{mn} = [\vec{B}]_{mn} \times [C]_{mn} \] (27)

where matrix \([\vec{B}]_{mn}\) is given as:

\[ [\vec{B}]_{mn} = D \begin{bmatrix} (r_{mn})_1 & -(r_{mn})_1 & (r_{mn})_2 & -(r_{mn})_2 \\ (p_{mn})_1 & (p_{mn})_1 & (p_{mn})_2 & (p_{mn})_2 \\ e^{-(k_{mn})_1 b} (r_{mn})_1 & -e^{-(k_{mn})_1 b} (r_{mn})_1 & e^{-(k_{mn})_2 b} (p_{mn})_2 & -e^{-(k_{mn})_2 b} (p_{mn})_2 \\ e^{-(k_{mn})_1 b} (p_{mn})_1 & e^{-(k_{mn})_2 b} (p_{mn})_2 & e^{-(k_{mn})_1 b} (p_{mn})_1 & e^{-(k_{mn})_2 b} (p_{mn})_2 \end{bmatrix} \] (28)

and

\[ (r_{mn})_1 = -(k_{mn})_1 + (2 - \nu)\alpha_m^2 (k_{mn})_1, \quad (r_{mn})_2 = -(k_{mn})_2 + (2 - \nu)\alpha_m^2 (k_{mn})_2 \]
\[ (p_{mn})_1 = (k_{mn})_1 - \nu\alpha_m^2, \quad (p_{mn})_2 = (k_{mn})_2 - \nu\alpha_m^2 \] (28)

The consideration of the sign convention in the matrix analysis compared to the classical mechanics of material leads to symmetry of the spectral element matrix in frequency-domain. Hence, in derivation of \([\vec{B}]_{mn}\) in Eq. (27), the following sign changes are applied:

\[ \text{at } y = 0 \rightarrow (\vec{V}_y)_{mn,1} = -(\vec{V}_y)_{mn}, \quad \text{and } (\vec{M}_y)_{mn,1} = -(\vec{M}_y)_{mn} \]
\[ \text{at } y = b \rightarrow (\vec{V}_y)_{mn,2} = (\vec{V}_y)_{mn}, \quad \text{and } (\vec{M}_y)_{mn,2} = -(\vec{M}_y)_{mn} \] (29)

Combining Eq. (23) and Eq. (27) yields to:

\[ [\vec{F}]_{mn} = [\vec{S}]_{mn} \times [\vec{U}]_{mn} \] (30)

where \([\vec{S}]_{mn}\) is the spectral element matrix or exact dynamic stiffness matrix of the plate strip which is defined as:
Using the same procedure which is used in conventional FEM, the global spectral element matrix, the global force and displacement vectors for a specific \( m \) and \( n \) are derived as:

\[
\begin{bmatrix}
\bar{S}^G_{mn}
\end{bmatrix} = \begin{bmatrix}
\bar{B}^G_{mn}
\end{bmatrix} \times \begin{bmatrix}
\bar{A}^G_{mn}
\end{bmatrix}^{-1}
\]

(31)

where the superscript \( G \) denotes the global system. As the exact dynamic stiffness matrix of global system is formulated, only one element (strip) is sufficient to model a uniform plate of any dimension, in absence of any discontinuity in material and geometrical properties of the plate. To calculate the natural frequencies (\( \omega_{\text{Natural}} \)) of the global system the dynamic stiffness matrix should vanish as follow:

\[
\text{det} \left( \begin{bmatrix}
\bar{S}^{-G}_{mn}
\end{bmatrix} \right) = 0 \rightarrow \omega_{\text{Natural}}
\]

(33)

And the spectral nodal displacement can be calculated exactly in the frequency domain as:

\[
\begin{bmatrix}
\bar{U}^G_{mn}
\end{bmatrix} = \left( \begin{bmatrix}
\bar{S}^{-G}_{mn}
\end{bmatrix} \right)^{-1} \times \begin{bmatrix}
\bar{F}^G_{mn}
\end{bmatrix}
\]

(34)

Having the nodal displacements in frequency domain and using IFFT the dynamic response of global system can be calculated in time domain. In the process of calculation, attention should be paid that the displacement in time-domain will be real only if the calculation of displacement vector in frequency-domain is performed only up to the Nyquist frequency and in other frequencies, displacement vector is obtained by imposing that they should be the complex conjugate of the beginning part, calculated by Eq. (34) [16].

4. NUMERICAL EXAMPLES AND DISCUSSION

4.1 Verification

To verify the developed formulations, a thin square plate with simply supported on all edges, subjected to a 8.89 \( N \) (2 lb) moving point load which moves along the middle line of the plate is studied. The plate has the dimension of 101.6 \( mm \) \( (4 \times 4 \text{ in}) \) with thickness of 25.4 \( mm \), the Young’s modulus of elasticity, Poisson ratio and the mass density are 206842.77 \( MPa \) \( (30 \times 10^6 \text{ psi}) \), 0.3, and \( 1.069 \times 10^{-8} N.\text{sec}^2/mm^4 \) \( (0.001 \text{ lb.sec}^2/\text{in}^4) \), respectively. The dynamic amplification factor (DAF) for central point of the plate, calculated using various numerical methods and various load speed and is
presented in Table 1. In this table, $T_1$ is the first fundamental period ($T_1 = 9.7172 \times 10^{-4} \text{ sec}$). The results computed by the presented method are compared to the results achieved by FEM with 25 elements in Ref. [6], the classical finite strip method developed by the authors with 5 strips [18], and FEM in Ref. [5]. As shown in Table 1, the presented method gives high accurate natural frequencies of the plate in comparison with other numerical methods, using only one strip.

<table>
<thead>
<tr>
<th>$\frac{T_1}{T_d}$</th>
<th>$v \ (\text{in/sec})$</th>
<th>$\frac{W_{\text{dyn}}}{W_{\text{sta}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8</td>
<td>515</td>
<td>1.033</td>
</tr>
<tr>
<td>1/4</td>
<td>1030</td>
<td>1.083</td>
</tr>
<tr>
<td>1/2</td>
<td>2060</td>
<td>1.221</td>
</tr>
<tr>
<td>1</td>
<td>4120</td>
<td>1.576</td>
</tr>
<tr>
<td>2</td>
<td>8240</td>
<td>1.402</td>
</tr>
</tbody>
</table>

Figure 3. Maximum dynamic response of the plate in central point ($v = 26.162 \text{ m/sec}$)

An important consideration in the analysis of dynamic response of a plate under moving load is the number of harmonic series in Fourier transform required for an accurate prediction. Maximum dynamic central deflection of the plate ($W_{\text{dyn}}$) using 1, 3, 5, 7 and 9 harmonic series in the presented method is shown in Figure 3 when the plate is under the moving load with speed of $v = 26.162 \text{ m/sec}$. As in Figure 3, only three harmonics solution is enough for acceptable response (error less than 1%).

4.2 Influence of damping

In this example the central displacement of a square plate with simply supported in all edges, subjected to a $10 \text{kN}$ moving load which moves along middle line of the plate with speed of $12.5 \text{ m/sec}$ is calculated using the presented method and the FSM developed by authors.
The plate has the dimension of $4 \times 4\ m$, thickness of $0.1\ m$, Young’s modulus elasticity of $3 \times 10^3\ MPa$, Poisson ratio of $0.4$, the mass density of $1300\ kg/m^3$ and the first fundamental frequency of $59.03\ rad/sec$. In the FSM and the presented method, the damping ratio ($\xi$) and the damping coefficient per unit volume ($\eta$) are used respectively to specify the damping characteristic of the dynamic system. The damping ratio is considered to be equal to 0.01 ($\eta = 1535\ N\ sec/m^4$) and 0.05 ($\eta = 7675\ N\ sec/m^4$). The well-known equation $\eta = 2 \rho \omega_0 \xi$ was used to calculate the corresponding damping coefficient per unit volume. The transverse deflection of the plate with damping ratio of 0.01 and 0.05 is shown in Figure 4, and Figure 5, respectively. The comparison of the calculated results using the presented method with the results calculated using classic FSM confirms the accuracy of the current equation to approximate the damping coefficient per unit volume by having the damping ratio.

![Figure 4](image1.png)

Figure 4. Central displacement of plate with $\xi = 0.01$ subjected to the moving load

![Figure 5](image2.png)

Figure 5. Central displacement of plate with $\xi = 0.05$ subjected to the moving load
4.3 Influence of moving-load speed

Here the influence of the moving load speed on the maximum displacement in central point of the plate is studied. A moving point load $P = 10\, kN$ moves along $x$-direction with different speeds, ranging from $v = 12.5\, m/sec$ to $v = 50\, m/sec$. The time history dynamic response of the plate in central point is shown in Figure 6. As it can be seen in this figure, the moving load speed has an important influence on the maximum central displacement and by increasing load speed; the maximum central displacement of the plate tends into right side, also the critical load velocity is near $v = 37.5\, m/sec$ in this example.

![Figure 6. Central displacement of plate subjected to the moving load with different velocity](image)

![Figure 7. Plats subjected to the different moving loads](image)
4.4. Influence of eccentricity of load(s)
In this example, the effect of eccentricity of load(s) on central point displacement of the plate is studied. As shown in Figure 7, five different cases with different eccentricity of load are considered. The total magnitude of moving loads for different cases is the same (P = 10 kN) and all the plates have the same geometrical and material properties which are presented in previous subsection.

As shown in Figure 8, the smaller average eccentricity leads to the larger central displacement. Among these cases, Case 1 has the minimum eccentricity; therefore, the central displacement for this case is more than other cases.

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![Figure 8. Central displacement of plate subjected to the different moving loads](image-url)
4.4 Applying presented method for continuous plate in $x$ direction

In this example, the efficiency of the presented method to obtain the non-dimensional free vibration frequency of the rectangular one direction-continuous plate is assessed. The non-dimensional free-vibration frequencies $\omega l^2 \sqrt{\rho h/D}$ of two continues plates illustrated in Figure 9 with various boundary condition along transverse direction are presented in Table and Table . The results calculated by the presented method are compared with the classical FSM in Ref. [19] and FSM results in Ref. [20]. In Ref. [20] single span beam functions are used to obtain the non-dimensional free vibration frequencies of continues plates. In these tables, the abbreviations S-S, C-C, and S-C indicate various boundary conditions combinations along two transverse edges in which S stands for simple boundary condition, C represents Clamped support, and F stands for free boundary condition.

![Figure 9](image)

**Figure 9.** (a) Four span continuous panel, and (b) continuous panels with unequal span lengths in longitudinal direction

**Table 2:** Non-dimensional frequencies $\omega l^2 \sqrt{\rho h/D}$ of four span continuous panels

<table>
<thead>
<tr>
<th>Boundary Conditions</th>
<th>Source of Results</th>
<th>Number of Elements</th>
<th>Mode Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>S-S</td>
<td>Present</td>
<td>4x1</td>
<td>19.73</td>
</tr>
<tr>
<td></td>
<td>FSM [19]</td>
<td>4x1</td>
<td>19.74</td>
</tr>
<tr>
<td></td>
<td>FSM [20]</td>
<td>12x1</td>
<td>19.74</td>
</tr>
<tr>
<td>C-C</td>
<td>Present</td>
<td>4x1</td>
<td>20.82</td>
</tr>
<tr>
<td></td>
<td>FSM [19]</td>
<td>4x1</td>
<td>20.84</td>
</tr>
<tr>
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<td>FSM [20]</td>
<td>12x1</td>
<td>20.83</td>
</tr>
<tr>
<td>C-S</td>
<td>Present</td>
<td>4x1</td>
<td>20.02</td>
</tr>
<tr>
<td></td>
<td>FSM [19]</td>
<td>4x1</td>
<td>20.02</td>
</tr>
<tr>
<td></td>
<td>FSM [20]</td>
<td>12x1</td>
<td>20.02</td>
</tr>
</tbody>
</table>

**Table 3:** Non-dimensional frequencies $\omega l^2 \sqrt{\rho h/D}$ of continuous panels with unequal span lengths in longitudinal direction

<table>
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<tr>
<th>Boundary Conditions</th>
<th>Source of Results</th>
<th>Number of Elements</th>
<th>Mode Number</th>
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</thead>
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<tr>
<td>S-S</td>
<td>Present</td>
<td>3x1</td>
<td>12.92</td>
</tr>
<tr>
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<td>FSM [19]</td>
<td>4x1</td>
<td>12.94</td>
</tr>
</tbody>
</table>
5. CONCLUSION

In this paper, spectral finite strip method has been developed to analyse the dynamic response of a rectangular Levy-type plate, subjected to moving loads. Dynamic results, achieved by presented method, compared with finite element method and finite strip method in literature. Moreover, the effect of damping, moving-load speed and eccentricity of load on the dynamic response of plate is studied. A well-known relation for damping coefficient evaluation is advised in comparing by damping ratio in other numerical results. Numerical results show that the proposed method can be used for dynamic problem with moving loads accurately and economically.

REFERENCES

9. Gopalakrishnan S, Martin M, Doyle JF. A matrix methodology for spectral analysis of wave propagation in multiple connected Timoshenko beam, Journal of Sound and