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DOLPHIN ECHOLOCATION OPTIMIZATION FOR DESIGN OF CANTILEVER RETAINING WALLS

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ABSTRACT

Dolphin Echolocation Optimization (DEO) is a newly developed meta-heuristic optimization method inspired from dolphin's rules for searching their environment. In this paper, reinforced cantilever retaining walls are designed by DEO. Results show that DEO not only leads to better results in comparison to the previously utilized algorithms but also optimality curves achieved with this method provides the engineers better understanding of the design.

Keywords: Optimal design; retaining walls; dolphin echolocation; meta-heuristic algorithms.

1. INTRODUCTION

Retaining walls are structures to restrain soil in unnatural slopes. These structures should also protect against erosion on susceptible sites. Retaining walls are normally designed to withstand lateral earth and water pressures, the effects of surcharge loads, self-weight of the wall and in special cases, earthquake loads. Concrete retaining walls are suitable solution for structures in contact with soil and exposed to constant wetting and drying. The following parameters influence the design of the retaining walls:

- Wall height,
- Soil type,

• Sloping land below and/or above the retaining wall,

• Loads above and behind the retaining wall, e.g. parked cars.

Changes in each of these parameters can affect the other parameters. Design process usually starts by selecting parameters according to engineering judgment and refining their values by trial and error. This process can be substituted by an optimization method in order to achieve better results with less time and effort.

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Initial optimum design methods applied for design of retaining walls were mathematical-Based optimization methods. Pochtman [1] presented a gradient-based method for optimum design of a retaining walls. Dembicki and Chi [2] determine optimum shape of a retaining wall by a nonlinear multi-objective optimization. Keskar and Adidam [3], minimized cost of a cantilever retaining wall by the sequential unconstrained optimization technique (SUMT). Sarıbaş and Erbatur [4] utilized a nonlinear programming method for design of a retaining wall; Basudhar and Lakshman [5] adopted the sequential unconstrained minimization technique along with Powell's algorithm for multi-dimensional search and quadratic interpolation technique for one dimensional search in optimum design of a retaining wall. Sivakumar and Munwar [6], described an approach for reliability-based design optimization of reinforced concrete cantilever retaining wall.

Nature inspired optimization algorithms namely meta-heuristic algorithms were then utilized for design of cantilever retaining walls. Yepes et al. [7] presented a parametric study of commonly used walls for different fills and bearing conditions by simulated annealing algorithm; Kaveh and Shakouri [8] minimized cost of retaining wall of a given height by Harmony search algorithm; Camp and Akin [9] performed retaining wall design optimization by Big Bang-Big Crunch method. Kaveh and Behnam [10] optimized design of retaining walls by charges system search algorithm. Pei et al. [11] applied three heuristic algorithms, including genetic algorithm (GA), particle swarm optimization (PSO) and simulated annealing (SA) to solve the constrained optimization of retaining wall cost minimization. Kajehzadeh et al. [12] applied a gravitational search algorithm for design of a retaining wall Kaveh et al. [13] utilized multi-objective genetic algorithm for design of a retaining wall under seismic loads. Kaveh and Khayatazad [14] presented a model to obtain the optimum cost of the cantilever retaining walls by Chaotic Imperialist optimization algorithm.

In this study, the recently developed dolphin echolocation algorithm is applied to design of cantilever retaining walls. DEO is selected here because it has the ability to be adjusted for a pre-determined computational cost and also it is self adaptive.

After introduction the natural behavior of dolphins is studied to show the source of inspiration. Then Dolphin Echolocation Optimization in continuous search space is described. In subsequent section the problem of retaining wall design is discussed. This section is accompanied by the results section. Final section is devoted to conclusion.

2. DOLPHIN ECHOLOCATION IN NATURE

Dolphins can detect, discriminate, and pursue preys by means of their biosonar systems. They are able to generate sounds in the form of clicks. When the sound strikes an object, some of the sound-wave energy is reflected back towards the dolphin. The time lapse between click and echo enables the dolphin to estimate the distance from the object. This process is depicted in Fig. The varying strength of the signal as it is received on both sides of the dolphin's head makes it possible for him to evaluate the direction. By continuously emitting clicks and receiving echoes in this way, a dolphin can track objects. The click production speeds up when approaching an object of interest [15, 16, 17].

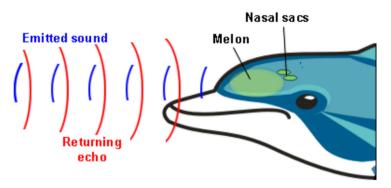


Figure 1. Dolphin echolocation in nature (Electronic science tutor n.d.)

3. DOLPHIN ECHOLOCATION OPTIMIZATION IN CONTINUOUS SEARCH SPACE

Dolphins take advantages of echolocation to discover their environment. The problem of finding suitable values for some variables' in a search space is like dolphin's search in their environment. This fact is simulated in an optimization method called Dolphin Echolocation Optimization, Kaveh and Farhoudi [18].

An optimization problem which is to choose the best answer is similar to dolphin's attempt to find the best target. Dolphins at the outset, look around the search space to find out where the preys are, subsequently they restrict the trace in order to locate the precise position.

The method simulates dolphin echolocation by decreasing size of the random search space proportional to the distance to the target. In the proposed method, the user defines a curve on which the optimization convergence should be performed. In this way the convergence criteria is dictated to the algorithm and also this process makes the algorithm's convergence less parameter dependent.

There is a unified method for parameter selection in meta-heuristics in discrete search space. In this method, an index of convergence factor is controlled during the optimization process, Kaveh and Farhoudi [19].

A Convergence Factor (CF) is defined as the average probability of the best answer. Here in continuous search space because of the inherently continuous characteristic of the variables it is not possible to calculate probability for the best answer as a single point, instead standard deviation is chosen to be a criterion for convergence. For variable j, CF is defined as follows:

$$CF_{j} = 1 - \frac{SD_{j}}{\left(UL_{j} - LL_{j}\right)/2} \tag{1}$$

where, SD_j is the standard deviation of values chosen for the j^{th} variable; UL_j is the upper limit of the j^{th} variable; and LL_j is the lower limit of the j^{th} variable.

A curve according to which the convergence factor should change during the optimization process must be assigned. Here, the change of *CF* is considered to be according to the following curve:

$$PP(Loop_i) = PP_1 + (1 - PP_1) \frac{Loop_i^{Power} - 1}{\left(Loops\ Number\right)^{Power} - 1}$$
(2)

PP: Predefined probability; PP_1 : Predefined probability of the first loop. This parameter is better to be set as convergence factor of the first loop in which the answers are selected randomly; $Loop_i$: Number of the loop in which optimization process is performing; Power: Degree of the curve. (As it can be seen, the curve in Eq. (2) is of Power degree). Loops Number: Number of loops in which the algorithm should converge to the final result. This number should be chosen by the user according to the computational effort that can be provided for the algorithm.

Fig. 2 shows the variation of *PP* by the changes of the *Power*, using the proposed formula, Eq. (2).

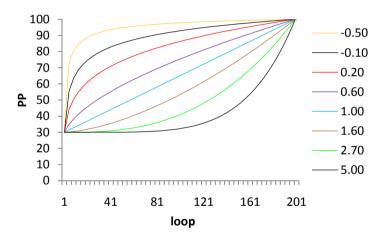


Figure 2. Sample convergence curves, using Eq. (2) for different values for power

3.1 Dolphin echolocation optimization algorithm

The flowchart of the DEO algorithm is depicted in Fig. Steps of the algorithm can be stated as follows:

- 1. Initiate NL locations for a dolphin randomly.
- 2. Calculate *PP* of the loop using Eq. (PP).
- 3. Calculate the fitness of each location.

Fitness should be defined in a manner that better answers receive higher values. In other words the optimization goal should be to maximize the fitness.

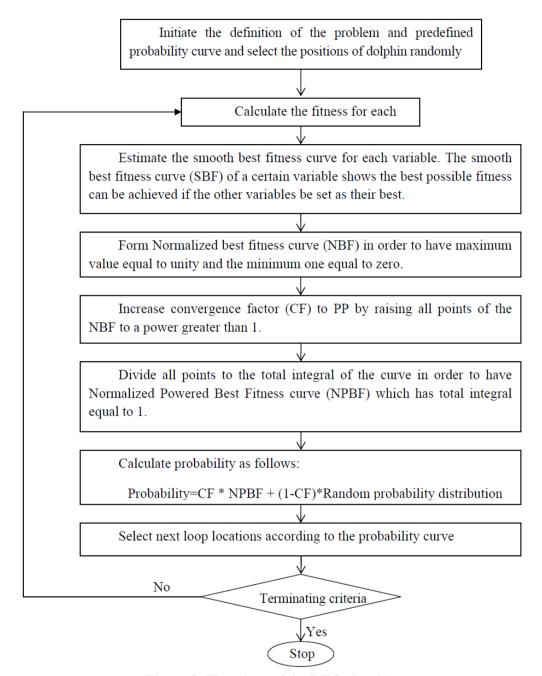


Figure 3. Flowchart of the DEO algorithm

- 4. Create the best fitness matrix (BF), Leading curve (LC) and Smooth Best Fitness curve (SBF) according to dolphin rules as follows:
- 4.1. Create the best fitness matrix (BF) and draw the leading curve (LC).

```
for j = 1 to NV
for i = 1 to NL
BF(L(i, j), j) = \max(Fitness(i), BF(L(i, j), j))
```

$$LC(L(i, j) + x, j) = \begin{cases} \max(\frac{1}{R_e} * (R_e - |x|) * Fitness(i), LC(L(i, j) + x, j)) & -R_e \le x \le R_e \\ 0 & otherwise \end{cases}$$
end
(3)

end end

where, NV is the number of variables; NL is the number of locations; L(i, j) is the jth variable's value in the ith location; BF contains the best ever achieved fitness for each variable. LC(x, j) is the maximum value obtained by producing an inverse V-type curve on all locations of this loop by considering y-axis as fitness and x-axis as available values for the jth variable. R_e is the effective radius which shows the distance around a selected alternative that its neighbors' probabilities are affected from its fitness R_e is recommended to be not more than 1/4 of the search space; Fitness(i) is the fitness of the ith location.

4.2. Draw the Smooth Best Fitness (SBF).

The Smooth Best Fitness (SBF) is a smooth curve for each variable which shows how each alternative is fitted for this variable. It passes through BF points that lay on /over LC curve. This can be performed by different methods; however, the one utilized here consists of the following steps for drawing the SBF curve for the jth variable:

1. For the first alternative, the magnitude of the SBF is the maximum value of LC and BF for this alternative, i.e. $SBF_{1i} = \max(LC_{1i}, BF_{1i})$;

Set alternative 1 to FiPo (First Point);

2

- a) Form a group of points of maximum PoNum numbers $X=\{X_1,X_2,...X_{LastPoint}\}|\ X_1>=$ FiPo+1 & $X_{LastPoint}<=UL_j$ (UL_j is the upper limit of the j^{th} variable) & LastPoint <= PoNum in which for each $X_i\in X$, BF(x_i,j) is greater or equal to LC(x_i,j).
- * It should be noted that for the point like p, if BF(p,j) is still zero it means that alternative p has not been used for the jth variable so far, then p cannot be one of the X group members.
- b) For each point of X group like X_i, calculate the slope of a line which connects FiPo to X_i.
- c) Define the point for which the slope is maximized and name it MaxPo.
- d) Draw a line from FiPo to MaxPo and set the value of SBF(FiPo+1, j) to SBF(MaxPo,j) to be on the line.
- 3. If MaxPo is not equal to UL_i set FiPo equal to FiPo+1 and repeat Steps 2 and 3.
- 5. Normalize the smooth best fitness curve in order to have maximum value equal to unity and the minimum one equal to zero. For j^{th} variable, if the maximum and minimum values of SBF are considered as Max_j and Min_j , respectively. Normalized SBF or NBF will be equal to:

$$NBF(x, j) = (SBF(x, j) - min_{j})/(max_{j} - min_{j})$$
 (4)

where, x belongs to the j^{th} variable domain.

- 6. Normalized best fitness (NBF) curve will be used in this step as probability distribution curve but before that, its convergence factor should be changed according to the predefined probability curve. An increase in convergence factor occurs when standard deviation decreases. Decreasing standard deviation is implemented in the algorithm by raising all points of normalized best fitness curve to a power greater than 1. The power should change till the achieved CF from Eq. (1) is equal to the predefined probability (PP).
- 7. Divide NBF to total area bounded by the curve and x axis in order to have a curve with total area enclosed by the curve equal to unity. Name the curve as the Normalized Powered Best Fitness (NPBF) curve.
- 8. Probability distribution curve is the CF * NPBF plus (1-CF)* Random distribution curve

(Random distribution curve is a curve with constant value of 1/(Domain length). It is obvious that its integral all around the domain will be equal to 1).

9. Select locations of the next loop according to the probability curve. In order to perform selection, calculate the cumulative probability curve for each dimension of specified location and choose a random number between 0 and 1. Select a point on cumulative probability curve the value of which is equal to the random number for the next loop.

The SBF at the end of algorithm is named optimality curve. Optimality curve of variable j at point x shows the best achievable fitness for the problem, if x be selected for the jth variable.

3.2 Input parameters

Input parameters for the algorithm are:

a) Loops number

For an optimization algorithm it is beneficial for the user to be able to dictate the algorithm to work according to the affordable computational cost. The number of loops can be selected by sensitivity analysis when high accuracy is required.

b) Convergence curve formula

This is another important parameter to be selected for the present algorithm. The curve should reach to the final point of 100% smoothly. If the curve satisfies the above mentioned criteria, the algorithm will perform the job properly, but it is recommended to start with a linear curve and try the curves that spend more time (more loops) in high values of PP. For example, if one is utilizing proposed curves of this paper, it is recommended to start with Power = 1 which usually gives good results and it is better to try some cases of the Power < 1 to check if it improves the results.

c) Effective Radius (R_e)

This parameter should be chosen according to the size of the search space and the sensitivity of the fitness to each variable.

d) Number of Locations (NL)

This parameter is the same as the population size in GA or number of ants in ACO. It should be chosen according the problem size.

f) Point Numbers (PoNum)

This parameter is used for constructing SBF, which is a smooth curve passing through peaks of LC curve. PoNum helps SBF to ignore some points and do not go up and down with every changes in LC. For example, when PoNum is equal to 5, the algorithm selects 5 peaks of LC curve and checks how to draw a line which starts from first point and ends to one of points in a way that all other points are located underneath the curve. In this way, for every 5 points, a line will be substituted all ups and downs in LC curve. Selection of this parameter does not have significant importance in optimization, but user should avoid large values which decreases the accuracy of the curve and final result. Usually, this parameter can be considered as 5, but user can increase it by an increase in Number of Locations. Obviously it cannot be more than Number of Locations.

4. PROBLEM DEFINITION

In this study, DEO is applied for optimum design of retaining wall of Fig. Dimensional properties of the wall depicted in Fig are design variables. Where T_1 is the thickness of top stem; T_2 is Thickness of key and stem; T_3 is the toe width; T_4 is heel width; T_5 is the top stem height; T_6 is the footing thickness; and T_7 is the key depth.

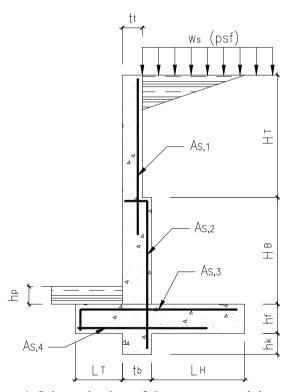


Figure 4. Schematic view of the concrete retaining wall

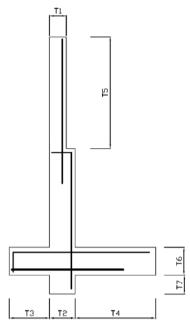


Figure 5. Design variables of optimization problem

- The optimized retaining wall data are as follows:
- The height of wall from top of the foundation is constant and equal to 6.1m.
- The unit breadth of retaining wall is considered for design.
- The level of ground water is under the level of wall, therefore does not affect the soil characteristics.
- The height of the backfill in front of the wall (h_p) is assumed as 0 and 2 m for each soil type.
- Surcharge load is 10 kN/m².
- The 28 days concrete cylinder strength is 25 MPa, Rebar yield stress is 300 MPa, and the allowable soil pressure is taken as $q_a=300 \text{kN/m}^2$.
- The clear concrete cover is 50 mm.
- Load factors are considered as $\gamma = 1.5$.

Properties of two types of soil are presented in Table 1. Lower and upper limits of the variables are shown in Table 2.

Table 1: Types of backfills

Type of	Description	Density	Internal friction
back fill		(kN/m^3)	angle (°)
F1	Coarse granular fills (GW, GP)	22	35
F2	Granular soils with more than 12% of fines (GW, GS, SM, SL) and fine soils with more than 25% of coarse grains (CL–ML)	20	30

		11					
Design	Thickness of	Thickness of the key	Toe	Heel	Height of	Footing	Key
variables	the top stem	and bottom stems	width	width	the top stem	thickness	depth
Upper bound	0.3 m	0.3 m	0.45 m	1.8 m	1.5 m	0.3 m	0.2 m
Lower bound	0.6 m	0.6 m	1.2 m	3 m	6.1 m	0.9 m	0.9 m

Table 2: Upper and lower bounds for design variables

4.1 Objective function

In the present study, cost is minimized during the optimization process. Following cost function has been shown to reach to proper optimum results

Minimize
$$Q = V_{conc}(C_1 + C_2) + W_{steel}(C_3 + C_4)$$
 (5)

This can be simplified by considering $\overline{Q} = \frac{Q}{C_1 + C_2}$

$$\overline{Q} = V_{conc} + W_{steel} \left(\frac{C_1 + C_2}{C_3 + C_4} \right)$$
(6)

where, is the cost of concrete; C_2 is the cost of reinforcing steel; C_3 is the cost of concreting; C_4 is the cost of erecting reinforcement; and steel V_{conc} (m³/m) and W_{steel} (kg/m) are the volume of the concrete and the weight of reinforcement steel, respectively. All parameters are in unit of length.

$$\left(\frac{C_1 + C_2}{C_3 + C_4}\right)$$
 may change by time and from one country to another but it has been shown

that its value can be in the range of 0.035-0.045.

Following requirements satisfied during the optimization process, according to ACI 318:

$$FS_{overturning} \ge 1.5$$
 (7)

$$FS_{Sliding} \ge 1.5$$
 (8)

$$FS_{Bearing capacity} \ge 2$$
 (9)

$$FS_{Bearing capacity} \ge 2$$
 (10)

$$M_u/(\phi_b M_n) \le 1 \tag{11}$$

$$V_{\nu}/(\phi_{\nu}V_{n}) \le 1 \tag{12}$$

where, $FS_{overturning}$, $FS_{Sliding}$ and are safety factors of overturning moment, sliding shear and bearing capacity of soil; M_u and V_u are ultimate flexure and shear; M_n and V_n are flexure and shear capacity; ϕ_b and are strength reduction factor for flexure and shear.

5. RESULTS

Results of optimization are presented in Table 3 for both types of soil. History of optimization for soil type F1 and F2 are also depicted in Figs. 6 and 7 respectively. As it can be observed, DEO achieves better results in comparison to HS, IHS and standard CSS. In addition, HIS and HS achieve the best result in 25000 iterations for both soil types, CSS achieve the result for F1 soil type in 25000 iterations and for F2 soil type in 20000 iterations but DEO reach to a better result in 10,000 iterations for both backfill types. Therefore DEO have higher convergence rate.

Table 3: Optimum results for two types of backfill

Type of fill F1			F2			
method	IHS	CSS	DEO	IHS	CSS	DEO
Thickness of top stem	0.33	0.31	0.30	0.34	0.31	0.31
Thickness of key and bottom stem	0.60	0.36	0.58	0.60	0.43	0.59
Toe Width	1.20	1.01	1.19	1.17	1.06	1.16
Heel width	2.56	2.40	1.83	2.13	2.71	2.62
Height of top stem	3.25	4.13	3.38	3.33	4.49	3.32
Footing thickness	0.57	0.34	0.40	0.56	0.30	0.51
Key depth	0.67	0.39	0.21	0.35	0.61	0.43
$As1 (mm^2/m)$	1033.0	1093.5	1167.5	926.0	1688.2	1216.4
$As2 (mm^2/m)$	3000.0	2083.3	2818.1	2634.0	2218.0	3158.3
$As3 (mm^2/m)$	2653.0	2574.8	2599.7	2148.0	2941.5	2868.8
$As4 (mm^2/m)$	1054.0	1200.6	1537.3	1034.0	1312.6	1044.3

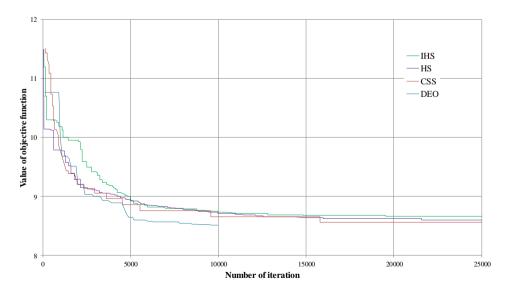


Figure 6. Optimization history for F1 backfill type

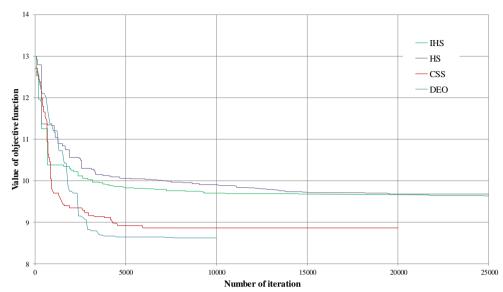


Figure 7. Optimization history for F2 backfill type

DEO has the capability to draw optimality curve for all design variables. These curves show how fitness is altered by changes in each variable. In Figs. 8 and 9, optimality curves of x1 and x2 variables in retaining wall design are depicted respectively for F1 and F2 soil types.

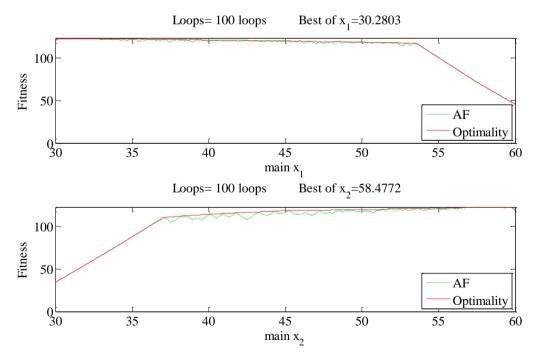


Figure 8. Optimality curve for variables x1 and x2 for F1 backfill type

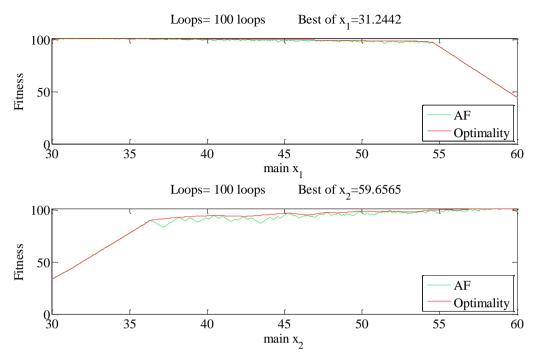


Figure 9. Optimality curve for variables x1 and x2 for F2 backfill type

6. CONCLUSIONS

In this study, the recently developed Dolphin Echolocation Optimization algorithm is applied to optimization of retaining wall in continuous search space. DEO achieved better results and higher convergence rate in comparison to HS, HIS and standard CSS.

The algorithm has the ability in developing optimality curves which can be utilized as a guide for designers. In these curves, not only the optimum choice for each variable is provided but also change in the fitness function due to changes in each variable is illustrated.

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APPENDIX: ANALYSIS AND DESIGN OF CONCRETE CANTILEVER **RETAINING WALL**

Analysis and design of retaining walls is performed according to (ACI 318-08) [20] and Das [21].

A.1. ACTIVE AND PASSIVE EARTH PRESSURE COEFFICIENTS

Active and passive earth pressure coefficients are computed according to the Coulomb's earth pressure theory, as follows:

$$K_{a} = \frac{\sin^{2}(\alpha + \phi)}{\sin^{2}(\alpha)\sin(\alpha - \delta)\left[1 + \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - \delta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^{2}}$$

$$K_{p} = \frac{\sin^{2}(\alpha - \phi)}{\sin^{2}(\alpha)\sin(\alpha + \delta)\left[1 - \sqrt{\frac{\sin(\phi + \delta)\sin(\phi + \delta)}{\sin(\alpha + \delta)\sin(\alpha + \beta)}}\right]^{2}}$$
(A-1)
(A-2)

$$K_{p} = \frac{\sin^{2}(\alpha - \phi)}{\sin^{2}(\alpha)\sin(\alpha + \delta)\left[1 - \sqrt{\frac{\sin(\phi + \delta)\sin(\phi + \delta)}{\sin(\alpha + \delta)\sin(\alpha + \beta)}}\right]^{2}}$$
(A-2)

Fig. A.1 shows the parameters of these equations.

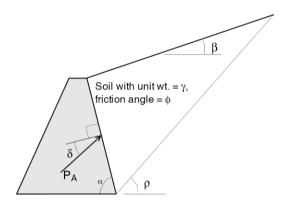


Figure A.1. Parameter definition of Coulomb's earth pressure formula

Loads acting on the considered cantilever retaining wall are shown in Fig. A.2.

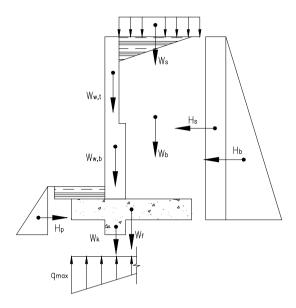


Figure A.2. Loads acting on cantilever retaining wall **A.2. STABILITY CONTROL**

A.2.1 Overturning moment control

Safety factor for overturning moment is calculated as follows:

$$FS_{overturning} = \frac{\sum Wx + M_{H_p}}{\sum Hy}$$
 (A-3)

A.2.2 Sliding Control

Safety factor for Sliding of base is calculated as follows:

$$FS_{Sliding} = \frac{\sum F_{R'}}{\sum F_{d}}$$
 (A-4)

Where $\sum F_R$ is the sum of horizontal resisting forces and $\sum F_d$ is sum of horizontal driving forces.

$$FS_{Sliding} = \frac{H_p + \mu \sum W}{H_b + H_s}$$
 (A-5)

A.2.3 Bearing capacity control

Safty factor for bearing capacity is caculated as follows:

$$FS_{Bearing capacity} = \frac{q_u}{q_{max}} \tag{A-6}$$

where q_u is the ultimate bearing capacity and q_{\max} is the maximum pressure acting on the footing. q_{\max} is calculated as:

$$q_{\text{max}} = \begin{cases} \frac{\sum W \left(1 + \frac{6e}{L}\right)}{BL}, & \text{for } e \le \frac{L}{6} \\ \frac{2\sum W}{3B(0.5L - e)}, & \text{for } e > \frac{L}{6} \end{cases}$$
(A-7)

$$e = \frac{L}{2} - \frac{\sum Wx - \sum Hy - M_{H_p}}{\sum \gamma W}$$
 (A-8)

A.3. STRENGTH CONTROL

A.3.1 Flexure capacity control

$$\frac{M_u}{\phi_b M_u} \le 1 \tag{A-9}$$

where, M_u is ultimate flexure; ϕ_b is load factor and M_n is flexure capacity

A.3.1.1 Stem flexure

$$M_{u} = \begin{cases} 1.6 \left(\frac{P_{a} H_{T}^{3}}{6} + \frac{P_{a} H_{T}^{2} w_{s}}{2\gamma_{b}} \right), & \text{for top stem} \\ 1.6 \left(\frac{P_{a} (H_{T} + H_{B})^{3}}{6} + \frac{P_{a} (H_{T} + H_{B})^{2}}{2\gamma_{b}} \right), & \text{for bottom stem} \end{cases}$$
(A-10)

$$P_{u} = \begin{cases} 1.2W_{w,t}, & \text{for top stem} \\ 1.2(W_{w,t} + W_{w,b}), & \text{for bottom stem} \end{cases}$$
 (A-11)

$$M_n = A_s f_y \left(d - \frac{A_s f_y - P_u}{1.7b f_c} \right)$$
 (A-12)

A.3.1.2 Heel flexure

Soil pressure under footing is depicted in Fig.

$$S = \frac{\sum \gamma W}{0.5q_{u,toe}} - L_T + t_b \tag{A-13}$$

$$M_{u,3} = \begin{cases} \frac{L_{H}}{2} \left(\gamma . w_{s} + \gamma . w_{b} + \frac{L_{H}}{L} \gamma . w_{f} \right) - \frac{\left(q_{u,3} + 2q_{u,heel} \right) b L_{H}^{2}}{6} & e_{u} \leq \frac{L}{6} \\ \frac{L_{H}}{2} \left(\gamma . w_{s} + \gamma . w_{b} + \frac{L_{H}}{L} \gamma . w_{f} \right) - \frac{q_{u,3} b s^{2}}{6} & e_{u} > \frac{L}{6} \end{cases}$$
(A-14)

$$\rho_{req,3} = \frac{0.85 f_c \left(1 - \sqrt{1 - \frac{M_{u,3}}{0.383bd^2 f_c}}\right)}{f_y}$$
(A-15)

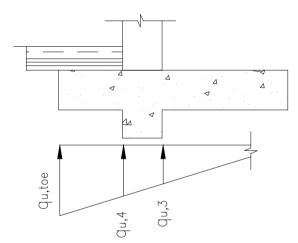


Figure A.3. Soil pressure under foundation.

A.3.1.3 Toe flexure

$$M_{u,4} = \frac{\left(q_{u,4} + 2q_{u,toe}\right)b \cdot L_T^2}{6} - \frac{L_T^2}{2L}\gamma w_f$$
 (A-16)

$$M_{u,4} = \frac{(q_{u,4} + 2q_{u,toe})b \cdot L_T^2}{6} - \frac{L_T^2}{2L} \gamma w_f$$

$$\rho_{req,4} = \frac{0.85 f_c \left(1 - \sqrt{1 - \frac{M_{u,4}}{0.383bd^2 f_c}}\right)}{f_y}$$
(A-17)

A.3.2 Shear capacity control

$$\frac{V_u}{\phi_v V_n} \le 1 \tag{A-18}$$

A.3.2.1 Stem Shear

$$V_{u} = \begin{cases} \gamma \left(\frac{P_{a} H_{T}^{2}}{2} + \frac{w_{s} P_{a} H_{T}^{2}}{\gamma_{b}} \right), & \text{for top stem} \\ \gamma \left(\frac{P_{a} (H_{T} + H_{B})^{2}}{2} + \frac{w_{s} P_{a} (H_{T} + H_{B})^{2}}{\gamma_{b}} \right), & \text{for bottom stem} \end{cases}$$

$$V_{u} = 2bd_{stow} \sqrt{f_{c}}$$

$$(A-19)$$

A.3.2.1 Heel Shear

$$V_{u} = \left[\gamma (h_{f} \gamma_{c} + (H_{T} + H_{B}) \gamma_{b} + w_{s}) - 0.5 (q_{u,3} + q_{u,heel}) \right] LH$$
 (A-21)

$$V_n = 2bd_{heel}\sqrt{f_c} \tag{A-22}$$

A.3.2.1 Toe Shear

$$V_u = 0.5(q_{u,toe} + q_{u,4})L_T$$
 (A-23)

$$V_n = 2bd_{toe}\sqrt{f_c}$$
 (A-24)