OPTIMAL DESIGN OF PLANAR FRAME STRUCTURES USING AN OPTIMIZATION ALGORITHM BASED ON GLOBAL SENSITIVITY ANALYSIS

A. Kaveh* and V.R. Mahdavi
Centre of Excellence for Fundamental Studies in Structural Engineering, Iran University of Science and Technology, Narmak, Tehran-16, Iran

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ABSTRACT

The main aim of this paper is to present a new single-solution metaheuristic algorithm so-called global sensitivity analysis based (GSAB) algorithm for discrete sizing optimization of steel frames. In the GSAB method, the single solution moves towards specified regions determined using the sensitivity indicator of the variables. In comparison to population-based meta-heuristic algorithms, where all the variables are simultaneously changed in the optimization process, in this approach first the highly sensitive variables of solution and then the less sensitive ones are iteratively changed in the search space, therefore the global convergence of algorithm is accelerated. In order to evaluate the performance of the GSAB algorithm, three planar frame benchmark problems are conducted. The experimental results show that the proposed method performs better than, or at least as well as other population-based meta-heuristic algorithms, especially in decreasing the number of fitness function evaluation.

Keywords: Optimal design; steel frame structures; global sensitivity analysis; meta-heuristic; sensitivity indicator.

1. INTRODUCTION

The objective of the optimum design of steel frame structures is to find the best set of cross-sections for its members that fulfill all the design limitations as well as having the lowest possible construction cost or weight for the structure. Since the structural members should be selected from a set of available sections in practical applications, this problem is referred to as discrete sizing optimization. It is recognized that achieving efficient methods to optimize the structural weight, generate a wide interest and challenge in the civil and mechanical engineering fields [1]. Finding the optimum design of steel frame problems is an

*E-mail address of the corresponding author: alikaveh@iust.ac.ir (A. Kaveh)
NP-hard combinatorial optimization problem, and the exact solution of the problem is complex and highly time consuming for structures with a large number of elements [2]. The optimization algorithms can be generally divided into two categories: deterministic and stochastic methods. The deterministic methods can obtain solution with higher convergence rate compared to stochastic approaches because they use the gradient information of the optimization problem to obtain the minima. However, the estimation of gradient information can be either costly or even impossible for discrete decision variables, and the optimum result obtained by using these methods is completely dependent on the use of a good starting point [3, 4]. Therefore, many metaheuristic algorithms (as a group of the stochastic methods) are developed for finding optimum design of planar and space structures, and some review papers on this topic are also available [2, 5, 6]. The basic purpose of these techniques is to achieve the natural phenomena driven algorithms such as immune system and swarm intelligence for optimizing the structures (for example, simulated annealing [7], genetic algorithm [8], ray optimization algorithm [9], dolphin echolocation optimization algorithm [10], charged system search [11] and colliding bodies optimization [12]).

A meta-heuristic is formally defined as an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploration (global search) and exploitation (local search) of the search space. Learning strategies are used to structure the information in order to find efficiently near-optimal solutions [13]. One of the disadvantages of the metaheuristic algorithms is that they do not consider the sensitivity information of decision variables to the objective function to push the populations into new positions. In other word, in two subsequent optimization iterations, all decision variables of a population have the same ranks for generating the new population. This makes the metaheuristic algorithms slow in convergence to global optima. However, one can extract the sensitivity information of the current population, before generation process, to speedily guide the populations to a near-optimal solution.

In this paper we apply a new single-solution metaheuristic optimizer, namely Global Sensitivity Analysis Based (GSAB) that uses a basic set of mathematical techniques, for optimal design of planar steel frame structures. Sensitivity analysis (SA) studies the sensitivity of the model output with respect to its input parameters [14]. This analysis is generally categorized as local SA and global SA techniques. While local SA studies sensitivity of the model output with respect to variations around a specific point, the global SA considers variations of the inputs within their entire feasibility space [15, 16]. One important feature of the GSA is Factor Priorization (FP), which aims at ranking the inputs in terms of their relative contribution to output variability. The GSAB comprises of a single-solution optimization strategy and GSA-driven procedure, where the solution is guided by ranking the decision variables using the GSA approach, resulting in an efficient and rapid search. In this method, the search process is conducted in the specified boundaries, where the well-known GSA approach is employed to decrease the search boundaries. The minimization of an objective function is then performed by moving the search space around the best global sample. Optimal design of truss structures using an optimization algorithm based on global sensitivity analysis is previously applied to optimal design of truss structure by Kaveh and Mahdavi [17].

The present paper is organized as follows: In the next section, the mathematical
formulations of the structural optimization problems are presented and a brief explanation of the LRFD-AISC is provided. Followed by a Section, we describe the well-known variance-based sensitivity approach. The new method is then presented. Three well-studied planar frame structural design examples are then studied. Conclusions are derived in the final section.

2. STRUCTURAL OPTIMIZATION

2.1 Objective function
The objective of this paper is to minimize the weight of structures while satisfying some constraints on strength and displacements. The mathematical formulation of these problems can be expressed as follows:

\[
\begin{align*}
\text{Find} & \quad \{X\} = [x_1, x_2, \ldots, x_{ng}] \\
\text{to minimize} & \quad W(\{X\}) = \sum_{i=1}^{nm} \rho_i A_i L_i \\
\text{subject to} & \quad \begin{cases} 
  g_j(\{X\}) \leq 0, & j = 1, 2, \ldots, nc \\
  x_{i_{\text{min}}} \leq x_i \leq x_{i_{\text{max}}} 
\end{cases}
\end{align*}
\]

where \(\{X\}\) is a vector containing the design variables, and \(ng\) is the dimension of design variables vector. \(W(\{X\})\) shows the weight of the structure, \(A_i\) and \(L_i\) and \(\rho_i\), represent the cross-sectional area, the length and the material density of the \(i\)th member, respectively. Here \(nm\) is the number of elements of the structure with \(x_{i_{\text{min}}}\) and \(x_{i_{\text{max}}}\) being the lower and upper bounds of the design variable \(x_i\), respectively. \(g_j(\{X\})\) denotes design constraints; and \(nc\) is the number of the constraints.

2.2 Constraint handling
For handling the constraints different methods are available. The mostliy used method is the penalty approach. Based on this technique, the objective function is expressed in the following form:

\[
f(\{X\}) = (1 + \nu) \times W(\{X\}), \quad \nu = \sum_{j=1}^{nc} \max\{0, g_j(\{X\})\}
\]

where \(g_j(X)\) with \(j=1, \ldots, nc\) are the design constraints, and \(\nu\) represents the sum of the violations.

According to Eq. (2), the value of \((1+\nu)\) is equal to unity if all the constraints are satisfied. Violation of any of the constraints results in a \((1+\nu)\) value bigger than one.

Strength and displacement constraints for steel frames are imposed according to the provisions of LRFD-AISC specification [18]. These constraints are briefly explained in the following:
(a) Maximum lateral displacement

\[ \frac{\Delta_T}{H} - R \leq 0 \]  \hspace{1cm} (3)

where \( \Delta_T \) is the maximum lateral displacement; \( H \) is the height of the frame structure; and \( R \) is the maximum drift index which is equal to 1/300.

(b) The inter-story displacements

\[ \frac{d_i}{h_i} - R_I \leq 0, \quad i = 1, 2, \ldots, ns \]  \hspace{1cm} (4)

where \( d_i \) is the inter-story drift; \( h_i \) is the story height of the \( i \)th floor; \( ns \) is the total number of stories; \( R_I \) presents the inter-story drift index (1/300).

(c) Strength constraints

\[ \begin{align*}
\frac{P_u}{2\phi_cP_n} + \frac{M_u}{\phi_cM_n} - 1 & \leq 0, \quad \text{for } \frac{P_u}{\phi_cP_n} < 0.2 \\
\frac{P_u}{\phi_cP_n} + \frac{8M_u}{9\phi_cM_n} - 1 & \leq 0, \quad \text{for } \frac{P_u}{\phi_cP_n} \geq 0.2
\end{align*} \]  \hspace{1cm} (5)

where \( P_u \) is the required strength (tension or compression); \( P_n \) is the nominal axial strength (tension or compression); \( \phi_c \) is the resistance factor (\( \phi_c = 0.9 \) for tension, \( \phi_c = 0.85 \) for compression); \( M_u \) is the required flexural strengths; \( M_n \) is the nominal flexural strengths; and \( \phi_b \) denotes the flexural resistance reduction factor (\( \phi_b = 0.90 \)). The nominal tensile strength for yielding in the gross section is calculated by

\[ P_n = A_g \cdot F_y \]  \hspace{1cm} (6)

and the nominal compressive strength of a member is computed as

\[ P_n = A_g \cdot F_{cr} \]  \hspace{1cm} (7a)

\[ \begin{align*}
F_{cr} &= (0.658 \lambda_c^2)F_y, \quad \text{for } \lambda_c \leq 1.5 \\
F_{cr} &= (0.877 \lambda_c^2)F_y, \quad \text{for } \lambda_c > 1.5
\end{align*} \]  \hspace{1cm} (7b)

\[ \lambda_c = \frac{kl}{r\pi} \sqrt{\frac{F_y}{E}} \]  \hspace{1cm} (7c)

where \( A_g \) is the cross-sectional area of a member and \( k \) is the effective length factor that is
calculated by:

\[ k = \sqrt{\frac{1.6G_AG_B + 4.0(G_A + G_B) + 7.5}{G_A + G_B + 7.5}} \]  

(8)

where \( G_A \) and \( G_B \) are stiffness ratios of columns and girders at two end joints \( A \) and \( B \) of the column section being considered, respectively.

### 3. BACKGROUND STUDY

The single-solution search algorithm proposed in this paper uses the SA theory. In this section the main strategies used for taking SA into account which are mainly based on the works of Zhai et al. [16], Saltelli et al. [19] and Archer et al. [20] are described.

#### 3.1 Variance-based sensitivity indices

Consider a numerical model in the form \( Y = g(X) \), with \( X = [x_1, x_2, \ldots, x_n] \) being the input vector, \( Y \) being the output scalar of the model, and \( g() \) being a deterministic mapping function. The uncertainty of \( X \) propagates through \( g() \) and resulting in the output model, \( Y \). As the uncertainty of the output model is represented by its variance, \( V(Y) \), to find the effect of an input \( X_i \) on the output, it is assumed that the true value of \( X_i \) can be determined by the variance reduction in the output, i.e., \( V(Y) - V(Y|X_i = x_i^0) \), where \( x_i^0 \) is the true value of \( X_i \) and \( V(Y|X_i = x_i^0) \) is the conditional expected value of \( V(Y) \). Since the true value is unknown, one can employ \( V(Y) - E_{X_i}(V(Y|X_i)) \) to evaluate the expected variance reduction in the output [16, 19, 20].

The variance of output model is calculated utilizing the following equation:

\[ V(Y) = V_{X_i}(E(Y|X_i)) + E_{X_i}(V(Y|X_i)) \]  

(9)

And the first order sensitivity indicator of \( X_i \), \( SI_i \), can be expressed as (Zhai et al. [16]):

\[ SI_i = \frac{V(Y) - E_{X_i}(V(Y|X_i))}{V(Y)} = 1 - \frac{E_{X_i}(V(Y|X_i))}{V(Y)} = \frac{V_{X_i}(E(Y|X_i))}{V(Y)} \]  

(10)

This represents the expected percentage reduction in \( V(Y) \) that is obtained when uncertainty in \( X_i \) is eliminated. In sensitivity analysis, \( SI_i \) varies between 0 and 1. The lower value of \( SI_i \) corresponds to the less influential \( X_i \), the higher value of \( SI_i \) corresponds to the much influential \( X_i \), and for \( SI_i = 0 \), the \( X_i \) will have no influence on \( Y \).

#### 3.2 The variance-based sensitivity analysis using space-partition method

The most well-known methods for calculating the variance-based sensitivity indicators are the Monte Carlo simulations; however they do not make full use of each output model.
evaluation. In order to calculate the variance-based sensitivity indicators from a given data the scatterplot partitioning method can be utilized [16]. For this method a single set of samples suffices to estimate all the sensitivity indicators. For estimating the variance-based sensitivity indices, a space-partition method is used in the following:

Suppose we have M points/samples \( \{X_1, ..., X^M\} \) and M model output samples \( \{y^1, ..., y^M\} \) obtained using the model \( y=g(X) \). The variance of \( Y \) can be calculated by the sample variance \( \bar{V}(y) \). For the sample bounds of \( X_i \) as \([b_1, b_2]\), let it be decomposed into \( s \) successive, equal-probability and non-overlapping subintervals \( A_k = [a_{k-1}, a_k) \), with \( k=1, ..., s \), \( b_1 = a_0 < a_1 < ... < a_k < ... < a_s = b_2 \), and \( Pr(A_k)=1/s \). Decompose the output samples \( \{y^1, ..., y^M\} \) into \( s \) subsets according to the decomposition of \( X_i \), where \( B_k = \{y^j | x^j_i \in A_k \}, \quad k = 1, ..., s \). The variance \( V(Y|X_i \in A_k) \) can then be estimated by

\[
\bar{V}(Y|X_i \in A_k) = V(B_k)
\] (11)

The expected conditional variance \( E_{x_i}(V(Y|X_i)) \) can now be approximately estimated using the following relationship:

\[
E_{x_i}(V(Y|X_i)) \approx \frac{1}{s} \sum_{k=1}^{s} V(B_k)
\] (12)

And ultimately, \( SI_i \) is estimated by

\[
\hat{SI}_i = 1 - \frac{E_{x_i}(V(Y|X_i))}{\bar{V}(Y)}
\] (13)

4. A GLOBAL SENSITIVITY ANALYSIS BASED ALGORITHM

This section introduces a global sensitivity analysis based (GSAB) algorithm, which is a single solution metaheuristic method. The proposed algorithm is named a global sensitivity analysis (GSA) because of determining the sensitivity indicator (SI) of the decision variables.

Metaheuristic algorithms can be divided into two categories based on their search mechanism: population-based and single-solution [13]. In first group, a number of populations/agents are first generated and then the positions of the agents are updated iteratively until the termination condition is satisfied. In the other hand, single-solution metaheuristics are also known as trajectory methods, which these algorithms produce single solution by exploring the search space efficiently while reducing the effective size of the search space. The GSAB algorithm consists of some samples for estimating the \( SI \) of decision variables. As these samples do not update iteratively and these are used only for calculating the SIs, the proposed GSBA is studied within the single-solution metaheuristic category. The feasibility space of samples in the GSAB algorithm updates for searching the
optimal solution over several iterations. In each iteration, the feasibility space is updated using two values: the sensitivity indicators and the global best sample. It is assumed that the problem is a minimization problem in $\mathbb{R}^D$. The notations used are as follows:

$S_t$: The sample matrix in the $t$th iteration, $S_t = [X^i_t | i = 1, 2, \ldots, N]$

$X^i_t$: The position of sample vector $i$ in the $t$th iteration, $X^i_t = \{x^i_{j,t} | j = 1, 2, \ldots, D\}$

$X_{\text{min}}$: The minimum allowable values vector of variables, $X_{\text{min}} = \{x_{\text{min},j} | j = 1, 2, \ldots, D\}$

$X_{\text{max}}$: The maximum allowable values vector of variables, $X_{\text{max}} = \{x_{\text{max},j} | j = 1, 2, \ldots, D\}$

$f(X_i)$: The fitness of vector $i$

$UB^t$: The upper boundary vector of variables in the $t$th iteration, $UB^t = \{u^i_{j,t} | j = 1, 2, \ldots, D\}$

$LB^t$: The lower boundary vector of variables in the $t$th iteration, $LB^t = \{u^i_{j,t} | j = 1, 2, \ldots, D\}$

$BW^t$: The band width of search space of variables in the $t$th iteration, $BW^t = \{b^i_{j,t} | j = 1, 2, \ldots, D\}$

$SF^t$: The scale factor of band width of search space in the $t$th iteration, $SF^t = \{s_{f,t}^j | j = 1, 2, \ldots, D\}$

$S_{\text{best}}$: The global best sample (i.e. with lower fitness), $S_{\text{best}} = \{s_{\text{best},j} | j = 1, 2, \ldots, D\}$

$R$: A random vector within $[0,1]$.

### 4.1 Methodology

The following steps outline the main procedure in the implementation of the GSAB.

**Step 1. Initialization:** The initial positions of samples are determined with random initialization in the search space:

$$X^0_i = X_{\text{min}} + R(X_{\text{max}} - X_{\text{min}}), \quad i = 1, 2, \ldots, N$$  \hspace{1cm} (14)

where $X^0_i$ determines the initial value vector of the $i$th sample; and $N$ is the number of samples. In the first step, some parameter settings must also be pre-defined for the proposed algorithm. There are two parameters: the number of samples, $N$, the number of subintervals for estimating the sensitivity indices, $s$. The number of samples is considered according to the problem’s complexity. More complex problems require a higher number of samples. The second parameter is the number of subintervals, $S$, which is used for GSA as mentioned in the previous section. These values affect the estimation of $SI$ (Equation (13)). A more detailed discussion of these parameters is given in the subsequent subsections.

**Step 2. Calculation of the sensitivity indices of variables:** In this step the output model, i.e. the objective function of optimization problem, is first calculated. The sensitivity analysis is performed next for the generated samples, and the sensitivity indicators ($SI$s) of variables are calculated through Equations (11) to (13).

**Step 3. Defining the search boundaries:** In the GSAB algorithm, the search boundaries are moved to the global best sample (which is updated and memorized in each iteration), $S_{\text{best}}$, to push the samples into feasible search space (as shown in Fig. 1). The search boundaries are also decreased based on the values of the sensitivity indices of variables, which is evaluated in the previous step. Hence, the upper boundary and lower boundary of
the search space of variables in $t+1$th iteration can be computed by:

$$UB^{t+1} = S_{best} + BW^t \times SF^t \leq X_{\text{max}}$$  
$$LB^{t+1} = S_{best} - BW^t \times SF^t \geq X_{\text{min}}$$  

(15)

where $BW^t$ and $SF^t$ are the band width and scale factor of boundaries in the $t$th iteration, respectively (Fig. 1). Equation (15) ensures that the current search space is moved around $S_{best}$ with the band width $BW^t$ in the D-dimensional space. The vector $BW^t$ can be calculated as:

$$BW^t = \max(S_{best} - LB^t, UB^t - S_{best})$$  

(16)

For the algorithm to converge to a near-optimal solution, further exploitation (strong locality) is required to move the current solution towards the optimal one. In the proposed GSAB algorithm, this is achieved by using a scale factor, $SF$. For this purpose, once the $SI$ values of the variables are obtained. Then the $SF$ is calculated as:

$$SF_j = 1 - \frac{SI_j}{\sum_{k=1}^{N} SI_k}, \quad \forall j = 1, \ldots, D$$  

(17)

This equation shows that the band width of the variables is decreased based on the amount of these sensitivity indices in the $t$th iteration.

Figure 1. An illustrative sketch of the search process

**Step 4. Replacement of the current samples:** In this step, the samples must be ensured to be inside the new search boundaries. For this purpose, the samples that exceed the boundaries are randomly regenerated into the new search boundaries, shown in Fig. 1, as:
\[ X_{i}^{t+1} = \begin{cases} X_{i}^{t}, & \text{if } LB_{i} \leq X_{i}^{t} \leq UB_{i}^{t+1} \\ LB_{i}^{t+1} + R(UB_{i}^{t+1} - LB_{i}^{t+1}), & \text{Otherwise} \end{cases} \]  

where \( i = 1, 2, \ldots, N \) and \( t \) represents the iteration index.

**Step 5. Termination:** The optimization process is repeated from Step 2 until a termination criterion, such as maximum iteration number or no improvement of the best sample, is satisfied. In the GSAB algorithm, if the maximum band width of search space, \( \text{max}(BW) \), becomes smaller than 0.000001, the optimization process will be stopped. For the sake of clarity, the flowchart of optimization procedure using the proposed GSAB is shown in Fig. 2.

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**5. NUMERICAL EXAMPLES**

In this section the efficiency of the proposed algorithm, GSAB, is illustrated through three well-studied planar frame structures under static loads taken from the optimization literature. The planar frame examples have been previously solved using a variety of other techniques, and are suitable examples to show the validity and effectiveness of the proposed algorithm.

The employed constraint handling is the penalty function approach. It should be noted that the output model of SA method is the penalized objective function. For all examples, the numbers of \( N = 40 \) samples are utilized. Also, these examples are independently optimized 20 times. The algorithm is coded in MATLAB. Structural analysis is performed with the direct stiffness method.
5.1 Example 1: A one-bay 8-story frame
The one-bay eight-story frame structure shown in Fig. 3 is considered as the first example. This problem was previously optimized by Khot et al. [21] using the optimality criterion method, Camp et al. [22] used genetic algorithm and Kaveh and Shojaei [23] utilized ant colony optimization. Application of other metaheuristic can be found in Kaveh [24].

The 24 members of the structure are categorized into eight groups. The members grouping and applied loads is indicated in Fig. 3. A set of 267 discrete W-sections from American Institute of Steel Construction (AISC) shapes database are used for the possible cross-sectional of each member. The lateral displacement at the top of the structure is the only constraint (limited to 2 in). The modulus of elasticity is taken as $E = 29,000$ Ksi.

![Figure 3. Schematic of the planar 8-story frame structure](image-url)
Twenty independent runs are conducted to obtain discrete optimum design of this problem by implementing the GSAB algorithm. Table 1 compares the best results obtained in this paper and those of other researches. The GSAB found the best cost as 7,040 lb after 2,319 fitness function evaluations. While the best cost found here is better, and it has the lowest fitness function evaluations amongst the existing literature results. Therefore, the GSAB algorithm is more reliable than other algorithms. The worst, mean, and standard deviation of the design weights attained by the GSAB are 8,041, 7,609, and 257 Ib, respectively. Top displacement of the found optimum structure is equal to 1.9595 in which is very close to the maximum allowable value of 2 in.

Table 1: Comparison of optimal design for the 8-story moment frame

<table>
<thead>
<tr>
<th>Group number</th>
<th>Kho et al. [21]</th>
<th>Camp et al. [22]</th>
<th>Kaveh and Shojae [23]</th>
<th>Present study</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>W14×34</td>
<td>W18×46</td>
<td>W21×50</td>
<td>W21×44</td>
</tr>
<tr>
<td>2</td>
<td>W10×39</td>
<td>W16×31</td>
<td>W16×26</td>
<td>W16×26</td>
</tr>
<tr>
<td>3</td>
<td>W10×33</td>
<td>W16×26</td>
<td>W16×26</td>
<td>W16×26</td>
</tr>
<tr>
<td>4</td>
<td>W8×18</td>
<td>W12×16</td>
<td>W12×14</td>
<td>W12×19</td>
</tr>
<tr>
<td>5</td>
<td>W21×68</td>
<td>W18×35</td>
<td>W16×26</td>
<td>W18×35</td>
</tr>
<tr>
<td>6</td>
<td>W24×55</td>
<td>W18×35</td>
<td>W18×40</td>
<td>W18×35</td>
</tr>
<tr>
<td>7</td>
<td>W21×50</td>
<td>W18×35</td>
<td>W18×35</td>
<td>W16×26</td>
</tr>
<tr>
<td>8</td>
<td>W12×40</td>
<td>W16×26</td>
<td>W14×22</td>
<td>W16×26</td>
</tr>
</tbody>
</table>

weight (lb)  | 9,221           | 7,380           | 7,100                 | 7,040        |

No. of analyses | -               | -               | 4,500                 | 2,391        |

In order to show the performance of the GSA method in the GSAB algorithm, a study is performed on the influence of the SIs on the results of the proposed algorithm. As described in Section 3.2, the GSA method requires two predefined parameters: the number of samples, \( N \), and the number of subintervals, \( s \). A larger number of samples leads to an increase of the accuracy of the sensitivity indicators. In the other hand, because of generating the output model of the GSA method, the fitness function (or output) evaluations increases with the number of samples. The number of subintervals can also affect the SI values. As Zhai et al. [16] indicate, the appropriate number of subintervals can be considered as \( s = \frac{N}{5} \). If we apply the space-partition variance-based sensitivity analysis approach for \( N = 100 \) samples, we obtain the sensitivity indicators, SIs, as given in Fig. 4. As it can be seen from this Fig, the SI of the first variable (the cross-sectional properties of first and second stories) is bigger than other variables; and then the most influential/sensitive variable is the first variable. Also, the SIs of the cross-sections of columns are bigger than the cross-sections of beams.

Fig. 5 shows the convergence rates of the upper and lower boundary of the search space and best ones in the optimization process. It should be noted that, as mentioned before, the number of samples is considered as \( N=40 \) in the optimization process. It can be seen, with respect to the 2-8th variables, that the search space of the first variable is rapidly decreased in the early iterations because it has higher sensitivity to the output (i.e. objective function). In contrast with the first variable, the 8th variable is converged in final iterations. Hence,
despite the fewer samples, the proposed GSA approach could appropriately rank the variables based on these sensitivities.

Figure 4. Sensitivity indicator of variables for penalized cost function of the first example
5.2 Example 2: A 3-bay 15-story planar frame

The second design example is a 15-story frame consisting of 64 joints and 105 members. The configuration, applied loads and the numbering of member groups for this problem are shown in Fig. 6. The modulus of elasticity is 29,000 ksi (200 GPa) and the yield stress is 36 ksi (248.2 MPa) for all members. The effective length factors of the members are calculated as \( k_x \geq 0 \) for a sway-permitted frame and the out-of-plane effective length factor is specified as \( k_y = 1.0 \). Each column is considered as non-braced along its length, and the non-braced length for each beam member is specified as one-fifth of the span length. The frame is designed following the LRFD specification using an inter-story drift displacement constraint [11]. Also, the sway of the top story is limited to 9.25 in (23.5 cm).

In this example, twenty independent runs are performed using the GSAB algorithm, resulting in an optimum design weight of 87607 lb in the best run, and 97176 lb in the worst one. The mean and standard deviation of the optimized weights were 91223 lb and 2303 lb, respectively. Table 2 indicates that better solution have been captured with the ECBO algorithm using much higher computational effort. Amongst the other solutions are 95,850 lb by hybrid algorithm based on particle swarm, ant colony and harmony search algorithms (HPSACO), 97,689 lb by hybrid big bang–big crunch and particle swarm optimization algorithms (HBB–BC), 93,846 lb by imperialist competitive algorithm (ICA), 92,723 lb by
charged system search (CSS) [11], and finally 93,795 and 86,986 lb that employ standard and enhance colliding bodies optimization (CBO and ECBO) [12]. The maximum number of structural analyses required to reach these solutions were 8800, 9900, 6000, 5500, 8280 and 15360 by HPSACO, HBB-BC, ICA, CSS, CBO and ECBO, respectively.

Figure 6. Schematic of the 3-bay 15-story frame structure
Table 2: Optimal design of the 3-bay 15-story frame

<table>
<thead>
<tr>
<th>Element group</th>
<th>Optimal W-shaped sections (Kaveh and Talatahari [11])</th>
<th>Optimal W-shaped sections (Kaveh and Ilchi [12])</th>
<th>Present work</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HPSACO</td>
<td>HBB-BC</td>
<td>ICA</td>
</tr>
<tr>
<td>1</td>
<td>W21×111</td>
<td>W24×117</td>
<td>W24×117</td>
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Weight (lb) | 95,850 | 97,689 | 93,846 | 92,723 | 93,795 | 86,986 | 87,607 |

Number of structural analyses | 8,800 | 9,900 | 6,000 | 5,500 | 8,280 | 15,360 | 5,945 |

Fig. 7 demonstrates the inter-story drifts for the best designs of the GSAB, the maximum value of which is 1.15 cm. This is less than the allowable value (1.17 cm). Fig. 8 shows the maximum stress ratios in frame group members obtained using the GSAB. The maximum stress ratio is 99.99%, which is closer to the limit line. Therefore, the maximum stress ratio constraint is the active constraint in the optimization process. The maximum stress ratios for all group members are also relatively identical, indicating an appropriate stress distribution.
Fig. 9 shows the band width of the search space of samples for variables $X_{i=1,2,9,10,11}$. The variables $X_1$ and $X_2$ indicate the cross-sectional of first to third stories external and internal columns, respectively. Similarity the variables $X_9$ and $X_{10}$ indicate the cross-sectional of 13th to 15th stories external and internal columns, and the variable $X_{11}$ indicates the cross-sectional of all the beams. As mentioned before, in the GSAB algorithm the band widths of high sensitive variables are speedily decreased compared to less sensitivity ones. It can be noticed that: (i) $X_{11}$ seems to be the most influential variable; (ii) $X_9$ and $X_{10}$ seem to be the lowest influential variables compared to the variables $X_1$ and $X_2$; iii) the variables associated to internal columns are much sensitive compared to the external variables. These results, obtained from the GSA method, are confirmed by the structural analysis and design. Since the active constraint in this example is the maximum stress ratio, one can conclude that: Firstly, the cross-sections of all the beams are affected by the stress distribution in the structural analysis, due to higher number of members of this group. Secondly, the columns of lower stories, especially internal columns are much sensitive in the structural design due to high external loads.
Figure 9. The convergence diagrams of the band width of variables $X_{1,2,9,10,11}$ for the 3-bay 15-story frame: a) all iterations; b) 50th-214th iterations

5.3 Example 3: A 3-bay 27-story frame
Fig. 10 shows the configuration and applied loads of a 3-bay 24-story frame structure [11]. This frame consists of 168 members that are collected in 20 groups (16 column groups and 4 beam groups). Each of the four beam element groups is chosen from all 267 W-shapes, while the 16 column element groups are limited to W14 sections. The material has a modulus of elasticity equal to $E = 29,732$ ksi (205 GPa) and a yield stress of $f_y = 33.4$ ksi
(230.3 MPa). The effective length factors of the members are calculated as \( k_x \geq 0 \) for a sway-permitted frame and the out-of-plane effective length factor is specified as \( k_y = 1.0 \). All columns and beams are considered as non-braced along their lengths. The frame is designed following the LRFD specification and uses an inter-story drift displacement constraint [18].

Figure 10. Schematic of the 3-bay 24-story frame structure

Table 3 shows the optimum design variables obtained using the GSAB algorithm, which are compared to the results of the other algorithms. The best result of the GSAB is 201506, while this is 201618, 215874, 212364, 212640, 214860 and 220465 lb for the ECBO, CBO, CSS, ICA, HS and ACO algorithms, respectively. Also, the number of analyses of the GSAB is 13124, while it is 15360, 8280, 5500, 7500, 13924 and 15500 for the ECBO, CBO,
CSS, ICA, HS and ACO algorithms, respectively. It is evident from Table 3 that although the number of function evaluations for the CBO, CSS and ICA algorithms are less than that of the GSAB, the best results of 20 independent runs for the GSAB is less than these algorithms.

Table 3: Optimal design of the 3-bay 24-story frame

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<td>13,924</td>
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</table>

Fig. 11 shows the inter-story drifts with maximum value being 0.04 ft that is the same as the allowable value. Fig. 12 illustrates the maximum stress ratios in frame group members obtained using the GSAB. It can be seen from these Figs that both constraints are active in the optimization process. Fig. 13 shows the convergence rate of the search space boundary for the variables $X_{i=1,2,3,4}$ of this example. It should be noticed that these variables correspond to the cross-sectional members of beams. Similar to the previous example, the variable $X_1$ is the sensitive variable compared to the other members, because these group of members can be affected by the stress distribution in the structural analysis and have high length in the frame structure. It can be also seen that the total number of iteration is 568 and the band width of variable $X_1$ reaches to zero in iteration 129. This indicates that one variable in the optimization process is decreased in iteration 129, and hence this can decrease the total number of function evaluations due to decrease the search space of the
optimization problem.

![Figure 11. The inter-story drifts for the best designs of the 3-bay 24-story frame](image1)

![Figure 12. The maximum stress ratio in the group elements for the best designs of the 3-bay 24-story frame](image2)

6. CONCLUDING REMARKS

In the present study, a metaheuristic algorithm, called as the global sensitivity analysis based (GSAB), is introduced and is utilized for optimal design of steel frame structures. From the results, the following conclusions are derived:
i) The population/agents of the GSAB is directly represented by the samples, which are used to find the sensitivity values of the decision variables as well as the optimization search in sequence at each iteration. Hence, one can consider the proposed algorithm as a single-solution metaheuristic category.

ii) Unlike the common metaheuristic algorithms where the agent of a population moves to the new positions without considering any information about the sensitivity of variables, in this algorithm the search boundaries are decreased based on the sensitivity indices of the variables, and this accelerates the converge of the solution.

iii) The GSAB algorithm is tested using three weight optimization of planar frame structures having different dimensions. The results are compared to those of some population based metaheuristics. This comparison reveals that besides its simplicity, the proposed GSAB algorithm is also competitive, especially from the number of functions evaluation point of view, when compared to the performance of some other algorithms.

iv) The results of sensitivity analysis are confirmed by the structural analysis and design. In the early iterations of the algorithm, the band width of search space of the sensitive variables reaches to zero and this can decrease the search space of the optimization problem, increasing the convergence rate of the algorithm.

Finally, further research remains to be conducted on incorporating our sensitivity analysis search strategy in to other swarm intelligence algorithms. It would be interesting to know how the SA affects the results of the other population-based evolutionary algorithms. The authors are also in the process of applying the GSAB algorithm for solving complex multi-objective optimization structural problems.

REFERENCES

A. Kaveh and V.R. Mahdavi