INTRODUCING A NEW WALL-FRAME EARTHQUAKE RESISTING SYSTEM

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ABSTRACT

The paper introduces a new earthquake resistant rocking-wall moment frame (RWMF) that is capable of damage reduction, collapse avoidance and self-centering due to strong ground motion. The system consists of a grade beam restrained moment frame, (GBRMF) attached to a co-planar, post tensioned (PT) rigid rocking core (RRC) by means of gap opening link beams (GOLBs) and buckling restrained braces (BRBs). Several practical details aiming at damage reduction have been presented. Worked examples have also been provided to demonstrate the validity of the proposed solutions. All results have been verified by independent computer analysis. The proposed formulae are ideally suited for preliminary design and teaching purposes.

Keywords: Rocking-wall; moment frame; initial imperfections; collapse prevention; self-centering; reparability.

1. INTRODUCTION

While fixed base moment frame–shear wall combinations are the most popular earthquake resisting systems worldwide, they are not free from technical flaws and economic drawbacks. Conventional fixed-base moment frame–shear wall or braced frame combinations rely on uncontrolled inelastic responses of their members to absorb seismic energy. These systems have served their functions rather well in the past. However, they are practically un-repairable and prone to catastrophic collapse due to major seismic events. Here, a relatively new, dual earthquake resisting system consisting of ductile moment frames (MFs) with rotationally controllable column supports in combination with post-tensioned RRCs is introduced Fig.1(b). The proposed configuration is capable of damage control, collapse prevention and self centering due to strong ground motion. The idea that rocking

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motions may reduce damage to MFs during earthquakes in not new, and was originally recognized by MacRae et al. [1, 2] Ajrab et al. [3], Panian et al. [4], Ji et al. [5] and Wada et al. [6]. The pioneering effort in developing design concepts for controlled rocking of self-centering cores are due to Christopoulos et al. [7,8], Deierlein et al. [9], Eatherton et al. [10,11], Takeuchi et al. [12], Janhunen et al. [13] and Grigorian et al. [14]. Knowledge gained from these and other sources has helped formulate the basis of the current article. The focus of the present paper is on the global response of the RWMF rather than member design. GOLBs used to connect the MF to the RRC, Fig. 1(b) act as gap opening PT moment connections, [15, 16]. Further research has shown that while these connections display excellent realignment abilities in laboratory testing, they tend to damage the adjoining columns and diaphragms in assembled systems; [17, 18], [10 and 19] have both proposed innovative detailing methods to prevent damage due to rocking. The interested reader in rocking frame innovations is referred to two well documented bibliographies [20 and 21].

ASCE 41 [22] defines specific performance levels for immediate occupancy, life safety, and collapse prevention, where collapse prevention, is defined as “the post-earthquake damage state in which the building is on the verge of partial or total collapse”. A well-designed RWMF can be expected to provide reliable support for the gravity systems, facilitate the rescue/evacuation effort and improve the repairability of the frame after a major seismic event. The present article focuses on collapse prevention employing RWMF technologies for both new as-well-as existing structures. This leads to the notion that if collapse prevention is feasible through reliable technologies, then life safety and immediate occupancy can also be achieved through similar strategies. To ensure that near perfect self-centering is achieved, design strategies need to focus on two basic issues: selection of a structural system that can dissipate the stipulated minimum seismic energy and retain little to no residual drifts and that its response conforms to a flag shaped hysteresis curve that agrees with observed behavior. The proposed configuration is composed of two types of structural systems-regular energy dissipating, perfectly elasto-plastic MFs, with or without auxiliary attachments, connected to co-planar cable stabilized RRCs, by means of GOLBs. The RRC tends to reduce axial and shear stresses in the MF. Since axial and shear stresses have little to no effect on drift concentration, the forthcoming arguments are advanced on the premise that racking moments dominate RWMF response and that such effects can be ignored for the purposes of this paper. The RRC also forces the combined structure to act as a unified system with a highly dominant first mode of vibration. This qualifies the system for nonlinear static analysis [23]. The step-by-step development of the structural system is accompanied by analytic formulations that are supported by two basic hypotheses that: the RWMF acts as an elastoplastic system and that the parallel combination of any number of bilinear-elastic components can be reduced to an equivalent multi-linear system. This allows the design of the RWMF to be driven by its global strength and stiffness, rather that the properties if its members. Worked examples have been provided to demonstrate the validity of the proposed solutions. All results have been verified by independent computer analysis. The proposed approach for the development of the new earthquake resistant system is introduced as follows.
2. THE MODELING APPROACH

The successful modeling of a new earthquake resistant RWMF depends largely on the approach and purpose of the analysis, [24]. An understanding of the characteristics of RWMFs can greatly reduce the analytic ambiguities and help develop closed form solutions. For instance, it has been observed that ductile MFs combined with RRCs adapt a unique mode of response where all beams and columns undergo the same rotation and drift respectively, regardless of their location within the framework. In other words the drift ratio remains constant for all horizontal subframes. This implies that the original scheme, Fig. 1(b), can be modeled as several equivalent systems [25, 26] e.g., a number of single story horizontal subframe stacked on top of each other [27] as single bay vertical subframes arranged in series or as options (c) and (d) in Fig. 1. While option (d), the basic module, is best suited for research and developmental purposes, the single bay model, Fig. 1(c), lends itself better for system introduction and teaching objectives. The analysis of the system is conducted following the design led method of approach. In design led analysis the known attributes of the structure are utilized as inherent properties rather than analytic results. For example, the lumping of similar and similarly behaving elements of the GBRMF of Fig. 1 (b), into equivalent single bay, Fig. 1(c), or single module, Fig. 1(d), models can help capture some of its unique attributes that would not be readily apparent from electronic data. In addition the methodology takes advantage of four important research findings:

1. The RRC-MF combination tends to acts as a SDOF system [3].
2. The RRC acts as an upright simply supported beam [28].
3. In rigid RWMFs the shear in the frame tends to be uniform over the height [29].
4. Very rigid RRCs absorb the entire external load applied to the RWMF [14].

The proposed approach is capable of treating collapse prevention, self-alignment, and uniform drift as inherent properties of the proposed system. Here, the same modeling approach is used to help track the elasto-plastic response of the structure, first as an elastic continuum under monotonically increasing lateral forces and then as a restrained MF at incipient collapse. In order to focus greater attention on the conceptual aspects of this presentation the bulk of the preliminary analysis has been excluded from this presentation.

2.1 Elastic analysis of the free standing RWMF (no supplementary devices)

The knowledge that the RWMF is forced to tilt as a rigid body helps reduce the task of
otherwise cumbersome analysis to manually manageable solutions. In other words, if the response of the three equivalent systems of Fig. 1 under similar lateral effects, can be related to the same single variable, the uniform drift ratio $\phi$, then the basic equations of bending of all three systems can be expressed as:

$$
\phi = \frac{M_0}{K_F} = \frac{M_0}{K_S} = \frac{M_0}{K}
$$

(1)

where $K_F$, $K_S$ and $K$ are the rotational stiffnesses of the prototype and the equivalent MFs of Figs. 1(c) and (d) respectively. If $M_0 = \sum_{i=1}^{m} F_i x_i$, then the expanded form of Eq. (1) can be expressed as [30]; i.e.

$$
\phi = \frac{M_0}{12E} \left[ \frac{1}{\sum_{j=0}^{n} \sum_{i=1}^{m} (I_{i,j} / h_i)} + \frac{1}{\sum_{j=1}^{n} \sum_{i=0}^{m} (I_{i,j} / l_j)} \right] = \frac{M_0}{K_F} \text{ or } \phi = \frac{M_F}{K_F}
$$

(2)

It is instructive to note that the two terms in the square brackets consist of sums of generalised quantities, and that there can be numerous combinations of beam and column sections that result in the same prescribed $\phi$. This is a hint that the use of repetitive members can lead to more repairable, highly constructible, minimum weight structures. The solution for the single bay model, directly transformed from Eq. (2), can be expressed as:

$$
\phi = \frac{M_0}{12E} \left[ \frac{1}{2(I / h)} + \frac{1}{(I / L)} \right] = \frac{M_0}{K_S}
$$

(3)

$$(I / L) = \sum_{j=1}^{n} (I_{i,j} / l_j), \quad 2(I / h) = \sum_{j=0}^{n} (I_{i,j} / h_j).$$

The solution to the single module can be written as:

$$
\phi = \frac{M_0}{12E} \left[ \frac{H}{2J} + \frac{L}{2I} \right] = \frac{M_0}{K} \text{ or } \phi = \frac{M_F}{K_F}
$$

(4)

Here, $M_0 = VH$, $2(I / L) = \sum_{i=0}^{m} \sum_{j=1}^{n} (I_{i,j} / l_j)$ and $2(J / H) = \sum_{i=1}^{m} \sum_{j=0}^{n} (J_{i,j} / h_j)$. Solutions (2), (3) and (4) are equally valid with mutually transferable results. The global $P$-delta effects and the total moment of resistance of all three models can be expressed as: $M_{PA}$ and $M_F = M_0 + M_{PA}$ respectively. The latter version lends itself better to describing the physical issues involved in the response of the RWMF. A summary of development of RWMF formulae, based on the response of the basic module is presented in Table 1.
2.2 Plastic limit state analysis of freestanding RWMF (no supplementary devices)

Global plastic failure patterns generally result in smaller maximum drift ratios and more economical solutions (Mazzolani and Piluso [31]). The use of RRCs in conjunction with reduced beam sections leads to preferred plastic collapse patterns, such as those shown in Fig. 2. Unsupplemented RRCs can prevent progressive collapse [32], due to soft story failure and impose uniform rotation on the MF but cannot increase the ultimate carrying capacity of the system. If the RWMFs of Figs. 2(a) and (b) with equal global P-delta moments are assumed to undergo a small virtual rotation \( \theta \), and that plastic hinge offsets are zero, then the corresponding virtual work equations can be expressed as:

\[
(M_0^p + M_{PA})\theta = \sum_{i=1}^{m} F_i x_i \theta = \sum_{i=1}^{m} \sum_{j=1}^{n} 2M^p_{i,j} \theta
\]

(5)

For case 2(b):

\[
(M_0^p + M_{PA})\theta = \sum_{i=1}^{m} F_i x_i \theta = \sum_{i=1}^{m} 2\bar{M}_i^p \theta
\]

(6)

For case 2(c):

\[
(M_0^p + M_{PA})\theta = \sum_{i=1}^{m} F_i x_i \theta = 4M^p \theta
\]

(7)

where \((M_0^p + M_{PA})\) is the total external moment that gives rise to the total plastic moment of resistance, \(M_F^p\) of the three models, in the absence of supplementary devices and can be assessed as:

\[
(M_0^p + M_{PA}) = M_F^p = \sum_{i=1}^{m} \sum_{j=1}^{n} 2M^p_{i,j} = \sum_{i=1}^{m} 2\bar{M}_i^p = 4M^p
\]

(8)

A parametric example of the applications of Eqs. (2) and (8) is presented as follows:

**Example 1: Preliminary design of RWMF**

Consider the preliminary design of the MF of the un-supplemented, \(m \times n\), RWMF of Fig. 1(b) under lateral loading \(F_j = F_i/m\), and total nodal floor and roof gravity loads \(np\) and \(np/2\) respectively, such that the uniform target drift does not exceed \(\phi_{all}\) at incipient collapse.

Given: \(I_{0,j} = I_{m,j} = I'/2\), \(I_{i,j} = I'\) for all other \(i\), \(J_{i,0} = J_{i,n} = J'/2 = \rho\ell'/2\), \(J_{i,j} = J' = \rho\ell'\) for all other \(j\), \(M_{0,j} = M^p_{m,j} = M^p \ell'/2\), \(M_{i,j} = M^p\) for all other \(i\). \(N^p_{i,0} = N^p_{i,n} = N^p \ell'/2 = \lambda M^p \ell'/2\) and \(N^p_{i,j} = N^p_{j,i} = \lambda M^p\ell'\) for all other \(j\). \(l_j = l\), \(h_i = h\).
Solution: \( M_0 = (m + 1)(2m + 1)Fh / 6 \) and \( M_{pA} = \sum_{i=1}^{n} n_p \phi_i = nmpH\phi / 2 \), \( P = np/2 \) and \( H = mh \). From Eq. (2): \( \sum_{j=1}^{n} \sum_{i=0}^{n} I_{i,j} = mnI' \) and \( \sum_{j=0}^{n} \sum_{i=1}^{n} J_{i,j} = mnJ' \),
\[
\frac{1}{K_F} = \frac{1}{12mnE} \left[ \frac{h}{J'} + \frac{l}{l'} \right].
\]
Hence \( \phi_{all} = \frac{M_0 + M_{pA}}{K_F} = \frac{M_0}{K_F} + \bar{\bar{\mu}} \phi \), where \( \bar{\bar{\mu}} = \frac{nmpH}{2K_F} \) may be looked upon as the global stability quotient of the subject structure. This leads to \( \phi_{all} = M_0/(1 - \bar{\bar{\mu}})K_F \) and \( (M_0 + M_{pA}) = M_0 + \bar{\bar{\mu}} \phi K_F = M_0/(1 - \bar{\bar{\mu}}) \) or \( M_0/(1 - \bar{\bar{\mu}}) = \sum_{j=0}^{m} \sum_{i=1}^{n} 2M_{i,j} = 2mnM'P \).

Therefore, the complete solution can be summarized as:
\[
\phi_{all} = \frac{(m + 1)(2m + 1)Fh}{72mn(1 - \bar{\bar{\mu}})E} \left[ \frac{h}{J'} + \frac{l}{l'} \right] \quad \text{and} \quad M'P = \frac{(m + 1)(2m + 1)Fh}{12(1 - \bar{\bar{\mu}})mn}.
\]

Note the distinct similarities between Eqs. (9) and (4) above. The plastic moment of resistance of the equivalent module can be estimated as \( 4M'P = 2mnM'P \). The RRC end reactions due to \( F_i \) can be computed as \( Q = M_0/H = (m + 1)(2m + 1)F/6m \) and \( R = \sum_{i=1}^{m} F_i - Q = (m^2 - 1)F/6m \). Solution (9) also reveals the following attributes associated with the subject MF, as part of the proposed RWMF:

- The framework is a system of uniform strength and stiffness and as such constitutes a structure of minimum weight [33].
- The MF lends itself to cost effective construction, since it can be constructed out of repetitive beams, continuous columns and identical connections.
- The structure is highly repairable since damage is concentrated at beam ends only, and that: \( I, J \) and \( M'P \) thus obtained constitute excellent data for the preliminary design of the example MF. Repairable moment connections can be envisaged for all beam ends.

The equivalency of the mathematical models of the proposed RWMF and the basic module implies that the three systems share the same general characteristics. Since the beams and columns of the subject module are of equal strength and stiffness, it may be looked upon as a structure of minimum weight and drift. Additionally, all three Eqs. (5), (6) and (7) satisfy the prescribed yield criteria, static equilibrium and the selected boundary conditions, and as such represent minimum weight, unique [33] plastic design solutions. It follows that if each minimum weight basic module is designed in accordance with the prescribed criteria, then the entire assembly could also be regarded as a minimum weight frame that complies with the same design rules, i.e., the design of one such module can be utilized to generate similar modules to reconstruct the prototype. The similarity of the load and deformation profiles, relates member forces and deformations, the failure load and the mode of collapse to the same normalized straight line that qualifies the MF as a structure of uniform response. Solutions (1)-(8) are significant in that they lead to powerful design formulae, include the effects of supplementary devices and provide insight into the behavior of the combined structure. A brief study of these relationships reveals that the response of
the subject RWMF depends mainly on the magnitudes of the global variables such as $K_F$ and $M^P$. In other words $\phi$ is not influenced by the properties of the individual elements or their distribution within the structure. This is a useful finding and leads to the conclusion that the beams and columns of the system can be selected in any preferred fashion that satisfies Eqs. (1), (8) and the pertinent code requirements. For instance, if ease of construction, material savings and post earthquake repairability are part of the design strategies, it would be logical to select identical beam sections $I_{i,j} = I'$ and $M^P_{i,j} = M'$, and identical columns $N^p_{i,j} = N' = \lambda M'$ and $J_{i,j} = J'$. Where $\lambda > 1$ is the column over-strength factor. Several sizing options come to mind, e.g., if progressive hinging is preferred over simultaneous failure, as in Example 1, then the strengths of the beams can be selected in any ascending or descending order of $M'$ provided their sum does not exceed $(M_0 + M_{ps})$. Or, in order to prevent system collapse, a number of strategically located members may be designed to remain elastic during a seismic event. Additionally the methodology allows the effects of $P$-delta moments, initial imperfections and tipping of supports to be directly reflected in the proposed formulae. The most notable aspect of the proposed concept is its ability to describe the behavior of all three models by a flag-shaped hysteresis diagram as presented in the following section.

### 3. BASIC HYPOTHESES

If a number of elasto-plastic assemblies are combined in parallel, then the resulting system can be represented by an equivalent multi-linear elasto-plastic structure. The maximum stiffness and ultimate strength of the combined structure would be the sum of the stiffnesses and ultimate strengths respectively of all such systems. Fig. 3(a) shows the moment-drift $(M - \phi)$ diagrams of four such components that constitute the lateral resisting system of the subject RWMF. The $(M - \phi)$ response diagram of the combined structure is shown in Fig. 3(b), where it is shown that the combined stiffness $K'$ degrades to $K'_k$ as the total moment of resistance increases from the plastic moment of resistance of the MF, $M^P_F$ to $M^Y_R$, the yield strength of the combined system at incipient collapse. The same diagram also indicates that if the applied moment $(M_0 + M_{ps})$ exceeds $M^P_F$, but is less than $M^Y_R$ then the RWMF will collapse due to plastic failure of the MF. In addition the system would tend to realign itself when the loading is removed. Self-centering can be achieved if the moment provided by the restoring components is larger than the moment provided by the energy dissipating elements. The idealized response modes of the proposed system are applicable to combinations of perfectly elastoplastic, energy dissipating MFs, BRBs and bi-linear elastic gap opening assemblies with zero residual displacements. In general, collapse prevention is a simpler proposition than self-centering. Collapse prevention can be achieved if the total moment of resistance of the combined system is larger than the plastic failure moment of the MF. If self-centering is also part of the design strategy then additional resilience may be
needed to help return the system to its original position. PT tendons are assumed to remain elastic throughout the history of loading of the structure. In general collapse prevention and self-centering are treated as complementary processes.

3.1 Collapse prevention and self-centering

Fig. 3(a), line abcde, shows the idealized static hysteretic response of a ductile MF or a perfect elasto-plastic device. Line afghk represents the response of a similar system or component such as a BRB. Lines mn and pq represent the linear elastic behavior of typical gap opening assemblies such as LBs and RRCs. $K_{F_{st}}$ refers to the strain hardening stiffness of the MF. The flag-shaped hysteretic plot of Fig. 3 (b) shows the combined response of three selected systems under reversible static loading. It also defines the ability of the combined structure to self-center as the drift ratio returns to zero at the end of the loading cycle. It may be seen from these diagrams that collapse prevention can be achieved if the total moment of resistance $M^R = (M_R + M^P_R) > (M_0 + M_{Pa})$ is larger than the total applied moment. The opportunity to incorporate different re-centering technologies in parallel provides the designer with a wider range of realignment strategies and a broader spectrum of target references than those utilized in common practice. The symbolic hysteretic plots of Fig. 3(b) are limited in nature to simple RWMFs with no initial imperfections, $P$-delta effects or sinking of the supports.

Figure 3. (a) Component static hysteretic response, (b) System static hysteretic response (*not to scale*)

4. CONCEPTUAL DEVELOPMENTS

The conceptual developments presented herein are based upon four simple principles, that:

- The net effects of all lateral forces, including those of the supplementary devices can be expressed in terms of their equivalent overturning moments. Conversely, the effect of any global moment can be simulated by an equivalent notional horizontal force acting at roof level,

- In design led development the structure and its elements are selected to perform as expected rather than tested for the same requirements [34].
• the stiffnesses of the supplementary devices work in parallel with the stiffness of the MF, and that:
• the viability of RWMFs as earthquake resistant system has been verified by time-history analysis [6]

Several technical as well as modeling issues can influence the development of a continuum formula for the elasto-plastic design of RWMFs during and after strong ground motion. These include but are not limited to the contributions of the supplementary devices, initial imperfections, settlements of supports, the P-delta effects, work point offsets, incompatible displacements between rocking and stationary components, loss of pre-stressing and inaccuracies involved in the backbone models, etc.

4.1 Initial imperfections
Pre-service out-of-plumbness of frames and out-of-levelness of supports are generally looked upon as initial imperfections, whereas the same effects caused by external forces are construed as signs of damage and or failure. In both cases, such deformations are best represented as vertical and/or horizontal drift ratios. GBRMF-RRC combinations are particularly sensitive to out-of-straightness, $\phi_0$, Table 1 - row 1, and sinking of supports $\phi_S$, Table 1 - row 8. These phenomena tend to accelerate the P-delta effects and hinder the collapse prevention and realignment processes. The assumption made here is that the initial imperfections are confined to the MF, and that the RRC is perfectly plumb. However, due to practical limitations, only one case of pre-service out-of-straightness and an independent case of tipping of supports due to lateral forces is examined in the present paper. The displacement and moment amplification factor $f_{cr}$, commonly utilized in elastic design manifests itself as the capacity reduction factor in limit state analysis. As illustrated in Table 1 - row 1, an out-of-plumb module under external forces $P$ and $V$ can be modeled as a perfectly straight equivalent frame under the same forces plus a notional lateral force $P\phi_0$, or as an equivalent reduced capacity notional frame with modified member properties $M_{Notional}^P = f_{cr}M^P$ and $N_{Notional}^P = f_{cr}N^P$, $f_{cr} = (1 - \mu)$, where $\mu = PH / K_F$ [35]. The permanent gravity load tends to increase the initial drift $\phi_0$ by as much as $\mu\phi_0 / (1 - \mu)$, thus:

$$\phi = \frac{PH(\phi_0 + \phi)}{K_F} = \frac{\mu\phi_0}{1 - \mu} \quad \text{and} \quad \phi_0 + \phi = \frac{\phi_0}{1 - \mu} \quad (10)$$

It is instructive to note that $M_0 = VH$ occurs after $P$ has given rise to $\phi$. $\phi_0$ is the total initial drift of the system before the activation of the supplementary devices and the sinking of the support. Following Eqs. (1) and (10), the corresponding drift ratio could be assessed as:

$$\phi = \frac{VH + PH(\phi_0 + \phi)}{K_F} = \frac{M_0}{f_{cr}K_F} + \frac{\mu\phi_0}{f_{cr}^2} \quad (11)$$
The total drift generated by all effects up to this stage can now be computed as:

\[
\bar{\phi} = \phi + \bar{\phi}_0 = \frac{M_0}{f_{cr}K_F} + \frac{\mu\phi_0}{f_{cr}^2} + \frac{\phi_0}{f_{cr}f_{cf}} = \frac{M_0}{f_{cr}K_F} + \frac{\phi_0}{f_{cr}^2}
\]  

(12)

This equation clearly describes the effects of \( P \) and \( \phi_0 \) on the response of the unsupplemented RWMFs. For \( P = \phi_0 = 0 \), Eq. (12) reduces to the basic Eq. (4) and corresponds to segment \( ab \) of Fig. 3(a). The contributions of the LBs, BRBs, RRC and other supplementary devices to the global stiffness of the subject module are studied under sections 4.2 through 4.4 below.

4.2 Gap opening link beam response

Gap opening LBs, Fig. 1(b), are prismatic, PT, collector elements that are designed to hold the RRC in place, reduce stresses on the MF, transfer interactive forces between the MF and RRCs, dissipate seismic energy and provide leverage for self-centering and collapse prevention. Despite their well publicized applications, the practical detailing of gap opening beams needs special considerations to mitigate unaccounted damage to the structure. Such damage is caused by the tilting or expansion of the rocking beams that impart unforeseen tresses and strains to the columns and floors of the system. The most practical and cost effective solution to this problem is due to Dowden and Bruneau [19] who recommend using discontinuous PT rods, anchored within the beam and replacing the traditional full contact end surfaces with top flange bearing ends. The response of gap opening LBs is sensitive to their geometries, cable layout, initial pre-stressing as well as their end offsets from the center lines of the adjoining walls and/or columns. However, the LBs of Fig. 4 have been developed to overcome such problems and achieve greater self-centering efficiency. The proposed PT profiles increase tendon extensions and reduce stressing losses due to decompression. The chamfered ends reduce initial joint stiffness to zero. Gap opening increases the tendon elastically and helps to realign the system after loading is removed. The slotted bolt-holes of Fig. 4(a) and the pivoting supports of Fig. 4(b) have been devised to facilitate the free rotations of the GOLBs. Lateral stability is provided by means of a single articulated connection, such as that shown in Fig. 5(c), installed at the rocking center of the GOLB, a distance \( \bar{y} = \bar{D}_{yl} + (D_{rt} + D_{yt}) \) from the center line of the left hand column. It is instructive to note that the coupling of the stressed tendons and the rigid core can be represented by equivalent rotational elastic springs of stiffness \( K_L \). The LB is activated in response to any drift angle \( \phi \) caused by the external forces \( F \). Upon activation, LBs transmit axial forces and generate equal end couples that oppose the external overturning moments. These moments tend to tilt and bend the elements of the adjoining systems in proportion to their flexural rigidities. Typically, a RRC would tend to tilt in the same sense as the LB moments, as in Table 1, rows 3 and 4, and a MF would deform as a racking system in the same direction, as in Table 1, row 4. If the LB is rigid enough, it will force its hinged ends to rotate with respect to the face of the wall or column, see Fig. 4, by as much as:
\[ \frac{\alpha d}{2} = T_L L / A_L E_L. \]  
\( \alpha \phi \) is the adjusted \( \phi \) due to magnifying effects of \( D_{lft} \) and \( D_{rt} \), and is equal to:

\[ \frac{(D_{lft} + D_{rt} + \tilde{I})}{\tilde{I}} \phi = \alpha \phi \]  
(13)

where \( E_L \) is the elastic modulus of the unbonded tendons. \( d \) is the vertical distance between the center lines of upper and lower sets of the tendons, Figs. 4(a) and (b). \( T_L \) and \( A_L \) represent the additional tendon force due to gap opening \( (\alpha d \phi / 2) \) and its sectional area respectively. \( L_L \geq \tilde{I} \) is the effective length of the continuous floor level cable under consideration. The gap opening moment and the equivalent rotational stiffness of the LB can be related to each other by \( \phi = M_L / K_L \) where, \( M_L = T_L d / 2 \) and \( K_L = \alpha A_L E_L d^2 / 4 L_L \). However, since the MF is not rigid, the stiffness of the connection may decrease in proportion to the stiffness of the adjoining element. The drift reduction on the equivalent module due to opposing moments, \( M_L \) Table 1, row 4, can be computed as:

\[ \psi = \frac{(2M_L) L}{24EI} = \frac{2M_L}{K_F'} = \frac{2K_L \phi}{K_F'} = \bar{K}_L \phi \]  
(14)

\[ \bar{K}_L = (\alpha A_L E_L d^2 L / 48EI L_L), \]  
is the characteristic parameter of the LBs. \( K_L \) and \( \bar{K}_L \) indicate that the re-centering ability of the LBs depend on the length, \( L_L \) of its tendons. The net effect of the LB moments \( M_L \) on the combined structure can be expressed as:
The first term, \( (M_0 + M_{PB} - 2M_L) / K_F \), describes the net effect of the external and LB moments acting on the RRC, whereas the second term, \( 2\phi K_L / K'_F \), describes the effects of the LB moments acting directly on the MF. Eq. (15) can be simplified to read:

\[
\phi = \frac{M_0 + PH\bar{\phi}_0}{(1 - \mu + K_L)K_F + 2K_L}
\]  

Here, \( (1 + K_L)K_F \) may be regarded as the adjusted stiffness of the module due to the added stiffness of the LBs. The net contribution of the LBs to the system in the absence of \( P \) can be computed as:

\[
\phi = \frac{M_0}{(1 + K_L)K_F + 2K_L}
\]  

Following Eq. (17) the total horizontal tendon force composed of initial tendon force \( T_{L,0} \) and that due to additional extensions can be shown to be equal to:

\[
T_L = T_{L,0} + d \left[ \frac{(E_F/A_F, / L_L)(E_L/A_L, / I_L)}{(E_F/A_F, / L_L) + (E_L/A_L, / I_L)} \right] a\phi
\]  

Subscript \( Fr \) refers to frame member containing the horizontal tendons of the LBs. It is instructive to note that as plastic hinges form at the ends of the beams, the relative stiffnesses, \( (I/L) \) of the beams become zero. In theoretical terms, while the rotational springs hold the module together, the stiffness of the MF tends toward zero and Eq. (16) reduces to:

\[
\phi = \frac{M_0^p + PH\bar{\phi}_0}{4K_L}
\]  

i.e., the two LBs of Fig. 2(c) sustain the total destabilizing moment \( M_0^p \) acting on the module. The significance of Eq. (19) is in that it demonstrates the ability of the LBs in providing kinematic stability for the failing MF, a hint that similar measures may help prevent catastrophic collapse of the entire system due to strong ground motion. The total accumulative drift including \( \bar{\phi}_0 \) can be computed as:

\[
\bar{\phi} = \phi + \bar{\phi}_0 = \frac{M_0 + [(1 + K_L)K_F + 2K_L] \bar{\phi}_0}{[(1 - \mu + K_L)K_F + 2K_L]}
\]
Since the total resistance of the LBs, is independent of their locations, it makes sense to use identical LBs along the height of the structure. This concept is further examined in Example 2 below, where it is shown how the LB parameters of the prototype, Eq. (9), $M'_{L,i}, k_L$ and $A'_{L}$ can be related to the corresponding items, $M_L, K_L$ and $A_L$ respectively of the current basic model.

Example 2: Transformation of results from basic model to prototype

Compute the equivalent LB moments $M'_{L,i}$, stiffness $k_L$ and tendon area $A'_{L}$ of the RWMF of Example 1.

Solution: Assuming:

$$mM'_{L} = 2M_L$$

$M'_{L} = 2M_L / m$. It follows therefore that, $I' = 2I / mn$.

Example 3: Contributions of LBs to the strength and stiffness of the RWMF

Given: $\mu = 0.2$, $\phi_0 = 0.1 \phi_{all}$, utilize generic LBs to upgrade the MF of Example 1, for increased demand $\gamma M_0$ ($\gamma = 1.5 > 1$) and the same allowable drift ratio $\phi_{all}$.

Solution: $\phi_0 = \phi_{all} (1 - \mu) = 0.1(1 - 0.2) = 0.125$ radians. Eqs. (4) and (16) give, respectively:

$$\phi_{all} = \frac{M_0}{K_F}$$

and

$$\phi_{all} = \frac{\gamma M_0 + \mu K_F \phi_0}{[(1 - \mu + K_L)K_F + 2K_L]}$$

Equating the two equations for $\phi_{all}$, gives after simplification:

$$\frac{K_L}{K_F} = 0.5[\gamma + \frac{\beta \mu}{(1 - \mu)}(1 - \mu + K_L)].$$

$K_L$ is a small number compared with unity and can be ignored for the first practical trial. Therefore: $K_L = 0.5[1.5 + 0.025 - 0.2]K_F = 0.6625K_F$, i.e., the stiffness is from $K_F$ to over $2.13K_F$.

4.3 Rigid rocking core response

The main function of the RRC, Fig. 1(b), also referred to as the building spine, is to reduce seismic forces on the MF, prevent soft story failure, impose uniform drift along the height of the system and to provide support for the supplementary devices, etc. The high rigidity of the core causes all added devices to undergo equal rotations and to absorb proportional amounts of energy. The base pivot provides two degrees of restraints. The MF provides lateral restraint and makes the core act as a statically determinate element. This forces the reference line of displacements to pass through both the pin and the free end of the core. In addition, the RRC tends to redistribute seismic moments rather evenly between groups of similar members such as beams and columns of equal spans and heights respectively. The physical behavior of the RWMF can best be visualized by the MF and the RRC resisting the
lateral forces together until the MF becomes a mechanism. The core can be designed in conjunction with other devices to absorb the entire seismic load after the MF has been incapacitated. Seismic shear is transferred to the RRC through stressed tendons, LBs, BRBs, as well as specially detailed connections between the slab and the wall, Fig.5(a, b). The challenge is the mitigation of seismic damage at RRC/diaphragm interfaces, where the incompatible movements of the two may disrupt the role of the RRC. Eatherton (2010) has suggested an innovative yoke and roller scheme which isolates the RRC from the gravity system. The author proposes the simple connections of Figs. 5(a) and 5(c) that are detailed to prevent the slab, the wall and the LBs from being damaged during earthquakes. Detail 5(a) allows horizontal shear transfer from slab to wall without inhibiting the rocking action of the wall at the same junction. The detail also provides out of plane stability at all floor levels. A typical RRC base detail is shown in Fig. 5(d). The lateral restoring capabilities of free standing RRCs are defined by their ultimate strength and base level rotational stiffness. The main function of the unbonded tendons, Figs.1 (b) and 5(d) is to stabilize and add strength and stiffness to the RRC. The pair of parallel tendons and the pivot at the base constitutes a rotational spring of stiffness $K_C$ designed to remain elastic during and after a major seismic event. In other words, instead of using axial springs at the ends of the RRC, an equivalent rotational scheme has been utilized to capture the restraining effects of the activated tendons. The stress-strain relationship of the wall base spring at any loading stage can be expressed as a linear function of $\phi = M_C / K_C$, where $M_C$ represents the moment of resistance of the equivalent spring due to $\phi$. Strain compatibility between tendon stretching and spring rotation requires that $\phi d' = cH$. Substituting for $c = T_C / A_c E_C$ and $M_C = T_C d'$ in the strain equation gives, $K_C = d'^2 A_c E_C / H$. In reality, $T_C$ should be large enough to return the collapsing MF to its original position. In order to appreciate the contribution of the RRC in the absence of other auxiliary devices, consider the static interaction of the single module and its core, as illustrated in Table1 row 5, Figs.1(d) and (e), which give:

$$\phi = \frac{M_0 + PH\phi_b}{(1 - \mu)K_F + K_C} \quad \text{and} \quad Q = \frac{M_C}{H} = \frac{K_C \phi}{H}$$

(22)

Figure 5. (a, b) Wall-slab shear connection, (c) LB to slab connection, (d) Rocking wall base and cables

Note that the magnitude of the interactive force $Q$ is also influenced by the gravity forces
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as well as the initial imperfection \( \phi_0 \). Since stiﬀnesses \( K_C \) and \( K_F \) are parallel and similar in nature, then Eq. (16) can be amended to reﬂect the contribution of the RRC to the response of the proposed RWWMF; as:

\[
\phi = \frac{M_0 + PH\bar{\phi}_0}{[(1 - \mu + \bar{K}_L)K_F + 2K_L + K_C]}
\]  

(23)

Similarly Eq. (20) can be amended to reﬂect the total accumulative drift including \( \bar{\phi}_0 \) as:

\[
\bar{\phi} = \phi + \bar{\phi}_0 = \frac{M_0 + [1 + \bar{K}_L]K_F + 2K_L + K_C]\bar{\phi}_0}{[(1 - \mu + \bar{K}_L)K_F + 2K_L + K_C]}
\]  

(24)

Realignment is sensitive to the magnitude of the PT forces. Therefore, for \( K_C \) to be effective, the shortening eﬀects of all prestressed members should be taken into consideration. Following activation of \( \phi \), tendon elongation produces an increase in strand force, which, in turn, causes the RRC to shorten/

The total tendon force composed of initial tendon force \( T_{0C} \) and that due to additional extensions can be shown to be equal to:

\[
T_C = T_{0C} + 2d\left[\frac{(E_A A_C / H)(E_C A_C / L_C)}{(E_A A_A / H) + (E_C A_C / L_C)}\right] \phi
\]  

(25)

Suffix \( W \) refers to the RRC. Once the RRC forces are known it can be designed for strength and stiﬀness as required. The RRC should be suﬃciently strong and rigid to perform its functions as part of the lateral load resisting system. However, as a precautionary measure it would be safe to assume that the RRC alone is capable of withstanding the ultimate earthquake induced and \( P-\delta \) effects.

**Example 4: Preliminary design of RRC**

Provide design formulae for the RRC of Example 1. All the Example 1 data are applicable.

Solution: Since the RRC acts as an upright simply supported beam or braced frame, the corresponding distribution of bending moments due to \( F_i \) and \( \bar{F_i} = P\phi = \bar{F} \) at any elevation \( i \times h \) can be expressed as:

\[
M_{cor,i} = \frac{F_i h}{6m} [m^2 - 1 - (i^2 - 1)] + \frac{F_i h}{2} [(m - 1) - (i - 1)]
\]  

(26)

If the strength design of the core is based on Eq. (26) and reactions of Example 1, then no
soft story failure of the MF can take place. To establish a sufficiently high rigidity, $E_w I_w$, for
the RRC, its maximum drift ratio $\varphi_w$ can be related to a fraction of the prescribed uniform

drift $\phi$ i.e., $\varphi_{\text{max}} = \alpha \phi$. The minimum equivalent section modulus of the core due to $F_i$ and
$\bar{F}_i$ can be expressed as:

$$\varphi_{\text{max}} = \frac{F_i h^2 (m-1)(2m-1)(2m^2 + 3m - 4)}{180 E_w I_w m} + \frac{\bar{F} h^2 (m-1)(m^2 + m - 2)}{24 E_w I_w}$$

$$I_{\text{min}} = \frac{F_i h^2 (m-1)(2m-1)(2m^2 + 3m - 4)}{180 E_w m\phi m} + \frac{\bar{F} h^2 (m-1)(m^2 + m - 2)}{24 E_w m\phi}$$

(27)

(28)

4.4 Use of buckling restrained braces

The parallel combination of MFs and BRB frames is included as a dual system option in
ASCE 7. The purpose of this section is to assess the contribution of the diagonal BRBs
of Fig. 1 (a) and Table1 row 6, to the global stiffness of the RWMF. The imaginary
braced frame may be construed as being composed of the right hand column of the
module and the left hand face of the RRC as its vertical chords, and the LBs and BRBs
as its horizontal and diagonal elements respectively, Fig. 2(d). In the current scheme the
BRBs are not considered as resisting gravity forces. The RRC and the MF undergo equal
drifts $\varphi$. As a result, each subframe, such as those shown in Figs. 1(b) and 2(d),
displaces an amount $\Delta_i = \phi h_i$, with respect to its lower chord. The challenge here is to
relate the brace force $T_{B,i}$ to the uniform drift ratio $\phi$. Following the principles of design
led analysis, the response of the BRBs can be related to the characteristics of the system
rather than the external loading. This is achieved by assuming all members of the MF are
axially rigid. The axial deformation $\delta_i$ of any such brace can be related to the uniform
drift ratio $\phi$, i.e.

$$\delta = \frac{\alpha \phi H \tilde{H}}{L_B} = \frac{T_B L_B}{A_B E_B}, \quad T_B = \frac{\alpha \phi H \tilde{H} A_B E_B}{L_B^2} \quad \text{or} \quad A_B = \frac{T_B L_B^2}{\alpha \phi H \tilde{H}}$$

(29)

$L_B$ is the effective length of the brace. $F_B = T_B \cos \theta = T_B \tilde{H} / L_B$, represent the equivalent
notional lateral shear force corresponding to BRB force $T_B$ of Table1 row 6. Next, consider
the relationship between the notional overturning moment $M_B = F_B H$ and the module
rotation $\phi$. Substituting for $F_B$ and rearranging, it gives:

$$M_B = \frac{T_B \tilde{H} H}{L_B} = \frac{\alpha H \tilde{H}^2 A_B E_B \phi}{L_B^3} = K_B \phi \quad \text{or} \quad \phi = \frac{M_B}{K_B}$$

(30)
This implies that the braced frame tends to oppose the external overturning moment by a notional global moment of resistance related to the axial resistance of the BRBs. Following Eq. (23), it gives:

$$
\phi = \frac{M_0 + PH\bar{\phi}_0}{[(1-\mu + \bar{K}_L)K_F + 2\bar{K}_L + K_C + K_B]}
$$

Following Eq. (24), Eq. (31) can be amended to reflect the total accumulative drift including $\bar{\phi}_0$ as:

$$
\bar{\phi} = \phi + \bar{\phi}_0 = \frac{M_0 + [(1 + \bar{K}_L)K_F + 2\bar{K}_L + K_C + K_B]\bar{\phi}_0}{[(1-\mu + \bar{K}_L)K_F + 2\bar{K}_L + K_C + K_B]}
$$

The response of the BRBs depends upon their location within the braced frame. Following Eq. (30), the characteristics of the brace at level $i$ can be expressed in terms of the single design variables $A_{B,i}$ as:

$$
T_{B,i} = \frac{\alpha \phi_i l_i A_{B,i} E_B}{L_{B,i}^2} \text{ or } A_{B,i} = \frac{T_{B,i} L_{B,i}^2}{\alpha \phi_i l_i E_B} \text{ and } F_{B,i} = T_{B,i} \cos \theta_i - \sum_{r=i+1}^{m-1} F_{B,r}
$$

$F_{B,i}$ is a function of $A_{B,i}$ and should be determined in such a way as to satisfy the notional moment equation; $M_B \leq \sum_{i=1}^{m} F_{B,i} x_i$, which is a simple task and can be performed in minutes. However, since in practice story heights are almost equal, i.e. $h_i = h$ and $L_{B,i} = L'$, it would be convenient to select $A_{B,i} = A_B'$ and $T_{B,i} = T_B$. This in turn leads to $F_{B,m} = \alpha \phi h T^2 A_B' E_B / L_B^3$ and $F_{B,i} = 0$ for $i < m$.

**Example 5: Preliminary design of BRBs**

Consider the MF of Example 1 with pin ended beams and no supplementary devices. Provide a temporary bracing system that is ¼ as strong and as stiff as the original MF.

**Solution**: The solution requires that $K_B = K_F / 4$ and $M_B^p = M_F^p / 4$. It follows from Solution 1 that:

$$
\frac{1}{K_B} = \frac{1}{3mnE} \left[ \frac{h}{J} + \frac{l}{I} \right], \quad M_B^p = \frac{mnM_F^p}{2} \quad \text{and} \quad T_B^p = \frac{M_B^p}{4H} = \frac{nM_F^p}{2h}
$$

4.5 Upgrading for target demand

For $P = \phi_0 = 0$, the elastic global stiffness of the RWMF in the elastic range can be
condensed as:

\[ K^* = \left[ (1 + K_L)K_F + 2K_L + K_C + K_B + K_M \right] \]

where, \( K_M \) represents the stiffness of all other or miscellaneous devices. It can be readily observed from Eq. (35) that the use of LBs, BRBs, RRCs and other auxiliary devices, such as viscous fluid or upgrading data \( P \), \( \phi_{all} \), \( \bar{\phi}_0 = \beta\phi_{all} \) and \( \gamma M_0 \) are known, then Eqs. (34) and (35) can be rearranged as:

\[ \phi_{all} = \frac{\gamma M_0 + \mu K_F \bar{\phi}_0}{K^* - \mu K_F} \quad \text{and} \quad \bar{\phi}_{all} = \frac{\gamma M_0 + K^* \bar{\phi}_0}{K^* - \mu K_F} \]

The contribution of the supplementary devices can be assessed by equating Eqs. (36) and (12), i.e.,

\[ \gamma = \frac{K^* - \mu (1 + \beta)K_F}{K_F - \mu (1 + \beta)K_F} \]

5. PLASTIC STATE AND COLLAPSE PREVENTION

Seismic collapse is generally triggered the \( P \)-delta phenomenon, preceded by the ductile failure of the MF. Plastic failure mechanisms often undergo large lateral displacements that lead to structural catastrophic collapse. A flexible foundation will also affect the global stability of a RWMF by reducing the overall stiffness of the structure. While gravity forces, as component of the \( P \)-delta effect, are constant quantities, lateral displacements can be controlled, even reversed by means of RWMF technologies as suggested by Eqs. (36) through (38). Collapse prevention and realignment are controlled design conditions that occur after strong ground motion. The premise is that the response of the RWMF is not affected by the flexibility of the foundations and that there are several sets of supplementary devices that are capable of preventing catastrophic collapse and pulling the system back to its original position. And if this is the case, then all or most of the supplementary devices should be capable of sustaining the deformations of the MF without yielding or loss of stiffness. \( \phi_{all} \) becomes \( \phi_{F} \) at first yield. Eqs. (36) are modified again to reflect \( \phi_{F} \) and \( \bar{\phi}_{F} \) at first yield, point 2, Fig. 3(b), i.e.:

\[ \phi_{F} = \frac{M_0^p + PH(\bar{\phi}_0 + \phi)}{K^*} = \frac{M_0^p}{K^* - \mu (1 + \beta_{F})K_F} \quad \text{and} \quad \bar{\phi}_{F} = \frac{M_0^p + K^* \bar{\phi}_0}{K^* - \mu K_F} \]

\( \bar{\phi}_0 = \beta_{F} \phi_{F} \cdot M_0^p \) is the moment that initiates first yield in the MF. The proportion of the external moments absorbed by the supplementary devices can be estimated as:
$M_C = K_C \phi_{fy}$, $M_L = K_L \phi_{fy}$, $M_B = K_B \phi_{fy}$ and $M_M = K_M \phi_{fy}$. The moment needed to induce plasticity in the MF is equal to:

$$M_p^R = [1 - \mu(1 + \beta_{fy}) + \bar{K}_L]K_F \phi_{fy} \quad (39)$$

Ductile MFs absorb considerable amounts of seismic energy through hysteretic deformations. Ductile deformations tend to accelerate the $P$-delta effects and result in residual deformations after major seismic events, and as such can adversely affect the self alignment and collapse inhibiting processes. Consider the plastic failure of the fully supplemented MF of Fig. 2(a) or its equivalent Fig. 2(c). The objective here is to first estimate the maximum elastoplastic displacement of the system after first yield and then to compute the restoring force needed to return the RWMF to its original position after strong ground motion. As the initial stiffness of the MF diminishes, $K_F$ becomes zero, and $K^*$ reduces to $K_R^* = (4K_L + K_C + K_B)$. $\bar{\phi}_{fy}$ becomes initial drift for segment 2-7 of Fig. 3(b). The corresponding drift equation can be expressed as:

$$\phi_{2-7} = \bar{\phi}_{fy} + (M_0 + M_{FA}) / K^*_R, \text{ or: } \phi_{2-7} = \frac{M_0 + (K^*_R + \mu K_F) \bar{\phi}_{fy}}{K^*_R - \mu K_F} \quad (40)$$

Introducing $\bar{\phi}_{fy} = \beta_y \phi_{yield}$ at $\phi_{2-7} = \phi_{yield}$, equation (37) can be re-written in the familiar form:

$$\overline{\phi}_{yield} = \frac{M_0}{[(1 + \beta_y)K^*_R - (1 - \beta_y)\mu K_F]} \quad (41)$$

As the monotonic loading increases, $K^*$ diminishes to $K^*_R = (4K_L + K_C + K_{FA})$. However, the designer may wish to further exploit the strength of the PT systems by allowing $K^*$ to degrade to $K^*_R = 4K_L + K_{FA}$ or $K^*_R = K_C + K_{FA}$, or whatever he/she deems appropriate for the case under study. The basic assumption made here is that there are no unaccounted residual deformations and that $\overline{\phi}_{yield}$ serves as the initial out-of-straightness of the surviving combination. It follows that the additional post-yield moment, $\partial M_0^R$, extends the ductile drift ratio to $(\nu + 1)\phi_{yield}$, where $\nu$ may be regarded as the equivalent ductility ratio of the system. Self centering occurs when $(\nu + 1)\phi_{yield}$ is overcome by the resilience of the tendons and the supplementary devices. Following Eqs. (38) the moment-drift relationship of segment 2-7 and the maximum system drift at point 7 of the hysteretic response curve of Fig. 3(b) can be written down by inspection as:
\[
\nu \phi_{yield} = \frac{\partial M_0^p + \mu K_F \phi_{yield}}{[K^*_R - \mu K_F]} \quad \text{and} \quad \bar{\phi}_{\text{max}} = \phi_{yield} + \nu \phi_{yield} = \frac{\partial M_0^p + K^*_R \phi_{yield}}{[K^*_R - \mu K_F]}
\]  

(42)

The maximum external moment corresponding to total drift \( \bar{\phi}_{\text{max}} \) can now be computed as:

\[
M_{\text{max}} = M_0^p + \partial M_0^p = [K^*_R + \nu K_R^* - \mu(1 + \nu + 2\beta)K_F] \phi_{yield}
\]  

(43)

**Example 6: Determination of the restoration factor**

Consider the plastic failure mode of the MF of Example 1, Fig. 3(a) or (c). Compute the tendon force needed to restore the RWMF to its original position after strong ground motion. Assume there are no residual displacements, the target drift is \( \nu \phi_{yield} \), \( P = \phi_0 = \mu = 0 \) and \( K_{Fst} = 0 \).

**Solution:** \( K_B = K_L = K_C = K_M = 0 \), i.e., \( K^*_R = [K_F + K_C] \). \( M_C^Y > M_F^p \) to prevent collapse. Eq. (38) gives \( \phi_{yield} = M_0^p / (K_F + K_C) \), at first yield. \( \phi_{yield} \) increases to \( \nu \phi_{yield} \). The proportion of the moment absorbed by the RRC is \( M_C = \nu K_C M_F / K_F \). Realignment takes place entirely elastically. The restoration process is enforced by the activated RRC against the frame resistance \( K_F \). Comparing the magnitudes of the restoring and resisting moments after the seismic event, it provides a measure of the re-centering capabilities of the system as:

\[
SC = \frac{\nu K_C M_F}{K_F M_F} = \frac{\nu K_C}{K_F} \quad \text{and} \quad T_C = \frac{\Omega M_C}{d'} = \frac{\Omega \nu K_C M_F}{d' K_F}
\]  

(44)

**6. KEY DESIGN ISSUES**

A discussion of the design concepts for controlled response of self-centering RWMF is outside the scope of the current article. However, some of the key design concepts are briefly touched upon in this section. A brief account of rocking frame design issues can be found in reference [10]. The satisfactory design of efficient, self-centering RWMFs is governed by four independent but related criteria:

- Determination of system strength and stiffness in accordance with the requirements of the prevailing codes of practice. In the current US practice, chapter 18 of ASCE 7 [36] is used for the design of earthquake resisting structures with energy dissipating devices, where specific guidelines have been provided for the modeling of displacement and velocity-dependent components. BRBs are modeled as regular structural components. For velocity-dependent devices a two step procedure is recommended. First, the components
of the MF are designed for strength only, using the base shear as given in Section 18.2.2.1, and the strength requirements of Section 12.2.1. Next, the damping system is selected to meet the prescribed criteria for the drift ratio, as computed from design earthquake ground motion.

- Given the overturning moment $M_0$, related to the maximum considered earthquake (MCE), the nominal moment of resistance $M_R^Y$, provided by the MF and the supplementary components, can be estimated as: $M_0 \leq \xi M_R^Y$. Here $\xi$ is a code assigned resistance factor typically taken as 0.9 in the American practice.

- Determination of system resistance to prevent plastic collapse of the MF. Briefly stated, collapse prevention is a function of the inequality $M_R^Y > M_0 > M_F^p$. i.e., if the applied moment $M_0$ exceeds the plastic moment of resistance of the MF, but is less than the total moment of resistance $M_R^Y$, then the incapacitated MF will not fail through a plastic collapse mechanism [36].

- Determination of system adequacy for self-centering. Self-centering capability is assessed by the commonly accepted self-centering index, $SC = (M_R = M_C)/M_F^p$, where $M_R$ is the available effective moment of resistance after loading is removed. Observing that $M_R^Y = M_F^p + M_C$ and $M_0 \leq \xi M_R^Y$, it can be shown that:

$$M_C \geq \frac{SCM_0}{(1+SC)\varphi}$$ (45)

- Determination of the energy dissipation ratio $ED = \alpha_d M_F^p / M_R^Y$. $ED$ is an index that is used to control strength degradation during strong ground motion. $\alpha_d$ is the degrading element component factor. Currently there are no specific guidelines for the lower limit on $\alpha_d$ in relation with RWMF design. $ED = 0.25$ as suggested in [10] is recommended for preliminary design purposes: Assuming $\alpha_d = 1$ for the subject MF, then it may be shown from $(ED = 0.25)(M_F^p + M_C) = M_F^p$, that $M_C \geq 3M_F^p$. At this stage foundations are still intact and damage is limited to formation of plastic hinges at beam ends and replaceable supplementary devices, except the RRC tendons and the BRBs. There are cases where collapse prevention can not lead to automatic realignment, e.g., tipping of supports and/or loss of reserve resilience.

### 7. EFFECTS OF FOUNDATION SETTLEMENT

Settlement of supports of earthquake resisting systems is a major concern that has not been adequately addressed in the literature [37]. However, the issue is important enough to warrant a brief introduction as part of the current article. The scope of the present section is limited to the most common type of foundation failure where the entire MF tends to undergo
a rigid body tilt in the same sense as the initial imperfections, Fig. (8), Table 1. While the sinking of supports has little to no effect on the ultimate carrying capacity of the RWMF, uniform tilting of the supports can exaggerate the P-delta effects and hinder collapse prevention and post earthquake realignment. Since the magnitude of the plastic failure load is independent of the actual rigidities of the joints, then Eqs.(38), (39) and (40) can be utilized directly by treating the rotation of the supports as additional angular out of straightness $\pm \phi_j$, in which case: $\phi_0 \rightarrow (\phi_0 + \phi_j)$ and $\phi_0 \rightarrow \phi_{0y} = (\phi_0 + \phi_j)/(1 - \mu_j)$. Two simple scenarios with regards to footing failure can take place.

**Table 1: Summary- development of RWMF formulae**

<table>
<thead>
<tr>
<th>MODEL AND LOADING</th>
<th>DESCRIPTION</th>
<th>MOMENT</th>
<th>STIFFNESS</th>
<th>$\phi$</th>
<th>$\bar{\phi} = \sum \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initial imperfection and $P$-delta Effect</td>
<td>$M_{ps} = P(\phi_0 + \phi)H$</td>
<td>$K_f \mu$</td>
<td>$\frac{M_{ps}}{K_f}$</td>
<td>$\phi = \frac{M_{ps}}{K_f}$</td>
</tr>
<tr>
<td>2</td>
<td>Lateral Force</td>
<td>$M_{l} = VH$</td>
<td>$K_f$</td>
<td>$\frac{M_{l}}{K_f}$</td>
<td>$\bar{\phi} = \frac{M_{l}}{K_f}$</td>
</tr>
<tr>
<td>3</td>
<td>Link Beam Moments on Wall</td>
<td>$2M_{l} = 2\frac{Fy}{d} - K_f \phi$</td>
<td>$K_f$</td>
<td>$2\frac{M_{l}}{K_f}$</td>
<td>$\bar{\phi} = \frac{M_{l} + \lambda_{l} + 2\lambda_{l}}{2\lambda_{l}}$</td>
</tr>
<tr>
<td>4</td>
<td>Link Beam Moments on Frame</td>
<td>$2M_{l} = 2\frac{Fy}{d} - K_f \phi$</td>
<td>$K_f$</td>
<td>$\frac{M_{l}}{K_f}$</td>
<td>$\phi = 2\lambda_{l} + \lambda_{l} + \frac{2\lambda_{l}}{K_f}$</td>
</tr>
<tr>
<td>5</td>
<td>Wall Tendon Moments on Wall</td>
<td>$M_{c} = T_c d^2 - K_c \phi$</td>
<td>$K_c$</td>
<td>$\frac{M_{c}}{K_c}$</td>
<td>$\phi = \frac{M_{c} + \lambda_{c} + 2\lambda_{c}}{2\lambda_{c}}$</td>
</tr>
<tr>
<td>6</td>
<td>Equivalent Frame Moments</td>
<td>$M_{e} = \frac{TH \cos \theta - K_d \phi}{H}$</td>
<td>$K_d$</td>
<td>$\frac{M_{e}}{K_d}$</td>
<td>$\bar{\phi} = \frac{M_{e} + \lambda_{e} + 2\lambda_{e}}{2\lambda_{e}}$</td>
</tr>
<tr>
<td>7</td>
<td>RWMF at Frame Failure</td>
<td>$M_{e} + M_{ps}$</td>
<td>$K_f^*$</td>
<td>$\frac{M_{e} + M_{ps}}{K_f^*}$</td>
<td>$\phi = \frac{M_{e} + M_{ps}}{K_f^*}$</td>
</tr>
<tr>
<td>8</td>
<td>Topping of Supports, Failure of Braces</td>
<td>$M_{e} = \frac{K_f^*}{K_f} \phi H$</td>
<td></td>
<td>$\phi = \frac{M_{e} + M_{ps}}{K_f^<em>} \frac{K_f^</em>}{K_f} \phi H$</td>
<td>$\bar{\phi} = \frac{M_{e} + M_{ps}}{K_f^<em>} \frac{K_f^</em>}{K_f} \phi H$</td>
</tr>
</tbody>
</table>

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7.1 Frame failure due to settlement of supports
If premature tipping of the supports leads to plastic failure of the MF, while supplementary
devices remain intact, Table 1, row 8, then following Eq. (38), the resulting drift ratio may be
estimated as:

$$
\phi_f = \frac{M_0^p + P H (\bar{\phi}_{0f} + \phi)}{K^*} = \frac{M_0^p}{K^* - \mu(1 + \beta_f) K_F} \quad \text{and} \quad \bar{\phi}_f = \frac{M_0^p + K^* \bar{\phi}_{0f}}{K^* - \mu K_F}
$$

(46)

where, $\bar{\phi}_{0f} = \beta_f \phi_f$. However if yielding of the tendons takes place at the same time as
the tipping of the supports, it may still be possible to prevent catastrophic collapse with little
to no options for realignment.

7.2 Settlement of supports after frame failure
As an alternative scenario consider the rotation of the footings after the MF has failed. The
maximum drift of the RWMF before the sinking of the support is given by Eq. (38). The
total out-of-straightness after the sinking of supports is equal to $\phi_{0s} = \phi_s + \bar{\phi}_{0f}$. The
additional moment $\partial M_0$, generated within the structure can be related to $\phi_s$ as:

$$
\phi_s = \frac{\partial M_0 + P H (\phi_{0s} + \phi)}{K_R^*} = \frac{\partial M_0}{K_R^* - \mu(1 + \beta_s) K_F}
$$

(47)

Here $\phi_{0s} = \beta_s \phi_s$. If collapse prevention is to be activated then $M_R^Y > M_0^p + \partial M_0, M_0^p$ and
$\partial M_0$ can be extracted from Eq. (38) and (47) respectively. Therefore:

$$
M_R^Y > [K^* - \mu(1 + \beta_f) K_F] \phi_{f_y} + [K_R^* - \mu(1 + \beta_s) K_F] \phi_s
$$

(48)

Example 7: Collapse prevention after sinking of supports
Consider the sinking of the supports of RWMF of Example 5 after plastic failure of the
MF. Given; $\mu = 0.1, \phi_0 = 0.0015, \phi_s = 0.002, \phi_{0s} = 0.0035, K_L = K_B = \bar{K}_L = 0,$
$K^* = K_F + K_C$ and $K_R^* = K_C$. Eq. (10) gives $\bar{\phi}_0 = 0.0015/0.9 = 0.00167$. From section
5.1; $\bar{\phi}_0 = \beta_f \phi_f$, i.e., 0.00167 = $\beta_f$ 0.0035 or $\beta_f = 0.4762$. Similarly
$\phi_{0s} = \phi_s + \bar{\phi}_{0f} = 0.0015 + 0.0035 = 0.005$. $\phi_{0s} = \beta_s \phi_s$ gives 0.005 = $\beta_s$ 0.002, i.e.,

$$
\beta_s = 2.5. \text{Eq. (45) gives} \quad T d l / 2 = [0.85238 K_F + K_C] 0.0035 + [K_C - 0.35 K_F] 0.002 =
$$

0.00228333K_F + 0.0055 K_C. The accuracy of this result is verified by Eq. (36), i.e.,

$$
M_F^p = [1 - 0.14762] 0.0035 K_F = 0.00298333 K_F.
$$
8. CONCLUSIONS

A new earthquake resistant RWMF, consisting of a GBRMF attached to a co-planar, post-tensioned RRC, by means of BRBs and gap opening LBs has been introduced. The proposed system is capable of damage reduction, collapse prevention and self-centering due to strong ground motion. The practical advantages of the proposed configuration over fixed base dual systems can be summarized as follows:

- The proposed RWMF, with or without supplementary devices, tends to deform with zero to negligible drift concentration along its height.
- The proposed RWMF tends to prevent soft story failure and the formation of column base plastic hinges [38].
- RRCs help generate seismic load profiles that are similar to code specified triangular distribution.
- The proposed RWMFs lend themselves well to self centering, with collapse prevention and damage avoidance strategies.
- The proposed RRCs reduce residual stresses and deformations due to strong ground motion.
- Overturning moments are transmitted to the footings only through axial reactions. No anchor bolt, base plate and footing damage can occur due to seismic action.
- In well proportioned RWMFs, the strength and ductility of all beams can be mobilized fully against seismic forces.
- The RRC tends to bend as a simply supported beam rather than a cantilever.
- Gap openings at the ends of the GOLBs and at the bottom of the RRC add natural damping and provide opportunities for self centering, damage reduction and collapse prevention.
- Gap opening post-tensioned LBs tend to reduce frame moments and drifts.
- RRCs can be used as elements of structural control for pre and post-earthquake conditions.
- The drift is not sensitive to minor changes in wall and supplementary device stiffnesses.
- In GBRMFs no moments are transmitted to the footings. The grade beams prevent the formation of plastic hinges at column supports. The grade beam provides means of controlling column base rotation and overall drift.
- For equal mass, RWMFs have longer natural periods of vibration and attract significantly smaller seismic forces than their conventional counterparts.
- RRCs suppress contributions of higher modes of vibrations. In other words the dominant mode shape remains unchanged during all phases of loading.
- Rocking walls tend to rotate as rigid bodies without significant in-plane deformations.
- The normalized displacement function is a straight line and remains unchanged throughout the loading history of the structure. Loss of stiffness changes only the value of the drift angle but not the drift profile.
- The proposed system is suitable for nonlinear static analysis.
- The displacement profile remains a function of the single variable for all loading conditions.
• The structure is a SDOF system, and as such lends itself well to equivalent energy studies.
• The lateral displacements of well proportioned GBRMFs are smaller than those of identical frames with fixed and pinned boundary support conditions.
• The magnitude and the distribution of the P-delta moments are more favorable in RWMFs than their fixed base counterparts.
• The proposed configuration is construction friendly and satisfies the theoretical conditions of minimum weight design.
• The use of repetitive members and connections increases the reparability of the structure. Despite many favorable attributes the proposed structural scheme is neither perfect nor complete. It is still under development and needs the test of time and scrutiny before it becomes a commonly recognized earthquake resisting system.

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INTRODUCING A NEW WALL-FRAME EARTHQUAKE RESISTING SYSTEM


