BUILDINGS WITH LOCAL ISOLATION SYSTEM: PERFORMANCE AND SIMPLIFIED METHOD OF DYNAMIC ANALYSIS

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ABSTRACT

Mass isolation is a method of structural vibration control against environmental loads such as strong earthquakes. Buildings with local isolation systems are a practical method of mass isolation. These buildings can be non-proportionally damped systems. Available methods for dynamic analysis of these buildings are complex and time-consuming. In this paper, the concept and efficiency of these buildings at seismic response reduction is examined. A simple method for their dynamic analysis is introduced. Proof of the accuracy of the method is also presented.

Keywords: Structural control, local isolation, mass isolation, floor isolation, dynamic reduction

1. INTRODUCTION

Floor isolation systems (FIS) have been applied to many types of construction, such as power plants, industrial structures and vibration-sensitive rooms [1-4]. The target of this isolation is to control the response of the floor and/or the equipment on it.

FIS (local isolation system) controls the response of the main structure of a building as well as reducing floor acceleration. The structural mass as the main source of vibration absorbs earthquake input energy in the structure. Controlling the vibration of the building requires the isolation of mass from the main structure and, subsequently, from the ground (base isolation). This is called “mass isolation” and is shown in Figure 1, Refs. [5-7].

It is clear that the main part of mass is concentrated in the floors of a building. A suitable approach for controlling its response is to use appropriated isolation between the floors’ slabs and the main structure of the building (structure frame). The construction of a building with FIS requires that the main structure (columns and beams) be constructed first. Isolators are then placed on the beams and finally the floor slab is constructed on the isolators. Figure 2 shows the structural details of such a building. Discrete and continuous floor isolators are

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two proposed types of design; in the slab floor around of discrete isolators must be considered for punching shear effects.

![Figure 1. A proposed general model of mass isolation [7]](image)

Buildings with FIS such as multiple tuned mass dampers and secondary systems are known as non-classically damped systems [8, 9]. The dynamic analysis of buildings with FIS under earthquake excitation can be obtained using the time integration method [10], complex mode-superposition method [11], or the pseudo-force method [12]. These methods are complex and require time-consuming analyses.

In this paper, the basic characteristics of such buildings at seismic response reduction and a simple method for their dynamic analysis are introduced. The proposed simple method retains
the advantages of the mode-superposition method with its dynamic analysis carried down for half of the dynamic degrees of freedom which reveals building performance. The detailed formulation is presented and a typical example is analyzed to demonstrate the improvement of the proposed method and efficiency of these buildings at seismic response reduction.

2. BASIC PRINCIPLES OF FLOOR ISOLATION

Compare the periodic shift of a one-story shear building with FIS to one without isolation and their effectiveness in reducing the earthquake response. One-story shear buildings with and without FIS are shown in Figure 3. $M$ is the total mass given by $M = m_f + m_s$; $m_f$ is the floor mass; $m_s$ is the structural mass; $c_f$ and $k_f$ are the damping and stiffness; $c_s$ and $k_s$ are the damping and stiffness of the floor system. $\mu$ is the mass ratio; $\beta$ is the stiffness ratio; $\xi_s$ is the structural damping ratio and $\xi_f$ is the damping ratio of the floor system. By considering the building with FIS to be a non-proportionally damped system, its complex eigenfunction can be written as:

$$\lambda^4 \left[ -2 \frac{\beta}{\mu} \xi_s - 2 \sqrt{\frac{\mu}{1-\mu}} \xi_f - 2 \frac{1}{\sqrt{1-\mu}} \xi_b \right] + \lambda^3 \left[ \frac{\mu + \beta}{\mu(1-\mu)} + 4 \frac{\beta}{\mu(1-\mu)} \xi_f \right] \lambda^2 + \lambda \left[ -2 \frac{1}{\sqrt{1-\mu}} \mu \xi_f - 2 \frac{\beta}{\mu(1-\mu)} \xi_b \right] + \frac{\beta}{\mu(1-\mu)} \omega_b^2 = 0$$

(1)

where $\lambda$ is the eigenvalue of the building with FIS and $\omega_b$ is the natural frequency of the building without FIS. Eq. 1 is solved and the modal periods of the building with FIS ($T_1, T_2$) are derived for the periods of the building without FIS ($T_b$).

Figure 3. Analytical models of one-story shear building (a) with FIS (b) without FIS
\[ T_1 = C_1 T_b \quad \quad T_2 = C_2 T_b \] (2)

Values for \( C_1, C_2, \xi_1 = 0.02, \) and \( \xi_2 = 0.2 \) are shown in Table 1. \( \xi_1 \) and \( \xi_2 \) are modal damping ratios for the building with FIS. For each \( \mu \) and \( \beta \), the values \( T_1 \) and \( T_2 \) are varied.

Table 1 indicates that the building with FIS in comparison with the building without FIS includes two sets of periods. These two sets are far from the region of higher acceleration, as shown schematically in Figure 4. Table 1 also shows that the first mode damping ratio is close to the damping ratio of the floor system. This causes the building with FIS to be divided into a soft part and a stiff part. The major mass of the building is concentrated in the soft part during low acceleration with the minor mass concentrated in the stiff part. Hence, equipping buildings with floor isolation can be an effective method of reducing structural seismic response.

![Schematic sketch of absolute acceleration spectrum](image)

Figure 4. Schematic sketch of absolute acceleration spectrum

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \mu=0.90 )</th>
<th>( \mu=0.95 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>( C_1 )</td>
<td>( \xi_1 )</td>
</tr>
<tr>
<td>0.001</td>
<td>30.126</td>
<td>0.1997</td>
</tr>
<tr>
<td>0.002</td>
<td>21.231</td>
<td>0.1994</td>
</tr>
<tr>
<td>0.003</td>
<td>17.343</td>
<td>0.1992</td>
</tr>
<tr>
<td>0.004</td>
<td>15.025</td>
<td>0.1989</td>
</tr>
<tr>
<td>0.005</td>
<td>13.445</td>
<td>0.1986</td>
</tr>
</tbody>
</table>
3. SIMPLIFIED METHOD OF DYNAMIC ANALYSIS

An analytical model of a typical shear building with the proposed FIS is shown in Figure 5. The number of dynamic degrees of freedom of this building is twice that of a building without FIS. This building has $2N$ dynamic degrees of freedom, where $N$ is the number of stories. The building is a combined system with a main linear structure (frame of structure) and isolated floors. The equations of the building can be expressed in the matrix form as:

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = P(t)$$

where $M$, $C$, and $K$ are $2N \times 2N$ mass, damping, and stiffness matrices of the building, respectively. $\ddot{X}(t)$, $\dot{X}(t)$, and $X(t)$ are $2N \times 1$ vectors of time-varying acceleration, velocities, and displacements, respectively and $P(t)$ is a $2N \times 1$ vector of time-varying nodal applied forces.

It is clear that available methods for dynamic analysis of the building are complex and time-consuming and the forms of the $C$ and $K$ matrices are complex. Furthermore, design
and control methods work best for a building with a small number of degrees of freedom. To overcome this difficulty, a simple method is carried out for the omitted, unnecessary, degrees of freedom. Eq. 3 can be expressed in matrix form as:

\[
\begin{bmatrix}
M_{ss} & M_{sf} \\
M_{fs} & M_{ff}
\end{bmatrix}
\begin{bmatrix}
\ddot{X}_s \\
\dot{X}_f
\end{bmatrix}
+\begin{bmatrix}
C_{ss} & C_{sf} \\
C_{fs} & C_{ff}
\end{bmatrix}
\begin{bmatrix}
\dot{X}_s \\
\dot{X}_f
\end{bmatrix}
+\begin{bmatrix}
K_{ss} & K_{sf} \\
K_{fs} & K_{ff}
\end{bmatrix}
\begin{bmatrix}
X_s \\
X_f
\end{bmatrix}
= \begin{bmatrix}
P_s \\
P_f
\end{bmatrix}
\tag{4}
\]

where matrices \(X\) and \(M\) are:

\[
X = \begin{bmatrix}
x_{s1} & x_{s2} & \ldots & x_{sN} & x_{f1} & x_{f2} & \ldots & x_{fN}
\end{bmatrix}
\tag{5}
\]

\[
M = \text{diag}\begin{bmatrix}
m_{s1} & m_{s2} & \ldots & m_{sN} & m_{f1} & m_{f2} & \ldots & m_{fN}
\end{bmatrix}
\tag{6}
\]

Sub-matrices \(M_{sf}\) and \(M_{fs}\) are zero, and sub-matrices of \(K\) and \(C\) are:

\[
K_{ff} = -K_{fs} = -K_{sf} = \text{diag}\begin{bmatrix}
-k_{f1} & -k_{f2} & \ldots & -k_{fN}
\end{bmatrix}
\tag{7}
\]

\[
C_{ff} = -C_{fs} = -C_{sf} = \text{diag}\begin{bmatrix}
-c_{f1} & -c_{f2} & \ldots & -c_{fN}
\end{bmatrix}
\tag{8}
\]

\[
K_{ss} = K_{ff} + K_s,
\]

\[
C_{ss} = C_{ff} + C_s,
\tag{9}
\]

where \(K_s\) and \(C_s\) are \(N \times N\) stiffness and damping matrices of the main structure (frame of the building). The simple method is used with dynamic reduction techniques [13] deciding which degrees of freedom are to be retained and which are to be omitted. The omitted degrees of freedom correspond to those at which the applied and inertial forces are negligible. The \(s\) subscript in Eq.4 shows displacements that also must be omitted. The reduced stiffness matrix shown below is obtained using the static condensation method [14].

\[
K_f = K_{ff} - K_{fs}K_{ss}^{-1}K_{sf}
\tag{11}
\]

The relationship between \(X_s\) and \(X_f\) can be express as:

\[
X_s = -K_{ss}^{-1}K_{sf}X_f.
\tag{12}
\]

To obtain an expression for the reduced mass matrix, the kinetic energy is considered as
\[ V = \frac{1}{2} \dot{X}^T M \dot{X} \]. Upon partitioning the mass matrix, using Eq. (12), we can write the kinetic energy as \[ V = \frac{1}{2} \dot{X}_f^T M_f \dot{X}_f \], where:

\[
M_f = M_f^{\text{ff}} - M_f^{\text{fs}} K_{ss}^{-1} K_{sf} - K_{fs} K_{ss}^{-1} M_f^{\text{fs}} + K_{fs} K_{ss}^{-1} M_{ss} K_{ss}^{-1} K_{sf}.
\]  

(13)

Eq.13 is the reduced mass matrix. Virtual work done by damping in the reduced system equals that of unreduced system, thus, an expression for the reduced damping matrix can be written as \[ \delta \dot{X}_s^T C_f \dot{X}_f = \delta \dot{X}^T C \dot{X} \] where:

\[
\delta \dot{X}^T C \dot{X} = \begin{bmatrix} \delta \dot{X}_s^T & \delta \dot{X}_f^T \end{bmatrix} \begin{bmatrix} C_{ss} & C_{sf} \\ C_{fs} & C_{ff} \end{bmatrix} \begin{bmatrix} \dot{X}_s \\ \dot{X}_f \end{bmatrix}. \]

(14)

Assuming velocity vector \( \dot{X}_s \) is related to velocity vector \( \dot{X}_f \), as well as \( X_s \) to \( X_f \) in Eq. 12, it can be written:

\[ \dot{X}_s = -K_{ss}^{-1} K_{sf} \dot{X}_f. \]  

(15)

Differential Eq.12 becomes:

\[ \delta X_s = -K_{ss}^{-1} K_{sf} \delta X_f. \]  

(16)

By substituting Eqs.15 and 16 into Eq. 14, the reduced damping matrix becomes:

\[
C_f = C_{ff}^{\text{ff}} - C_{fs}^{\text{fs}} K_{ss}^{-1} K_{sf} - K_{fs}^{\text{fs}} K_{ss}^{-1} C_{sf} + K_{fs}^{\text{fs}} K_{ss}^{-1} C_{ss} K_{ss}^{-1} K_{sf}.
\]  

(17)

Thus, the reduced dynamic equilibrium equation with degrees of freedom only for the floor is written:

\[ M_f \ddot{X}_f (t) + C_f \dot{X}_f (t) + K_f X_f (t) = P_f (t). \]  

(18)

The reduced dynamic equilibrium equation can also be written by omitting the floor’s degrees of freedom and considering the degrees of freedom of the main structure. This induces an equation similar to Eq. (18) for the dynamic analysis of the main structure. Since the mass of the main structure is small, dynamic effects can be neglected and the response of the main structure obtained by the pseudo-static method as:

\[ X_s = K_s^{-1} F_f, \]

(19)
where $F_I$ is the interaction forces vector between the floors and main structure.

Considering the basic physical properties of a building with FIS, the proposed natural period for the system is 1.5 to 3 sec, this being the range at which there is a decrease in earthquake acceleration. Thus, by these properties for structures designed with earthquake codes, the ratio stiffness of the floor system stiffness to a story stiffness is approximately 0.001 to 0.005. This physical property is used in the simplification of the reduced matrices.

Submatrix $K_{ss}$ includes two parts; the stiffness matrix of the main structure, $K_s$, and the stiffness matrix of the floor system, $K_{ff}$:

\[
K_{ss}^{-1} = (K_s + K_{ff})^{-1} = \left[K_s \left(I + K_s^{-1} K_{ff}\right)\right]^{-1} \Rightarrow K_{ss}^{-1} = (I + K_s^{-1} K_{ff})^{-1} K_s^{-1}.
\]  (20)

The term $(I + A)^{-1}$ is expanded as:

\[
(I + A)^{-1} = I - A + A^2 - A^3 + \cdots.
\]  (21)

In this equation, the matrix $K_s^{-1}$ is the inverse matrix of $K_s$, the flexibility matrix of the main structure. The physical properties of this type of building make the value of $A$ small, so powers of two or higher are disregarded.

\[
(I + A)^{-1} \approx I - A
\]  (22)

Eq. (22) substitutes into Eq. (20) making:

\[
K_{ss}^{-1} = (I - K_s^{-1} K_{ff})K_s^{-1} \Rightarrow K_{ss}^{-1} = K_s^{-1} - K_s^{-1} K_{ff} K_s^{-1}
\]  (23)

The values of the matrix $B$ are very small since the values of the $K_s^{-1}$ and $K_s^{-1} K_{ff}$ matrices are small. Thus, we can write:

\[
K_{ss}^{-1} \approx K_s^{-1}.
\]  (24)

Replacing $K_{ff}$ with $-K_{ff}$ in Eq. 11 leads to:

\[
K_f = K_{ff} (I - K_s^{-1} K_{ff}).
\]  (25)

Eq. (17) multiplies the two matrices with small values before and after $C_{ss}$. These can be disregarded, making:
\[ C_f = C_f - C_f K_s^{-1} K_f - K_f K_s^{-1} C_f. \] (26)

Eq. (13) multiplies two matrices with small values before and after \( M_f \). These can be disregarded, and, by then replacing \( M_f \) and \( M_f \) with zero:

\[ M_f = M_f. \] (27)

Thus, the simple form for damping and stiffness matrices are introduced where their terms are obtained by the dynamic properties of the floor system, adding small values to them [15].

4. NUMERICAL EXAMPLE

An ordinary 10-story building was designed with and without floor isolation. A parametric study for this building was carried out incorporating the differing floor characteristics of the building with FIS. Floors damping ratio \( \xi_f \) of 0.10, 0.20 and 0.40 with floors natural period \( T_f \) of 1.5, 2, 3 and 4 seconds were separately considered and then stiffness and damping of floors are obtained with the single degree of freedom model for each floor. A list of the dynamic properties of the main structure is seen in Table 2.

<table>
<thead>
<tr>
<th>Story</th>
<th>Stiffness (kN/m)</th>
<th>Damping (kN.s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>800</td>
<td>5.165</td>
</tr>
<tr>
<td>4-6</td>
<td>700</td>
<td>4.519</td>
</tr>
<tr>
<td>7-9</td>
<td>550</td>
<td>3.551</td>
</tr>
<tr>
<td>10</td>
<td>350</td>
<td>2.260</td>
</tr>
</tbody>
</table>

In order to demonstrate the improvement of the proposed method, a typical 10-story shear building with FIS was analyzed using a natural period of 1.5 seconds and a damping ratio of 0.4 for the floors. The dynamic response of this building to the El-Centro NS earthquake was derived using the proposed method and the time integration method was obtained using records from that quake’s similarity of response with other earthquakes.

The natural period and damping ratios for the first 10 modes of this building are listed in Table 3. It can be observed from Table 3 that natural periods and damping ratios obtained by the proposed method have good accuracy and are close to the complex mode-superposition
To demonstrate the accuracy of this method, compare the displacement time history of the floor and the main structure at the top story obtained using the proposed method and time integration method. The time-history curves of the floor and the main structure at the top story are shown in Figures 6 and 7. It can be seen that the floor displacements of the proposed method are close to the floor displacements of the time integration method. Also, the main structure displacements of the proposed method are close to the main structure displacements of the time integration method because of the omission of higher modes effects. The acceleration time history of the floor and the main structure at the top story of this building are shown in Figures 8 and 9. These figures indicate that the floor acceleration of the proposed method are close to the floor acceleration of the time integration method, but there was not sufficient accuracy to determine the acceleration of the main structure which has no effect in design.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Period (s) Without isolation</th>
<th>Period (s) With FIS</th>
<th>Damping Ratio Without FIS</th>
<th>Damping Ratio With FIS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Complex mode method</td>
<td>Proposed method</td>
<td>Error (%)</td>
<td>Complex mode method</td>
</tr>
<tr>
<td>1</td>
<td>1.0143</td>
<td>1.7139</td>
<td>1.8013</td>
<td>5.1</td>
</tr>
<tr>
<td>2</td>
<td>0.3664</td>
<td>1.5186</td>
<td>1.5419</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>0.2256</td>
<td>1.5066</td>
<td>1.5161</td>
<td>0.6</td>
</tr>
<tr>
<td>4</td>
<td>0.1702</td>
<td>1.5037</td>
<td>1.5092</td>
<td>0.4</td>
</tr>
<tr>
<td>5</td>
<td>0.1340</td>
<td>1.5023</td>
<td>1.5057</td>
<td>0.2</td>
</tr>
<tr>
<td>6</td>
<td>0.1137</td>
<td>1.5017</td>
<td>1.5041</td>
<td>0.2</td>
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<tr>
<td>7</td>
<td>0.0987</td>
<td>1.5013</td>
<td>1.5031</td>
<td>0.1</td>
</tr>
<tr>
<td>8</td>
<td>0.0914</td>
<td>1.5011</td>
<td>1.5027</td>
<td>0.1</td>
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<tr>
<td>9</td>
<td>0.0827</td>
<td>1.5009</td>
<td>1.5022</td>
<td>0.1</td>
</tr>
<tr>
<td>10</td>
<td>0.0762</td>
<td>1.5008</td>
<td>1.5019</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Figure 6. Displacement time history of floor at top story due to El Centro NS earthquake

Figure 7. Displacement time history of main structure at top story due to El Centro NS earthquake

Figure 8. Acceleration time history of floor at top story due to El Centro NS earthquake
Linear analysis of a typical 10-story shear building with and without FIS was carried out to demonstrate the efficiency of these buildings at seismic response reduction. The dynamic responses to the El-Centro NS, Taft EW, Bam, Kobe, San Fernando, and Hachinohe earthquake records were obtained using the time integration method. The maximum displacement of the main structure is shown in Figures 10 to 12 for a building with and without FIS for the El-Centro NS record only because of its similar response to other earthquakes. The figures indicate that the displacement of the building with FIS compared to the building without FIS has been substantially reduced. The maximum displacement of the main structure could be reduced more by reducing the stiffness of the floor systems (increase floor periods) and increasing their damping ratio. The value of reduction for higher damping ratios (0.20 and 0.40) is not considerable.
Safety is the most important criteria in the design of buildings, and this will be provided by a low level of floor acceleration. The maximum acceleration of the floor at the top story is shown in Figures 13-15 for buildings with and without FIS. It can be observed from the figures that the floor acceleration of the building with FIS, compared to the building without FIS, is reduced, which is essential. The maximum floor acceleration at the top story could be further reduced by reducing the stiffness of the floor system (increase floor periods) as well as increasing their damping ratio. The value of reduction for higher damping ratios (0.20 and 0.40) is not considerables.
Figure 13. Maximum floor acceleration at top story with $\xi_f = 0.1$

Figure 14. Maximum floor acceleration at top story with $\xi_f = 0.2$

Figure 15. Maximum floor acceleration at top story with $\xi_f = 0.4$
5. CONCLUSIONS

The use of a floor isolation system (local isolation system) causes a building to be divided into a soft and a stiff part. The major mass of the building is concentrated in the soft part during low accelerations and the minor mass of the building is concentrated in stiff part. Thus, buildings with floor isolation can effectively reduce the structural seismic response.

The proposed method uses a much smaller number of degrees of freedom, an advantage over the standard mode-superposition method, and a simple model of mass damping and stiffness matrices. The dynamic response of the building with FIS is that the proposed method resembles the time integration method in the essential physical properties of the building.

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