LARGE DEFLECTION OF ARBITRARY THIN PLATES USING SUPERPARAMETRIC FINITE ELEMENT

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ABSTRACT

Different arbitrary shapes of plates are used in civil, marine and aerospace engineering. They may undergo large deflections due to large transverse loads and hence, the nonlinear analysis must be carried out on the structures. In view of the above an elegant finite element formulation is developed for analyzing the geometrically nonlinear static behavior of arbitrary shaped thin plates using superparametric element. This element is capable of accommodating different geometries just like isoparametric element. The efficacy of the element is shown by presenting different numerical examples.

Keywords: Finite element analysis; large deflection; nonlinear plate theory; arbitrary thin plates; superparametric element.

1. INTRODUCTION

The arbitrary shaped plates especially in civil, marine and aerospace engineering may undergo large deflections under transverse loads. The value of lateral displacement obtained by the large deflection analysis will be significantly less than that obtained with linear analysis shown in Fig.1 as the structure becomes stiffer. Hence the additional effects due to large deflection must be considered in the analysis.


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using a Mindlin formulation. As usual, they faced the latent problem of isoparametric formulation like shear locking and the spurious energy modes. Though they have tried to alleviate the above problems by reduced and selective integration, the behavior of the elements were inconsistent depending on whether a boundary is straight or curved. Cheung and Dashan [6] considered finite strip method to study arbitrary shaped thin plates. Annular sector plate is analyzed by Turvey and Salehi [7] using a dynamic relaxation finite difference procedure. Singh and Elaghabash [8] proposed a numerical method (Ritz type) for the linear and geometrically non-linear static analysis of rhombic plates. The plate geometry is defined by a quadrangular boundary with four straight edges and the natural coordinates in conjunction with the Cartesian coordinates are used to map the geometry. Spline finite strip method is used by Sheikh and Mukhopadhyay [9] to analyze plates of different geometries. They have discretized mapped domain into a number of strips using cubic serendipity function and used two different displacement interpolation function in longitudinal and the other direction. A mathematical model is formulated by Shahidi et al. [10] based on elastic Cosserat theory for analysis of arbitrary quadrilateral plates. Das et al. [11] studied the isotropic skew plates under uniformly distributed load using variational principle. The effect of transverse shear was included in a shell element for geometric nonlinear analysis by Wankhade [12]. His work provided brief study of skew plates with variable parameters.

In the present analysis, the geometrically nonlinear static behavior of arbitrary shaped thin plates is studied using superparametric element. The cubic serendipity function is used to represents the arbitrary geometry of the plate. An ACM plate ([13], [14]) bending element along with the in-plane deformations is considered for the displacement function. This element is capable of accommodating different geometries just like isoparametric element. As this element considers only thin plates and hence, does not consider the shear deformation, thus eliminating the shear locking problem and generation of spurious mechanisms. In the formulation the arbitrary planform of the whole plate is mapped into a square domain and the nonlinear formulation uses Von-Karman's nonlinear equation in the total Lagrangian coordinate system using [N]-notation. The nonlinear governing equations are solved by Newton Raphson iterative method. The deflections and stresses at critical points of the plates of different shapes are compared with the published results. The new results are compared with the SAP 2000 results wherever possible.

Figure 1. Small and large deflection theory
2. MAPPING OF THE PLATE

The arbitrary shape of the plate is mapped [15] approximately into a \([-1,+1]\) region in the \(s-t\) plane as shown in Fig. 2 with the help of the cubic serendipity shape function [16].

\[
x = \sum_{i=1}^{12} N_i(s,t)x_i \\
y = \sum_{i=1}^{12} N_i(s,t)y_i
\]

where \((x_i, y_i)\) are the co-ordinates of the i-th boundary node of the plate and \(N_i\) is the corresponding cubic serendipity shape function. The mapped square plate is now discretized into a number of elements and each element is being mapped with the same cubic serendipity shape function to a natural coordinate element of domain \([-1,+1]\) in \(\xi-\eta\) plane as shown in Fig. 3.

![Figure 2. Mapping of arbitrary geometry into a square domain in \(s-t\) plane](image1)

![Figure 3. Mapping of element into a square domain in \(\xi-\eta\) plane](image2)
From the mapping we have,

\[
\begin{bmatrix}
\frac{\partial w}{\partial x} \\
\frac{\partial w}{\partial y} \\
\frac{\partial w}{\partial \eta}
\end{bmatrix} = [J]\begin{bmatrix}
\frac{\partial w}{\partial \xi} \\
\frac{\partial w}{\partial \eta}
\end{bmatrix}
\]

(3)

where, \([J]\) = \[
\begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} \\
\frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta}
\end{bmatrix}
\]

(4)

3. DISPLACEMENT INTERPOLATION FUNCTION

For the proposed element, the four noded rectangular ACM plate bending element with five degrees of freedom \((u,v,w,\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y})\) at each node is considered. The interpolation functions for the in-plane and bending are the usual ones presented in detail in [17].

4. STRESS-STRAIN RELATIONSHIP

The generalized stress-strain relation is given by

\[
\{\sigma\} = [D]\{\varepsilon\}
\]

(5)

where, \(\{\sigma\}\) is the stress resultant given by

\[
\{\sigma\} = \begin{bmatrix}
\{\sigma^p\} \\
\{\sigma^b\}
\end{bmatrix} = \begin{bmatrix}
N_x \\
N_y \\
N_{xy} \\
M_x \\
M_y \\
M_{xy}
\end{bmatrix}
\]

(6)

and \([D]\) is the rigidity matrix given by
\[ [D] = \begin{bmatrix} [D^p] & [0] \\ [0] & [D^b] \end{bmatrix} \]  

(7)

where, \[ [D^p] = \frac{Eh}{1-v^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix} \]

\[ [D^b] = \frac{Eh^3}{12(1-v^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix} \]

The superscripts \( p \) and \( b \) used in the equations are for the inplane and bending part respectively. Taking the mid-plane of the plate as the reference plane for the analysis, strain matrix is given by

\[
\begin{bmatrix} \varepsilon \\ \varepsilon_{NL} \end{bmatrix} = \begin{bmatrix} \varepsilon^p_L \\ \varepsilon^b_L \\ \varepsilon^p_{NL} \\ \varepsilon^b_{NL} \end{bmatrix}
\]

\[
\begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ \frac{\partial^2 w}{\partial x \partial y} \\ -2 \frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} = \begin{bmatrix} 1 \left( \frac{\partial w}{\partial x} \right)^2 \\ 2 \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial y^2} \\ 1 \left( \frac{\partial w}{\partial y} \right)^2 \\ 2 \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial x^2} \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

(8)

5. STRAIN DISPLACEMENT RELATIONSHIP

The strain displacement relation is given by
\[
\{\epsilon^p\}_L = \{B^p\}_L \{\delta\} \\
\{\epsilon^p\}_NL = \{B^p\}_NL \{\delta\} \\
\{\epsilon^b\}_L = \{B^b\}_L \{\delta\}
\]

The inplane linear strain is given by

\[
\begin{bmatrix}
\frac{\partial u}{\partial y} \\
\frac{\partial v}{\partial x} \\
\frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} \\
\frac{\partial w}{\partial y}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} & 0 & 0 \\
0 & 0 & \frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y} \\
\frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y} & \frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x}
\end{bmatrix} \begin{bmatrix}
\frac{\partial \xi}{\partial u} \\
\frac{\partial \eta}{\partial u} \\
\frac{\partial \xi}{\partial \eta} \\
\frac{\partial \eta}{\partial \eta}
\end{bmatrix}
\]

\[
\{\epsilon^p\}_L = [T^p_L] \{\xi(\xi, \eta)\}
\]

\[
\{B^p\}_L = [\xi(\xi, \eta)]
\]

The linear strain due to bending [15] action is given by

\[
\begin{bmatrix}
-\frac{\partial^2 w}{\partial x^2} \\
-\frac{\partial^2 w}{\partial y^2} \\
2 \frac{\partial^2 w}{\partial x \partial y}
\end{bmatrix} = \begin{bmatrix}
T^{\xi} & T^{\eta}
\end{bmatrix} \begin{bmatrix}
\frac{\partial w}{\partial \xi} \\
\frac{\partial w}{\partial \eta} \\
\frac{\partial w}{\partial \xi} \\
\frac{\partial w}{\partial \eta}
\end{bmatrix}
\]
where, \([T^{b2}] = [J2]^{-1} [J1][J]^{-1}\)

\[ \{B^b\}_L = [T^{b1} \quad T^{b2}] \begin{bmatrix}
\frac{\partial N_u}{\partial \xi} \\
\frac{\partial N_v}{\partial \eta} \\
\frac{\partial^2 N_u}{\partial \xi^2} \\
\frac{\partial^2 N_v}{\partial \eta^2} \\
\frac{\partial^2 N_u}{\partial \xi \partial \eta} \\
\frac{\partial^2 N_v}{\partial \xi \partial \eta}
\end{bmatrix} \tag{16} \]

6. FORMULATION OF THE GEOMETRIC NON-LINEARITY

The geometrically nonlinear formulation [18] is done using [N]-notation formulated in [19] and the displacements referred to the original configuration following the Lagrangian method.

6.1 Equilibrium equation

The equilibrium equation is obtained by application of virtual principle as

\[ \{\Psi\} = \int_V [B]^T \{\sigma\} dV - \{P\} = \{0\} \tag{17} \]
where \( \{ \psi \} \) denotes the sum of external and internal generalized forces and \( [B] \) is obtained from the following relationship.

\[
d\{ \varepsilon \} = [B]d\{ \delta \}
\]  

(18)

Again matrix \( [B] \) may be expressed as

\[
[B] = [B]_L + [B]_{NL}
\]  

(19)

where \( [B]_L \) is the linear part and \( [B]_{NL} \) is the nonlinear part and is dependent on displacements. If the strains are reasonably small, we can still write the elastic relationship

\[
\{ \sigma \} = [D] \{ \varepsilon \}
\]  

(20)

Referring Eq.(8),

\[
\{ \varepsilon \}^p_{NL} = \frac{1}{2} \begin{bmatrix}
\frac{\partial w}{\partial x} & 0 \\
0 & \frac{\partial w}{\partial y}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial w}{\partial x} \\
\frac{\partial w}{\partial y}
\end{bmatrix} = \frac{1}{2} [A] \{ \theta \}
\]  

(21)

\[
\{ \theta \} = \begin{bmatrix}
\frac{\partial w}{\partial x} \\
\frac{\partial w}{\partial y}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial}{\partial x} [N^p] \\
\frac{\partial}{\partial y} [N^p]
\end{bmatrix} \{ \delta \} = [G] \{ \delta \}
\]  

(22)

\[
\{ \varepsilon \}^p_{NL} = \frac{1}{2} [A] [G] \{ \delta \} = \frac{1}{2} [B]^p_{NL} \{ \delta \}
\]  

(23)

\[
\{ \varepsilon \} = \left( [B]_L + \frac{1}{2} [B]_{NL} \right) \{ \delta \}
\]  

(24)

Referring Eqs.(21) and (24), Eq.(18) becomes

\[
d\{ \varepsilon \} = \left[ [B]_L \; d\{ \delta \} \right] + \left( \frac{1}{2} d[A] \{ \theta \} + \frac{1}{2} [A] \; d\{ \theta \} \right)
\]  

(25)

\[
= \left( [B]_L + [B]_{NL} \right) d\{ \delta \}
\]  

(26)

as,  \( d[A] \{ \theta \} = [A] \; d\{ \theta \} = [A] [G] \; d\{ \delta \} = [B]_{NL} d\{ \delta \} \)
With the help of Eqs. (19)-(24), Eq. (17) becomes

$$[K_s][\delta] - \{P\} = \{0\}$$

(27)

where \([K_s]\) is the secant stiffness matrix and is given by

$$[K_s] = \int_V (([B]_L + [B]_{NL})^T [D] ([B]_L + \frac{1}{2}[B]_{NL}) ) dV$$

(28)

$$[K_s] = \int_V ([B]_L^T [D] [B]_L ) dV + \frac{1}{2} \int_V ([B]_L^T [D] [B]_{NL} ) dV + \frac{1}{2} \int_V ([B]_{NL}^T [D] [B]_L ) dV + \frac{1}{2} \int_V ([B]_{NL}^T [D] [B]_{NL} ) dV$$

(26)

$$= [K]_L + [K_{NL}]$$

(30)

where,

- \([K]_L\) = Linear part of the stiffness matrix

- \([K_{NL}]\) = Nonlinear part of the stiffness matrix

6.2 Incremental equilibrium equation

The tangent stiffness matrix is obtained by taking appropriate variation of Eq. (17) with respect to \(\{\delta\}\).

$$d\{\psi\} = \int_V d([B]_L + [B]_{NL})^T \{\sigma\} dV + \int_V ([B]_L + [B]_{NL})^T d\{\sigma\} dV - d\{P\}$$

(31)

Substituting Eq. (19), (24) and (26) into Eq. (31), it yields to

$$\int_V d([B]_L + [B]_{NL})^T \{\sigma\} dV + \int_V ([B]_L + [B]_{NL})^T [D] ([B]_L + [B]_{NL}) dV d\{\delta\} - d\{P\} = \{0\}$$

(32)

$$[K_{\sigma}] d\{\delta\} + ([K]_L + [K_{\sigma}]) d\{\delta\} = d\{P\}$$

(33)

$$[K_{\sigma}] = [K]_L + [K_{\sigma}] + [K_{\sigma}]$$

(34)

$$[K_{\sigma}] d\{\delta\} = \int_V ([B]_{NL})^T \{\sigma\} dV$$

(35)

where,

- \([K]_L\) = Linear part of the stiffness matrix

$$= \int_V [B]_L^T [D] [B]_L dV$$
\[ [K_{\sigma}]_{NL} = \text{Nonlinear part of the tangent stiffness matrix} \]
\[ = \int_V \left( \left[B^T\right][D][B]_{NL} + [B]_{NL}[D][B] + [B]_{NL}^T[D][B]_{NL} \right) dV \]
\[ [K_{\sigma}] = \text{Initial stress matrix or Geometric stiffness matrix} \]

6.3 Large deflection analysis in N-notation

Referring Eq.(23),

\[ [B^p]_{NL} = [A][G] \]  \hspace{1cm} \hspace{1cm} (36) \]

\[ [B]_{NL} = \begin{bmatrix} [B^p]_{NL} \\ [B^p]_{NL} \end{bmatrix} = \begin{bmatrix} [A][G] \\ [0] \end{bmatrix} \]  \hspace{1cm} \hspace{1cm} (37) \]

The volume integral is replaced by area integral, as the contribution across the thickness direction is considered in the rigidity matrix.

Using Eq.(6) and (36), Eq.(35) becomes

\[ [K_{\sigma}] d\{\delta\} = \int_V [B]_{NL}^T[\sigma] dV \]
\[ = \int_A [G] \begin{bmatrix} d \hat{w} \\ d \hat{w} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix} dA \]
\[ = \int_A [G] \begin{bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix} dA \]
\[ = \int_A [G]^T \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}^{T} \begin{bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{bmatrix} dA \]
\[ = \int_A [G]^T [S] \begin{bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{bmatrix} dA \]
\[ = \int_A [G]^T [S] [G] d\{\delta\} dA \]
\[ [K_{\sigma}] = \int_A [G]^T [S] [G] dA \]  \hspace{1cm} \hspace{1cm} (44) \]

where, \([S] = \begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix}\)
The secant stiffness matrix \([K_3]\) derived in Section-6.1 is unsymmetric in B-notation. In this context, N-notation [20] is used which gives symmetric secant stiffness matrix and is expressed as
\[
[K_3] = [N_0] + \frac{1}{2} [N_1] + \frac{1}{3} [N_2]
\]
\[
[N_0] = \int_A [B]^T[D][B]_L dA
\]
\[
[N_1] = \int_A \left([B]^T[D] [B]_{NL} + [B]_{NL}^T[D][B]_L + [G]^T[S]_L[G]\right) dA
\]
\[
[N_2] = \int_A \left([B]_{NL}^T[D][B]_{NL} + [G]^T[S]_{NL}[G]\right) dA
\]
where, \([S]_L\) and \([S]_{NL}\) are the linear and nonlinear parts of the initial stress matrix.

### 7. NEWTON-RAPHSON METHOD

The nonlinear equations can be expressed as
\[
\Psi\{\delta\} = [K_3\{\delta\}][\{\delta\}] - \{P\} = \{0\}
\]
(45)

Ignoring higher order terms, the function \(\Psi\{\delta\}\) is expressed in terms of Taylor series as,
\[
\Psi\left(\{\delta\}^{n+1}\right) = \Psi\left(\{\delta\}^n\right) + \left(\frac{d\Psi}{d\{\delta\}}\right)^n [\Delta\delta]^n = 0
\]
(46)

where, \(\{\delta\}^{n+1} = \{\delta\}^n + [\Delta\delta]^n\)
(47)

\[\{\Delta\delta\}^{n+1} = ([K_T]^n)^{-1}\left(\{P\} - [K_T]^n\{\delta\}^n\right)\]
(48)

where,
- \(\{P\}\) = Load Level; \(\{\Delta\delta\}^n = \) Incremental Displacement; \(K_T = \frac{d\Psi}{d\{\delta\}}\)

### 8. BOUNDARY CONDITIONS

As a general case, the stiffness matrix for a curved boundary supported on elastic springs continuously spread along the boundary line is used. Considering a local axis system \(x_i - y_i\) at a point \(P\) on a curved boundary along the direction of the normal to the boundary at that point as shown in the Fig.4, the displacement components along it can be obtained. Let \(\beta\) be the angle made by the local axis \(x_i - y_i\) with the global axis \(x - y\) in the anticlockwise direction. Hence the relationship between the two axes is given by
Figure 4. Co-ordinate axes at any point of an elastically restrained curved boundary

\[
\begin{bmatrix}
  x \\
  y 
\end{bmatrix} =
\begin{bmatrix}
  \cos \beta & -\sin \beta \\
  \sin \beta & \cos \beta 
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  y_1 
\end{bmatrix}
\]  

(49)

The displacements at \(P\) which may be restrained can be expressed as

\[
\begin{bmatrix}
  u \\
  v \\
  w \\
  \frac{\partial w}{\partial x} \\
  \frac{\partial w}{\partial y} 
\end{bmatrix}
= \frac{1}{\rho}
\begin{bmatrix}
  \cos \beta & \sin \beta & 0 & 0 & 0 \\
  -\sin \beta & \cos \beta & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & \cos \beta & \sin \beta \\
  0 & 0 & 0 & -\sin \beta & \cos \beta 
\end{bmatrix}
\begin{bmatrix}
  u \\
  v \\
  w \\
  \frac{\partial w}{\partial x} \\
  \frac{\partial w}{\partial y} 
\end{bmatrix}
\]  

(50)

Expressing the above equation in terms of the shape function,

\[
\{f_b\} = [N_b]\{\delta\}
\]  

(51)

where

\[
[N_b] =
\begin{bmatrix}
  \cos \beta & \sin \beta & 0 & 0 & 0 \\
  -\sin \beta & \cos \beta & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & \cos \beta & \sin \beta \\
  0 & 0 & 0 & -\sin \beta & \cos \beta 
\end{bmatrix}
\]  

(52)
Let \( k_u, k_v, k_w, k_\alpha \) and \( k_\beta \) be the spring constants or restraint coefficients corresponding to the direction of \( u, v, w, \theta_\alpha \) and \( \theta_\beta \) respectively. The reaction components per unit length along the boundary line due to the spring constants corresponding to the possible boundary displacements are given by

\[
\begin{bmatrix}
k_uu_1 \\
k_vv_1 \\
k_ww_1 \\
k_\alpha \frac{\partial w}{\partial x_1} \\
k_\beta \frac{\partial w}{\partial y_1}
\end{bmatrix}
\]

\[
\{ f_b \} = [ N_b ] \{ \delta \}
\tag{54}
\]

where

\[
[N_b] = \begin{bmatrix}
k_u \cos \beta & k_u \sin \beta & 0 & 0 & 0 \\
- k_v \sin \beta & k_v \cos \beta & 0 & 0 & 0 \\
0 & 0 & k_w & 0 & 0 \\
0 & 0 & 0 & k_\alpha \cos \beta & k_\alpha \sin \beta \\
0 & 0 & 0 & - k_\beta \sin \beta & k_\beta \cos \beta
\end{bmatrix}
\tag{55}
\]

Using Eqs.(51) and (54), the stiffness matrix can be obtained by the virtual work principle and is expressed as

\[
[K_b] = \int [N_b]^T [N_b] J_b |d\lambda_1|
\tag{56}
\]

where \( \lambda_1 \) is the direction of the boundary line in the \( \xi - \eta \) plane and \( J_b = \text{Jacobian} = ds/d\lambda_1 \). The Jacobian is the ratio of actual length to the length of mapped domain at any segment of boundary length.
9. LOAD VECTOR

The consistent load vector [9] can be calculated from the principle of virtual work.

\[
\{P\} = \int [N_e]^T q \, J \, d\xi \, d\eta \tag{57}
\]

where \( q \) is the intensity of load acting on the plate.

<table>
<thead>
<tr>
<th>Table 1: Different parameters used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central deflection ( w )</td>
</tr>
<tr>
<td>Thickness ( h )</td>
</tr>
<tr>
<td>Uniformly distributed load ( q )</td>
</tr>
<tr>
<td>Concentrated load ( P )</td>
</tr>
<tr>
<td>Extreme fiber stress ( \sigma(x, y) )</td>
</tr>
</tbody>
</table>

10. RESULTS AND DISCUSSION

The proposed formulation is validated through a number of numerical examples. In each of the examples, the iteration process is continued until the total residual norm is within the prescribed tolerance limit as expressed by

\[
\left(\frac{\Delta P^T \Delta P}{\|P^T P\|}\right)^{0.5} \times 100 \leq \gamma \tag{58}
\]

where \( \gamma \) is the tolerance for the convergence and it is taken as 0.1%. The analyses for plate of different planforms such as square, skewed, circular, elliptical, annular, trapezoidal, semicircular, right angled triangular, equilateral triangular, semi-circular semi-elliptical, diamond and axe-head shaped are carried out with mesh divisions for the whole plate. The deflections and stresses obtained at critical points are compared with the published and SAP 2000 results wherever possible. Also, some examples of clamped square, rectangular, skew and circular plate under uniformly distributed load by the same authors using the present method can be found in [21].

The different parameters used are tabulated in Table 1 and the non-dimensional form for loads, deflections and stresses are given in Table 2. The boundary conditions applied are

\[
\begin{align*}
\begin{array}{l}
u = w = \theta_x = \theta_y = 0, \text{ for clamped edge and} \\
v = w = 0, \text{ for simply supported edge.}
\end{array}
\end{align*}
\]

10.1 Rectangular plate

Square plate in Fig.5 with two edges simply supported and other two free is analyzed and compared in Table 3. Different aspect ratios (1.0, 1.5 and 2.0) are considered for all the edges simply supported boundary conditions and the results for 32×32 mesh division are shown in Table 4-5.
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Table 2: Non-dimensional form

<table>
<thead>
<tr>
<th></th>
<th>Central deflection $W$</th>
<th>$w/h$</th>
<th>Uniformly distributed load $Q$</th>
<th>$(qa^4)/Eh^4$</th>
<th>Concentrated load $P$</th>
<th>$(pa^2)/Eh^4$</th>
<th>Extreme fiber stress $\sigma'(x, y)$</th>
<th>$(\sigma(x, y)a^2)/(Eh^2)$</th>
</tr>
</thead>
</table>

Figure 5. A typical $8 \times 8$ mesh discretization with boundary nodes of square plate

Table 3: Square plate with the two edges simply supported and the other two free

<table>
<thead>
<tr>
<th>Load $(N/cm)$</th>
<th>Central Deflection $W$</th>
<th>Moment $(N \cdot cm)$</th>
<th>Inplane Force $(N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>Milasinovic [22]</td>
<td>Present SAP</td>
<td>Present SAP</td>
</tr>
<tr>
<td>1.0</td>
<td>0.3318</td>
<td>0.3449</td>
<td>0.3449</td>
</tr>
<tr>
<td>2.5</td>
<td>0.5537</td>
<td>0.5780</td>
<td>0.5779</td>
</tr>
<tr>
<td>5.0</td>
<td>0.7686</td>
<td>0.7954</td>
<td>0.7951</td>
</tr>
<tr>
<td>7.5</td>
<td>0.9179</td>
<td>0.9422</td>
<td>0.9420</td>
</tr>
<tr>
<td>10.0</td>
<td>1.0352</td>
<td>1.0569</td>
<td>1.0567</td>
</tr>
</tbody>
</table>

Table 4: Deflection $W = w/h$ at the center of the simply supported rectangular plate

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>$b/a = 1.0$</th>
<th>$b/a = 1.5$</th>
<th>$b/a = 2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.79</td>
<td>0.5450</td>
<td>0.7533</td>
<td>0.8104</td>
</tr>
<tr>
<td>38.3</td>
<td>0.8281</td>
<td>1.0592</td>
<td>1.1081</td>
</tr>
<tr>
<td>63.4</td>
<td>1.0416</td>
<td>1.2902</td>
<td>1.3365</td>
</tr>
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<td>95</td>
<td>1.2309</td>
<td>1.4976</td>
<td>1.5442</td>
</tr>
<tr>
<td>134</td>
<td>1.4102</td>
<td>1.6966</td>
<td>1.7454</td>
</tr>
<tr>
<td>184</td>
<td>1.5826</td>
<td>1.8902</td>
<td>1.9424</td>
</tr>
<tr>
<td>245</td>
<td>1.7547</td>
<td>2.0853</td>
<td>2.1419</td>
</tr>
<tr>
<td>318</td>
<td>1.9239</td>
<td>2.2786</td>
<td>2.3400</td>
</tr>
<tr>
<td>402</td>
<td>2.0871</td>
<td>2.4663</td>
<td>2.5327</td>
</tr>
</tbody>
</table>
### Table 5: Stress $\sigma' = \sigma a^2/Eh^2$ at the center of the simply supported rectangular plate

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>$b/a = 1.0$</th>
<th>$b/a = 1.5$</th>
<th>$b/a = 2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.79</td>
<td>4.3881</td>
<td>5.8130</td>
<td>6.1066</td>
</tr>
<tr>
<td>38.3</td>
<td>7.1796</td>
<td>8.9021</td>
<td>9.1177</td>
</tr>
<tr>
<td>63.4</td>
<td>9.5160</td>
<td>11.5466</td>
<td>11.7503</td>
</tr>
<tr>
<td>95</td>
<td>11.7789</td>
<td>14.1736</td>
<td>14.4051</td>
</tr>
<tr>
<td>134</td>
<td>14.1106</td>
<td>16.9354</td>
<td>17.2225</td>
</tr>
<tr>
<td>184</td>
<td>16.5418</td>
<td>19.8560</td>
<td>20.2192</td>
</tr>
<tr>
<td>245</td>
<td>19.1660</td>
<td>23.0388</td>
<td>23.4963</td>
</tr>
<tr>
<td>318</td>
<td>21.9443</td>
<td>26.4296</td>
<td>26.9949</td>
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<tr>
<td>402</td>
<td>24.8184</td>
<td>29.9505</td>
<td>30.6322</td>
</tr>
</tbody>
</table>

10.2 Skew plate

The deflection and central stress of simply supported skewed plate shown in Fig.6 are compared with published results in Fig.7-8 respectively.

\[ a = 300\text{cm}; \ h = 3.0\text{cm}; \ \nu = 0.316; \ E = 0.3\times10^8 \text{N/cm}^2 \]

**Figure 6.** A typical $8\times8$ mesh discretization with boundary nodes of skew plate

**Figure 7.** Central deflection of simply supported skew plate

**Figure 8.** Central stress of simply supported skew plate
10.3 Circular plate

A circular plate shown in Fig. 9 is analyzed and results for clamped case are compared with published ones for concentrated load in Fig. 10-11. The present results for simply supported circular plate under uniformly distributed load (udl) and point load are given in Table 6 for 24×24 mesh division.

\[
\begin{align*}
r &= 100cm; \\ h &= 2.0cm; \\ \nu &= 0.3; \\ E &= 0.1 \times 10^6 N/cm^2
\end{align*}
\]

Figure 9. A typical 8×8 mesh discretization with boundary nodes of circular plate

Figure 10. Central deflection of clamped circular plate under concentrated load

Figure 11. Edge stress of clamped circular plate under concentrated load

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>Central Deflection</th>
<th>Central Stress</th>
<th>Load Factor</th>
<th>Central Deflection</th>
<th>Central Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>1.0348</td>
<td>1</td>
<td>0.4365</td>
<td>2.3556</td>
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<tr>
<td>2</td>
<td>0.7035</td>
<td>1.6261</td>
<td>2</td>
<td>0.6863</td>
<td>4.3564</td>
</tr>
<tr>
<td>3</td>
<td>0.8517</td>
<td>2.053</td>
<td>3</td>
<td>0.8631</td>
<td>6.1561</td>
</tr>
<tr>
<td>6</td>
<td>1.1385</td>
<td>2.9678</td>
<td>4</td>
<td>1.004</td>
<td>7.8342</td>
</tr>
<tr>
<td>10</td>
<td>1.3826</td>
<td>3.8515</td>
<td>5</td>
<td>1.1232</td>
<td>9.426</td>
</tr>
<tr>
<td>15</td>
<td>1.6008</td>
<td>4.7342</td>
<td>6</td>
<td>1.2275</td>
<td>10.951</td>
</tr>
</tbody>
</table>
### 10.4 Elliptical plate

Table 7 presents the results with $24 \times 24$ mesh size for both clamped and simple supports of elliptic plate shown in Fig.12 under uniformly distributed load where $a$ and $b$ are taken as semi-minor and semi-major axes of the plate respectively.

$$a = 100\text{cm}; b = 200\text{cm}; h = 2.0\text{cm}; \nu = 0.3; E = 0.1 \times 10^6 \text{N/cm}^2$$

Figure 12. A typical $8 \times 8$ mesh discretization with boundary nodes of elliptical plate

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Weil [23]</td>
<td>Present</td>
</tr>
<tr>
<td>Clamped</td>
<td>1</td>
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<td>0.3505</td>
<td>0.9</td>
<td>1.705</td>
<td>1.6023</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.600</td>
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<td>3.322</td>
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<tr>
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<td>3</td>
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<td>0.8086</td>
<td>2.077</td>
<td>2.2268</td>
<td>4.76</td>
</tr>
<tr>
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<td>1.163</td>
<td>1.1945</td>
<td>3.217</td>
<td>3.429</td>
<td>8.289</td>
</tr>
<tr>
<td></td>
<td>10</td>
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<td>1.5205</td>
<td>4.333</td>
<td>4.519</td>
<td>11.970</td>
</tr>
<tr>
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<td>15</td>
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<td>1.8088</td>
<td>5.490</td>
<td>5.5656</td>
<td>15.656</td>
</tr>
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<td>Simply Supported</td>
<td>1</td>
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<td>1.3994</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
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<td>2.0384</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
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<td>2.5069</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.4869</td>
<td>3.5477</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>10</td>
<td>1.777</td>
<td>4.593</td>
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<td></td>
<td></td>
</tr>
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<td>15</td>
<td>2.0441</td>
<td>5.6611</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 10.5 Annular plate

An annular plate (Fig.13) of sector angle ($\theta = 60^\circ$) is analyzed under uniformly distributed load and the deflection, membrane forces and bending moments at center are compared for clamped boundary conditions in Fig.14 -16. $r_o$ and $r_i$ are outer and inner radius of the plate respectively. The results for the case of simply supported boundary condition are given in Table 8 for $24 \times 24$ mesh division.
\[
\frac{r_o}{r_i} = 0.5 \quad a = r_o - r_i \quad h = 2.0 \text{cm} \quad \nu = 0.3 \quad E = 0.1 \times 10^8 \text{N/cm}^2
\]

Figure 13. A typical $8 \times 8$ mesh discretization with boundary nodes of annular plate

Figure 14. Central deflection of clamped annular plate

Figure 15. Membrane forces at center of clamped annular plate

Figure 16. Bending moments at center of clamped annular plate
Table 8: Central deflection and forces in simply supported annular plate

<table>
<thead>
<tr>
<th>Load Factor $h/w$</th>
<th>Central Deflection $N_1 r^2/(Eh^3)$</th>
<th>Membrane Force $N_0 r^2/(Eh^3)$</th>
<th>Bending Moment $M_1 r^2/(Eh^4)$</th>
<th>$M_0 r^2/(Eh^4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>666.7</td>
<td>1.1496</td>
<td>13.7285</td>
<td>8.121</td>
<td>4.0567</td>
</tr>
<tr>
<td>1333.3</td>
<td>1.4986</td>
<td>23.2935</td>
<td>14.143</td>
<td>5.0545</td>
</tr>
<tr>
<td>2666.7</td>
<td>1.9163</td>
<td>38.2685</td>
<td>23.753</td>
<td>6.1317</td>
</tr>
<tr>
<td>4000</td>
<td>2.2013</td>
<td>50.82</td>
<td>31.832</td>
<td>6.8112</td>
</tr>
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</table>

10.6 Trapezoidal plate

In Table 9, results are presented for central deflection and stress for a clamped and simply supported trapezoidal plate (Fig. 17) for 24×24 mesh division. $\alpha$ and $\beta$ are the angles made by left and right edge of trapezoid to $X$-axis. $L_x = $ base length of trapezoid and $L_y = $ slant length of left edge of trapezoid.

![Trapezoidal plate diagram](image)

$\alpha = 60; \beta = 120; \frac{L_x}{L_y} = 1.5; \nu = 0.3; E = 0.1 \times 10^8 \text{ N/cm}^2$

Figure 17. A typical 8×8 mesh discretization with boundary nodes of trapezoidal plate

Table 9: Deflection and stress in trapezoidal plate

<table>
<thead>
<tr>
<th>Boundaries</th>
<th>Central Deflection</th>
<th>Central Stress</th>
<th>Mid-edge Stress</th>
<th>Central Deflection</th>
<th>Central Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.0422</td>
<td>0.0429</td>
<td>0.578</td>
<td>0.5872</td>
<td>1.2493</td>
</tr>
<tr>
<td>10</td>
<td>0.0842</td>
<td>0.0855</td>
<td>1.168</td>
<td>1.1875</td>
<td>2.5023</td>
</tr>
<tr>
<td>50</td>
<td>0.3908</td>
<td>0.3961</td>
<td>5.834</td>
<td>5.9146</td>
<td>12.2128</td>
</tr>
<tr>
<td>100</td>
<td>0.678</td>
<td>0.686</td>
<td>10.589</td>
<td>10.7238</td>
<td>22.7819</td>
</tr>
<tr>
<td>200</td>
<td>1.0521</td>
<td>1.064</td>
<td>17.124</td>
<td>17.337</td>
<td>39.8275</td>
</tr>
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</table>
10.7 Semicircular Plate
A semicircular plate (Fig. 18) is analyzed for a clamped and simply supported boundary conditions and the results for 24x24 mesh division are shown in Table 10 and 11 respectively. The deflection and stress at coordinate (0, 56.66) and mid stress at bottom edge are calculated. $r =$ radius of the semicircle.

$$r = 100 \text{cm}; \; h = 2.0 \text{cm}; \; \nu = 0.3; \; E = 0.1 \times 10^8 \text{N/cm}^2$$

Figure 18. A typical 8x8 mesh discretization with boundary nodes of semicircular plate

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>Deflection Present</th>
<th>SAP</th>
<th>Stress Present</th>
<th>SAP</th>
<th>Mid-edge Stress Present</th>
<th>SAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
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<td>0.470</td>
<td>4.977</td>
<td>5.024</td>
<td>10.136</td>
<td>8.667</td>
</tr>
<tr>
<td>50</td>
<td>0.773</td>
<td>0.781</td>
<td>8.719</td>
<td>8.782</td>
<td>18.367</td>
<td>16.010</td>
</tr>
<tr>
<td>75</td>
<td>0.988</td>
<td>0.999</td>
<td>11.496</td>
<td>11.554</td>
<td>25.189</td>
<td>21.328</td>
</tr>
<tr>
<td>100</td>
<td>1.154</td>
<td>1.169</td>
<td>13.733</td>
<td>13.770</td>
<td>31.155</td>
<td>25.735</td>
</tr>
<tr>
<td>125</td>
<td>1.292</td>
<td>1.309</td>
<td>15.638</td>
<td>15.645</td>
<td>36.549</td>
<td>29.560</td>
</tr>
</tbody>
</table>

Table 11: Deflection and stress in simply supported semicircular plate

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>Deflection Present</th>
<th>SAP</th>
<th>Stress Present</th>
<th>SAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.855</td>
<td>0.829</td>
<td>7.046</td>
<td>6.839</td>
</tr>
<tr>
<td>50</td>
<td>1.134</td>
<td>1.109</td>
<td>10.173</td>
<td>9.920</td>
</tr>
<tr>
<td>75</td>
<td>1.322</td>
<td>1.297</td>
<td>12.518</td>
<td>12.214</td>
</tr>
<tr>
<td>100</td>
<td>1.468</td>
<td>1.443</td>
<td>14.490</td>
<td>14.132</td>
</tr>
<tr>
<td>125</td>
<td>1.590</td>
<td>1.565</td>
<td>16.241</td>
<td>15.824</td>
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</table>
10.8 Right-angled Triangular Plate

The deflection and stress at coordinate (37.5, 37.5) and (56.25, 37.5) for b/a = 1.0 and 1.5 respectively and mid stress at bottom edge of right angled plate (Fig. 19) are presented for clamped and simply supported boundary conditions in Table 12-15 for 24×24 mesh division.

Figure 19. A typical 8×8 mesh discretization with boundary nodes of right-angled triangular plate

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>Central Deflection</th>
<th>Central Stress</th>
<th>Mid-edge Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present</td>
<td>SAP</td>
<td>Present</td>
</tr>
<tr>
<td>300</td>
<td>0.411</td>
<td>0.418</td>
<td>11.583</td>
</tr>
<tr>
<td>350</td>
<td>0.464</td>
<td>0.471</td>
<td>13.339</td>
</tr>
<tr>
<td>400</td>
<td>0.514</td>
<td>0.521</td>
<td>15.015</td>
</tr>
<tr>
<td>450</td>
<td>0.560</td>
<td>0.568</td>
<td>16.615</td>
</tr>
<tr>
<td>500</td>
<td>0.630</td>
<td>0.611</td>
<td>18.143</td>
</tr>
</tbody>
</table>

Table 13: Deflection and stress in simply supported right-angled triangular plate for b/a = 1

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>Deflection</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present</td>
<td>SAP</td>
</tr>
<tr>
<td>300</td>
<td>0.794</td>
<td>0.783</td>
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<tr>
<td>350</td>
<td>0.849</td>
<td>0.838</td>
</tr>
<tr>
<td>400</td>
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<tr>
<td>450</td>
<td>0.943</td>
<td>0.932</td>
</tr>
<tr>
<td>500</td>
<td>0.985</td>
<td>0.973</td>
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</table>
Table 14: Deflection and stress in clamped right-angled triangular plate for $b/a = 1.5$

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>Deflection</th>
<th>Stress</th>
<th>Mid-edge Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present</td>
<td>SAP</td>
<td>Present</td>
</tr>
<tr>
<td>300</td>
<td>0.640</td>
<td>0.648</td>
<td>17.464</td>
</tr>
<tr>
<td>350</td>
<td>0.707</td>
<td>0.716</td>
<td>19.607</td>
</tr>
<tr>
<td>400</td>
<td>0.768</td>
<td>0.777</td>
<td>21.605</td>
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<td>500</td>
<td>0.876</td>
<td>0.885</td>
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</tr>
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</table>

Table 15: Deflection and stress in simply supported right-angled triangular plate for $b/a = 1.5$

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>Deflection</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present</td>
<td>SAP</td>
</tr>
<tr>
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<td>1.008</td>
<td>1.010</td>
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<td>1.073</td>
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<tr>
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<td>1.227</td>
<td>1.228</td>
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</table>

10.9 Equilateral Triangular Plate

The results of equilateral triangular Plate (Fig. 20) are shown for a clamped and simply supported boundary conditions in Table 16 and 17 respectively for $24 \times 24$ mesh size. The deflection and stress at coordinate $(56.25, 32.475)$ and mid stress at bottom edge are presented.

![Figure 20](image_url)

$a = 100\text{cm}; \ h = 2.0\text{cm}; \ \nu = 0.3; \ E = 0.1\times10^8 \text{N/cm}^2$

Figure 20. A typical $8 \times 8$ mesh discretization with boundary nodes of equilateral triangular plate
Table 16: Deflection and stress in clamped equilateral triangular plate

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>Deflection</th>
<th>Stress</th>
<th>Mid-edge Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present</td>
<td>SAP</td>
<td>Present</td>
</tr>
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</tr>
<tr>
<td>350</td>
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<td>16.123</td>
</tr>
<tr>
<td>400</td>
<td>0.549</td>
<td>0.569</td>
<td>18.097</td>
</tr>
<tr>
<td>450</td>
<td>0.600</td>
<td>0.621</td>
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</tr>
<tr>
<td>500</td>
<td>0.648</td>
<td>0.669</td>
<td>21.747</td>
</tr>
</tbody>
</table>

Table 17: Deflection and stress in simply supported equilateral triangular plate

<table>
<thead>
<tr>
<th>Load Factor</th>
<th>Deflection</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present</td>
<td>SAP</td>
</tr>
<tr>
<td></td>
<td>Present</td>
<td>SAP</td>
</tr>
<tr>
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<td>0.819</td>
</tr>
<tr>
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<td>0.900</td>
<td>0.879</td>
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<tr>
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<td>0.953</td>
<td>0.933</td>
</tr>
<tr>
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<td>0.982</td>
</tr>
<tr>
<td>500</td>
<td>1.046</td>
<td>1.027</td>
</tr>
</tbody>
</table>

10.10 Semi-circular Semi-elliptical Plate

A plate (Fig. 21) consisting of a semicircle and semi-ellipse is analyzed. In Table 18, results for central deflection and stresses in a clamped and simply supported boundary conditions are shown for 16×16 mesh division. Here, \( r \) = radius of the semicircle, \( a \) and \( b \) are semi-minor and semi-major axis of the plate.

\[
a = r = 200cm; \ b = 250cm; \ h = 2.0cm; \ \nu = 0.3; \ E = 0.1 \times 10^8 N/cm^2
\]

Figure 21. A typical 8×8 mesh discretization with boundary nodes of semi-circular semi-elliptical plate
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Table 18: Central deflection and stress in semi-circular semi-elliptical plate

<table>
<thead>
<tr>
<th>Boundaries</th>
<th>Central Deflection</th>
<th>Central Stress</th>
<th>Mid-edge Stress</th>
<th>Simply Supported</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2056</td>
<td>0.5919</td>
<td>0.8767</td>
<td>0.5435</td>
</tr>
<tr>
<td>2</td>
<td>0.3888</td>
<td>1.1653</td>
<td>1.7287</td>
<td>0.7784</td>
</tr>
<tr>
<td>3</td>
<td>0.5437</td>
<td>1.6737</td>
<td>2.5245</td>
<td>0.9333</td>
</tr>
<tr>
<td>6</td>
<td>0.8832</td>
<td>2.8378</td>
<td>4.5955</td>
<td>1.2339</td>
</tr>
<tr>
<td>10</td>
<td>1.1818</td>
<td>3.9084</td>
<td>6.8788</td>
<td>1.4909</td>
</tr>
<tr>
<td>15</td>
<td>1.4451</td>
<td>4.8997</td>
<td>9.3054</td>
<td>1.7215</td>
</tr>
</tbody>
</table>

10.11 Diamond Shaped Plate

A diamond shaped plate (Fig. 22 and Fig. 23) are analyzed for \( r_2/r_1 = 1.0 \) and 1.5 respectively. In Table 19 and 20, results for central deflection and stresses in a clamped and simply supported boundary conditions are shown for \( 32 \times 32 \) mesh division. Here, \( r_1, r_2 \) = radii of the arcs.

![Diagram of diamond shaped plate](image)

\( r_2/r_1 = 1.0; \quad r_1 = 100 \text{cm}; \quad h = 2.0 \text{cm}; \quad \nu = 0.3; \quad E = 0.1 \times 10^8 \text{N/cm}^2 \)

Figure 22. A typical \( 8 \times 8 \) mesh discretization with boundary nodes of diamond shaped plate for \( r_2/r_1 = 1.0 \)

Table 19: Central deflection and stress in diamond shaped plate for \( r_2/r_1 = 1.0 \)

<table>
<thead>
<tr>
<th>Boundaries</th>
<th>Central Deflection</th>
<th>Central Stress</th>
<th>Mid-edge Stress</th>
<th>Simply Supported</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>0.8676</td>
<td>18.6576</td>
<td>45.1430</td>
<td>1.1941</td>
</tr>
<tr>
<td>350</td>
<td>0.9546</td>
<td>20.6878</td>
<td>51.1530</td>
<td>1.2709</td>
</tr>
<tr>
<td>400</td>
<td>1.0329</td>
<td>22.5304</td>
<td>56.8642</td>
<td>1.3397</td>
</tr>
<tr>
<td>450</td>
<td>1.1042</td>
<td>24.2230</td>
<td>62.3197</td>
<td>1.4021</td>
</tr>
<tr>
<td>500</td>
<td>1.1696</td>
<td>25.7914</td>
<td>67.5537</td>
<td>1.4595</td>
</tr>
</tbody>
</table>
$r_2/r_1 = 1.5; \ r_1 = 100\text{cm}; \ h = 2.0\text{cm}; \ \nu = 0.3; \ E = 0.1 \times 10^8 \text{N/cm}^2$

Figure 23. A typical $8 \times 8$ mesh discretization with boundary nodes of diamond shaped plate for $r_2/r_1 = 1.5$

| Table 20: Central deflection and stress in diamond shaped plate for $r_2/r_1 = 1.5$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Boundaries      | Clamped         | Simply Supported |
| Load            | Central Deflection | Central Stress | Mid-edge Stress | Central Deflection | Central Stress |
| 300             | 0.7733          | 22.3848         | 51.1662         | 1.1114          | 25.2125         |
| 350             | 0.8493          | 24.8401         | 57.7238         | 1.1780          | 27.2463         |
| 400             | 0.9177          | 27.0040         | 63.9263         | 1.2380          | 29.1343         |
| 450             | 0.9799          | 29.0333         | 69.8305         | 1.2927          | 30.9060         |
| 500             | 1.0371          | 30.9201         | 75.4810         | 1.3432          | 32.5822         |

10.12 Axe-head Shaped Plate

An axe-head shaped plate (Fig.24) is analyzed and results for central deflection and stresses in a clamped and simply supported boundary condition for $32 \times 32$ mesh size are shown in Table 21. Here, $r =$ radius of the arc.

$\ r_2/r_1 = 1.0; \ r_1 = 100\text{cm}; \ h = 2.0\text{cm}; \ \nu = 0.3; \ E = 0.1 \times 10^8 \text{N/cm}^2$

Figure 24. A typical $8 \times 8$ mesh discretization with boundary nodes of axe-head shaped plate
11. CONCLUSIONS

The formulation for large deflection of thin plates of arbitrary shape is generalized by means of a mapping technique so that the analysis is performed in a square domain. Many researchers have used different elements to analyze plates but these elements are limited to solve a particular type of geometry only. The isoparametric element though elegant in its formulation to accommodate different geometries but is deficient with regard to its behavior because of the presence of shear locking and spurious mechanisms and cannot be fully alleviated even if with reduced and selective integration. In the present investigation, the element has all the advantages of the isoparametric element to model an arbitrary plate shape and without the disadvantages of the shear locking problem etc. The versatility of the element is proved by undertaking different plate geometries such as square, rectangle, skew, circle, ellipse, annular, and trapezoidal. Also, semicircular, right angled triangle, equilateral triangle shaped geometries along with some complex geometries are analyzed and compared with SAP 2000. In this context, the formulation is done in the total Lagrangian co-ordinate system and Newton-Raphson technique is used to solve nonlinear governing equations. The results obtained for various geometries of the plate are validated with the available ones which are found to be in excellent agreement.

REFERENCES

1. Levy S. Square plate with clamped edges under normal pressure producing large deflections, NASA Technical Note 847.