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SENSITIVITY ANALYSIS FOR NONLINEAR RESPONSE OF RCMRF USING PUSHOVER ANALYSIS

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ABSTRACT

Design sensitivity analysis is a necessary task for optimization of structures. Methods of sensitivity analysis for linear systems have been developed and well documented in the literature; however there are a few such research works for nonlinear systems. Nonlinear sensitivity analysis of structures under seismic loading is very complicated. This paper presents an analytical sensitivity technique for Reinforcement Concrete Moment Resisting Frames (RCMRF) that accounts for both material and geometric nonlinearity under pushover analysis. The results of proposed method are compared with the results of finite difference method. A three-story, two bays moment frame example is used to illustrate the efficiency of the method. This technique can be very useful and efficient for optimal performance-based design of RC buildings.

Keywords: sensitivity analysis, material nonlinearity, geometric nonlinearity, RCMRF, pushover analysis

1. INTRODUCTION

Structural optimization for linear response is a well-defined problem and a large number of research works have been carried out on this subject. In recent years, performance-based seismic design has become a necessity for design of new structures. Generally pushover analysis, which is a simplified static nonlinear procedure is used for performance-based design [1]. In the pushover analysis a predefined pattern of earthquake loads is applied incrementally on structure until a predefined target displacement is reached or a plastic collapse mechanism is occurred [2,3]. Accordingly for performance-based design structural optimization involves in nonlinear analysis. structural design optimization involves response sensitivity analysis in explicitly formulating constraint functions [4]. Considerable part of computational effort in an optimization problem is usually allocated to the sensitivity analysis. Each sensitivity coefficient defines the amount of change in a structural response due to a unit change in a design variable, such as sensitivity of displacement to change in cross-sectional dimensions or reinforcement ratio.

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While considerable research effort has been put on developing the sensitivity analysis techniques, there are a few research works in the literature that has focused on the theory of sensitivity analysis for nonlinear structural systems. Ryu et al. [5] proposed a general nonlinear sensitivity analysis accounting for geometric and material nonlinearity. They used modified Newton-Raphson method for their nonlinear analysis and used the same procedure for their sensitivity analysis. They presented the formulation for a truss example. Choi and Santos [6] were two of researchers that developed variational formulations for nonlinear design sensitivity analysis. They used linearized equilibrium equations to obtain first variations of the governing nonlinear equilibrium equations with respect to design variables. Gopalakrishna and Greimann [7] differentiated the equilibrium equation in each Newton-Raphson iteration to obtain incremental gradients and this method was used for the nonlinear sensitivity analysis of the plane trusses. Santos and Choi [8] presented a unified approach for shape sensitivity analysis of trusses and beams accounting for both geometric and material nonlinearities. They utilized the adjoint variable and direct differentiation methods. Ohsaki and Arora [9] presented an accumulative and incremental algorithm for the design sensitivity analysis of elastoplastic structures including geometrical nonlinearity. They performed the sensitivity analysis of trusses but they reported that the method is extremely time consuming for large structures. Lee and Arora [10] investigated the effect of discontinuity in yield surface on the sensitivity analysis and presented a procedure for their treatments. They developed design sensitivity analysis of structural systems having elastoplastic material behavior using the continuum formulation and illustrated the sensitivity analyses for a truss and a plate by this technique. Barthold and Stein [11] presented a continuum mechanical-based formu-lation for the variational sensitivity analysis accounting for nonlinear hyperelastic material behavior using either the lagrangian or eulerian description. Szewczyk and Ahmed [12] presented a hybrid numerical/-neurocomputing strategy for evaluation of sensitivity coefficients of composite panels subjected to combined thermal and mechanical loads. They pointed out that this method reduces the number of full-system analysis. Yamazaki [13] suggested a direct sensitivity analysis technique for finding incremental sensitivities of the path-dependent nonlinear problem based on the updated lagrangian formulation. Employing this method they performed the sensitivity analysis of a plate. Bugeda and et al. [14] proposed a direct formulation for computing the structural shape sensitivity analysis with a nonlinear constitutive material model. It was reported that their proposed approach was valid for some specific nonlinear material models. Schwarz and Ramm [15] proposed the variational direct method for sensitivity analysis of structural response accounting for geometrical and material nonlinearity with Prantel-Reuss plasticity model. Gong et al. [16] presented a procedure for sensitivity analysis of planar steel moment frameworks accounting for geometric and material nonlinearity. In their work, analytical formulations defining the sensitivity of displacement were derived. They used the incremental nonlinear method for pushover analysis.

While many researches have been carried out for nonlinear sensitivity analysis of trusses, single beams, single plates and shells, study of literature reveals two point. First, there are a few researches on nonlinear sensitivity analysis of structures under variable loading such as seismic loading. Secondly, there are a few researches about nonlinear sensitivity analysis of frameworks; and there is no research work on nonlinear sensitivity analysis of reinforced concrete frameworks. The objective of this study is to develop a formulation for sensitivity analysis of planar RCMRF accounting for both material and geometric nonlinearity under pushover analysis using Newton-Raphson iterations. The proposed procedure can be efficiently used for optimal performance-based design of RC frameworks. A three-story RCMRF has been used as an example to illustrate the applicability and efficiency of the developed sensitivity formulations.

2. PUSHOVER ANALYSIS OF RCMRF

The first step in design optimization is the calculation of sensitivity analysis that in turn depends on the method of structural analysis. The simplest recommended method for nonlinear static analysis is Pushover method. This method of analysis that is recommended by FEMA273 [2] and ATC40 [3] is a popular tool for evaluation of seismic performance of existing and new structures. Many researchers such as Saiidi and Sozen [17], Bracci and et al. [18], Kilar and Fajfar [19], Gupta and Krawinkler [20], Mwafy and Elnashai [21], Hassan and et al. [22] and Chopra and Goel [23] have used this analysis method. In the pushover analysis, it is necessary to specify a proper material behavior model for elements. This is explained in the next two sections.

2.1 Moment curvature relation

The moment-curvature relation of every RC structural element has a definitive effect on the behavior of the structure. In this research the trilinear moment curvature relation, as shown in Figure 1, is used for expressing the nonlinear behavior of reinforced concrete sections. The moment curvature relation of a structural element highly depends on its cross-section. In this study, the column sections are limited to rectangle and that of beams can assume rectangle , T or L shaped.

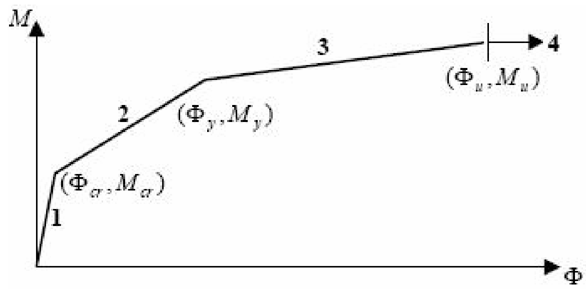

Figure 1. Trilinear moment curvature curve

To determine moment-curvature relation of RC members some assumptions should be maid that best fits with the test results. In this study, by ignoring the effect of axial force in moment-curvature relation, the following limitations for various states of behavior of RC elements are used [24,25]:

a) Cracking state

$$M_{cr} = f_r I/(h-y) \tag{1}$$

$$\phi_{cr} = f_r / E_c(h-y) \tag{2}$$

b) Yielding state

$$M_y = 0.5 f_c b_t (h-c)^2 [(2-\eta)p + (\eta - 2\beta)\alpha p'] \tag{3}$$

$$\phi_y = 1.05\varepsilon_y / (1-k)(h-c) \tag{4}$$

c) Ultimate state

$$M_u = (1.24 - 0.15p)M_y \tag{5}$$

$$\phi_u = 1.05\beta_1\varepsilon_u \big/ [(R^2+S)^{0.5} - R] \tag{6}$$

Where $R = (\rho'\varepsilon_u E_s - \rho f_y)(h-c)/(1.7 f_c)$; $S = \rho'\varepsilon_u E_s \beta_1 c(h-c)/(0.85 f_c)$; f_r is the modulus of rupture of concrete; f_c is the cylinder strength of concrete; f_y is the yield strength of steel; E_c is the modulus of elasticity of concrete; E_s is the modulus of elasticity of steel; ε_u is the ultimate strain of concrete; ε_y is the yield strain of steel; β_1 is a coefficient that depends on the strength of concrete; h is the overall height of section; c is the cover to steel centroid; b_t is the top width of section; y is the distance from the neutral axis of the section to the extreme fiber in tension/.n and is obtained from (7); I is the moment of inertia of the section and is obtained from (8). other parameters are defined from (9) to (14).

$$y = \{0.5b_t t^2 + 0.5b_b(h^2 - t^2) + (n_{sc}-1)[A'_s c + A_s(h-c)]\} \big/ [b_t t + b_b(h-t) + (n_{sc}-1)(A_s + A'_s)] \tag{7}$$

$$\begin{aligned} I = {} & b_t t^3/12 + b_t t(y-0.5t)^2 + b_b(h-t)^3/12 + b_b(h-t)(0.5h+0.5t-y)^2 + (n_{sc}-1) \\ & \times [A_s(h-c-y)^2 + A'_s(y-c)^2] \end{aligned} \tag{8}$$

$$k = \left\{ \begin{array}{ll} \sqrt{(p+p')^2/4\alpha^2 + (p+\beta p')/\alpha} - (p+p')/2\alpha & ,k \le \dfrac{t}{(h-c)} \\ \{[n_{sc}(\rho+\rho') + t(b_t/b_b - 1)/(h-c)]^2 + 2n_{sc}(\rho+\rho'\beta) + t^2(b_t/b_b - & \\ 1)/(h-c)^2\}^{0.5} - n_{sc}(\rho+\rho') - t(b_t/b_b - 1)/(h-c) & ,k \succ \dfrac{t}{(h-c)} \end{array} \right\} \tag{9}$$

$$\eta = 0.75\left((\phi_y(h-c) - \varepsilon)/\varepsilon_0\right)^{0.7} \big/ (1 + \varepsilon_y/\varepsilon_0) \tag{10}$$

$$\alpha = (1-\beta)(\phi_y (h-c) - \varepsilon_y)/\varepsilon_y - \beta \leq 1 \tag{11}$$

$$\rho = A_s / b_b (h-c); \rho' = A'_s / b_b (h-c) \tag{12}$$

$$p' = \rho' f_y / f_c ; p = \rho f_y / f_c \leq 1.6 \tag{13}$$

$$\beta = c/(h-c) \tag{14}$$

Where n_{sc} is the ratio of the modulus of elasticity of steel to that of concrete; ε_0 is the strain at maximum strength of concrete; b_b is the width of the section at the bottom; t is the flange thickness of T or L beam; A_s is the area of bottom bars and A'_s is the area of top bars. For the case of positive moment (1) to (14) are valid for all types of sections. For rectangular sections these equations can be simplified by substituting $b_b = b_t = b$ and $t = 0$. Also for the case of negative moment, these equations can be used by substituting b_t, b_b, $h-t$, A'_s and A_s instead of b_b, b_t, t, A_s and A'_s, respectively. Now by having the moment-curvature relations, the flexural stiffness can be specified for ends of the element as follows:

$$EI_p = M_{crp} / \phi_{crp} \leq EI_{pm} \tag{15-a}$$

$$EI_p = (M_{yp} - M_{crp})/(\phi_{yp} - \phi_{crp}) \leq EI_{pm} \tag{15-b}$$

$$EI_p = (M_{up} - M_{yp})/(\phi_{up} - \phi_{yp}) \leq EI_{pm} \tag{15-c}$$

Where (15-a), (15-b) and (15-c) are used for zones of 1, 2 and 3 of $M-\phi$ curve, respectively. In (15) EI_p and EI_{pm} are the stiffness and minimum stiffness of any section; M_{crp}, M_{yp} and M_{up} are the cracking, yielding and ultimate moments; and ϕ_{crp}, ϕ_{yp} and ϕ_{up} are corresponding curvatures.

2.2. Material nonlinearity

In this paper, a spread plasticity model that has been proposed by park et al. [24] is utilized. This plasticity model, that considers the cracking behavior of RC, can also account for material nonlinearity of RC elements with a very good approximation. It has been implemented in IDARC software [26]. In this model, attention is paid to the distribution of curvature along an element. For an element subjected to earthquake loading, the distribution of curvature may follow Figure 2-a. Since both flexural deformation and flexibility have reciprocal relation with stiffness of element there will be an analogy between flexibility of the section and its flexural deformation. Accordingly Figure 2-b can be considered for the flexibility distribution in the RC elements. Where EI_A and EI_B are the flexural stiffness of the section at end 'A' and 'B', respectively, and EI_0 is the initial stiffness of the element; α_A and α_B are the yield penetration coefficients and L' is the length of the element.

Adopting the above Plasticity Model for RC elements, the tangent stiffness matrix of each element can be derived. General derivation of tangent stiffness matrix is given by Park et al. [26] and detail derivation is carried out by Habibi [27]. Considering a RC element with six degrees of freedom, with its rigid parts, as shown in Figure 3, the tangent stiffness matrix of element can be found as follows:

$$\mathrm{K_{te}} = \mathrm{K_a} + \mathrm{K_b} \tag{16}$$

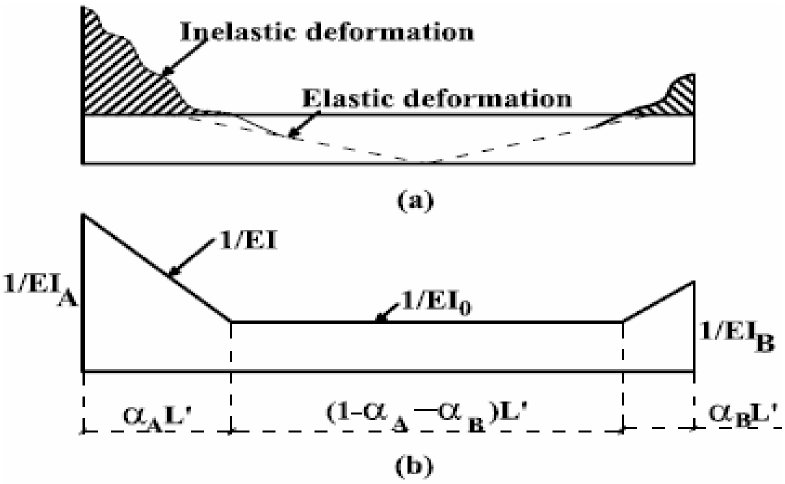

Figure 2. (a) Curvature distribution along a RC element (b) Flexibility assumption along a RC element

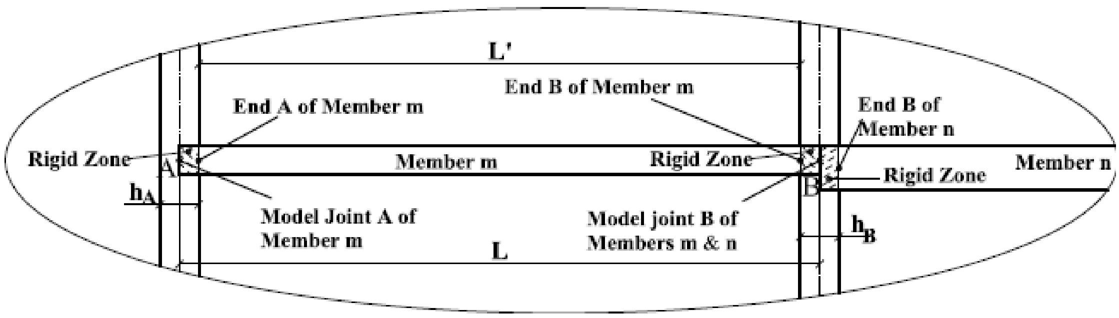

Figure 3. Rigid zone and ends definitions

In Eq. (16), K_a and K_b are axial stiffness matrix and bending stiffness matrix respectively and must be assembled in the element stiffness matrix K_e. K_a can be determined from:

$$\mathrm{K_a} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \frac{\mathrm{EA}}{\mathrm{L}} = \mathrm{AK_{a0}} \tag{17}$$

Where EA/L is the axial stiffness of the element, A is the area of cross section and K_{a0} is a constant matrix. K_b, is obtained from following equation:

$$K_b = R_E K_S R_E^T \tag{18}$$

Where:

$$R_E^T = \begin{bmatrix} -1/L & 1 & 1/L & 0 \\ -1/L & 0 & 1/L & 1 \end{bmatrix}; \quad K_S = \tilde{L} K_S' \tilde{L}^{-1} \tag{19}$$

Where L is length of element and $\tilde{L}$ can obtained from the following equation.

$$\tilde{L} = \frac{1}{1-\lambda_A-\lambda_B}\begin{bmatrix} 1-\lambda_B & \lambda_A \\ \lambda_B & 1-\lambda_A \end{bmatrix}; \quad \tilde{L}^{-1} = \begin{bmatrix} 1-\lambda_A & -\lambda_A \\ -\lambda_B & 1-\lambda_B \end{bmatrix} \tag{20}$$

Where λ_A and λ_B are the portions of rigid zone at the element ends. The elements of K_S' in (19) are obtained from following equations:

$$\mathrm{K_{S11}} = 12\mathrm{EI_0 EI_A EI_B L' f_b / D_e};\ \mathrm{K_{S12}} = \mathrm{K_{S21}} = \mathrm{K_{S11} f_c / f_b};\ \mathrm{K_{S22}} = \mathrm{K_{S11} f_a / f_b} \tag{21}$$

Where:

$$\begin{aligned} &\mathrm{f_a} = 4\mathrm{S_1} + \mathrm{S_2}\left(6\alpha_\mathrm{A} - 4\alpha_\mathrm{A}^2 + \alpha_\mathrm{A}^3\right) + \mathrm{S_3}\alpha_\mathrm{B}^3;\ \mathrm{f_b} = 4\mathrm{S_1} + \mathrm{S_2}\alpha_\mathrm{A}^3 + \mathrm{S_3}\left(6\alpha_\mathrm{B} - 4\alpha_\mathrm{B}^2 + \alpha_\mathrm{B}^3\right) \\ &\mathrm{f_c} = -2\mathrm{S_1} - \mathrm{S_2}\left(2\alpha_\mathrm{A}^2 - \alpha_\mathrm{A}^3\right) - \mathrm{S_3}\left(2\alpha_\mathrm{B}^2 - \alpha_\mathrm{B}^3\right);\ \mathrm{D_e} = \left(\mathrm{f_a f_b} - \mathrm{f_c^2}\right)\mathrm{L'^2};\ \mathrm{L'} = (1-\lambda_\mathrm{A}-\lambda_\mathrm{B})\mathrm{L} \end{aligned} \tag{22}$$

Where $S_1 = EI_A EI_B$; $S_2 = (EI_0 - EI_A)EI_B$; and $S_3 = (EI_0 - EI_B)EI_A$. The yield penetration parameters used in (22), specify the portion of the element where it cracks. When the moment of the section under consideration is less than cracking moment, Values of these parameters are considered to be equal to zero. For the single curvature of the element, when its moments are greater than cracking moment, value of the parameters are considered to be equal to 0.5. For other cases, assuming linear moment distribution along the element, these parameters can be determined from:

$$\alpha_p = \max\left\{\min\left\{\left(\left|M_p\right| - M_{crp}\right)/\left(\left|M_A - M_B\right|\right), 1\right\}, \alpha_{pm}\right\} \tag{23}$$

Where subscript p specifies any general point p and cr represents the cracking state under consideration. α_{pm} is the maximum yield penetration parameter, obtained in previous load steps. Usually, the flexural stiffness at the center of the element is equal to elastic stiffness and is determined from (24). For the case of single curvature if moments are greater than cracking moments, their values can be modified by (25).

$$\mathrm{EI_0 = 2EI_{A0}EI_{B0} / (EI_{A0} + EI_{B0})} \tag{24}$$

$$\mathrm{EI_0 = 2EI_A EI_B / (EI_A + EI_B)} \tag{25}$$

Where EI_{A0} and EI_{B0} are the elastic stiffness at ends ‘A’ and ‘B’, respectively; and EI_A and EI_B are the inelastic stiffness at ends ‘A’ and ‘B’, respectively. For a member that yield penetration spreads over the whole element $(when\ \alpha_A + \alpha_B \geq 1)$, EI_0 is obtained from (25). In this case, α_A and α_B and initial flexibility are modified to capture the actual flexibility distribution. These modifications are:

$$\alpha'_A = (f_A - f_B + (f_B - f_0)/\alpha_B)/((f_A - f_0)/\alpha_A + (f_B - f_0)/\alpha_B) \tag{26-a}$$

$$\alpha'_B = 1 - \alpha'_A \tag{26-b}$$

$$f'_0 = f_A - (f_A - f_0)\alpha'_A / \alpha_A \tag{26-c}$$

Where:

$$f_A = 1/EI_A; f_B = 1/EI_B; f_0 = 1/EI_0 \tag{27}$$

2.3 Geometric nonlinearity

When large deflections are present, the equations of force equilibrium must be formulated for the deformed configuration of the structure. The nonlinear terms in the strain-displacement equations modify the element stiffness matrix [28]:

$$K'_e = K_{te} + K_g \tag{28}$$

In (28) K_g is called geometrical stiffness matrix. This matrix for the element, as shown in Figure 4, is obtained by Przemieniecki [28]:

$$K_g = \frac{N}{L}\begin{bmatrix} 0 & & & & & \\ 0 & 6/5 & & symmetric & & \\ 0 & L/10 & 2L^2/15 & & & \\ 0 & 0 & 0 & 0 & & \\ 0 & -6/5 & -L/10 & 0 & 6/5 & \\ 0 & L/10 & -L^2/30 & 0 & -L/10 & 2L^2/15 \end{bmatrix} = N(K_{g0}) \tag{29}$$

Where N is the axial force of the element and K_{g0} is the constant matrix. The stiffness matrix calculated from (28) must be transformed to global system:

$$\mathrm{K_e = T_e^T K'_e T_e} \tag{30}$$

where T_e is the transformation matrix of the element.

2.4 Lateral Loading

In the pushover analysis, the load vector must incrementally be increased. At each load step, the base shear increment is applied to the structure with a predefined profile over the height of the structure. The incremental lateral load vector can be computed as:

$$\mathrm{\Delta P_E = \Delta V_b . [c_{v,1} \quad c_{v,2} \quad \cdots \quad c_{v,ns}]^T = \Delta V_b . C_v} \tag{31}$$

Where ΔV_b is the incremental base shear and C_v is the vector of lateral load distribution factors $c_{v,s}(s = 1,\ldots, number\ \ of\ \ stories)$, which is determined from FEMA273 [2]:

$$\mathrm{c_{v,s} = W_s H_s^k \Big/ \sum_{s=1}^{ns} W_s H_s^k \quad , s = 1,\ldots, ns} \tag{32}$$

Where W_s is the portion of the building seismic weight at story level s; H_s is the vertical distance from base of the building to story level s; ns is the number of stories; and k is a parameter that has been recommended by FEMA273 (1997) as follows:

$$\mathrm{k} = \begin{cases} 1 & \mathrm{T \prec 0.5} \\ 0.5\mathrm{T} + 0.75 & 0.5 \le \mathrm{T} \le 2.5 \\ 2 & \mathrm{T \succ 2.5} \end{cases} \tag{33}$$

Where T is the fundamental period of the building.

2.5 Internal forces

In each Newton-Raphson iteration i, considering equilibrium, the internal forces of the element at load step l can be determined from (34).

$$\Delta \mathrm{F}_{\mathrm{e}}^{(\mathrm{l},\mathrm{i})} = \mathrm{K}_{\mathrm{e}}^{(\mathrm{l})} \mathrm{d}_{\mathrm{e}}^{(\mathrm{i}-1)} \tag{34}$$

Where $K_e^{(l)}$ is the tangent stiffness matrix of the element and $d_e^{(i-1)}$ is the displacement vector of the element at load step l and Newton-Raphson iteration $(i-1)$. The internal forces of the element at iteration i of load step l must be updated by following equation:

$$\mathrm{F}_{\mathrm{e}}^{(\mathrm{l},\mathrm{i})} = \mathrm{F}_{\mathrm{e}}^{(\mathrm{l}-1)} + \Delta \mathrm{F}_{\mathrm{e}}^{(\mathrm{l},\mathrm{i})} \tag{35}$$

At every Newton-Raphson iteration, values of internal forces must be modified according to displacement vector. In order to modify the internal forces, first the increments in internal forces at two end joints of members are obtained from (34) and then the moments are transferred to the two ends of the element excluding rigid zones by (36).

$$\Delta M^{(i)} = \tilde{L}^{-1} \Delta m^{(i)} \tag{36}$$

With increment of moments in hand, the curvatures of the element are calculated from (37).

$$\phi_p^{(i)} = \phi_p^{(i-1)} + \Delta M_p^{(i)} / EI_p^{(l)} \tag{37}$$

In Eq. (37), $\phi_p^{(i-1)}$ is the curvature at end ‘p’ (either A or B) of the element in previous iteration; $\Delta M_p^{(i)}$ is the incremental moment at end ‘p’ of the element at current iteration i. With the obtained curvature, the nonlinear moment value can be obtained from the M-Φ relations:

$$M_p^{(i)} = EI_p^{(l)} \phi_p^{(i)} \tag{38-a}$$

$$M_p^{(i)} = M_{crp}^{(l)} + EI_p^{(l)} \left(\phi_p^{(i)} - \phi_{crp}^{(l)} \right) \tag{38-b}$$

$$M_p^{(i)} = M_{yp}^{(l)} + EI_p^{(l)} \left(\phi_p^{(i)} - \phi_{yp}^{(l)} \right) \tag{38-c}$$

$$M_p^{(i)} = M_{up}^{(l)} \tag{38-d}$$

Where (38-a), (38-b), (38-c) and (38-d) are used for zones of 1, 2, 3 and 4 of $M-\phi$ curve, respectively. In (38), $M_{crp}^{(l)}$, $M_{yp}^{(l)}$, $M_{up}^{(l)}$, $\phi_{crp}^{(l)}$ and $\phi_{yp}^{(l)}$ are the moment-curvature characteristics at end ‘p’ of the element at load step l. In the next step moments are transferred back to the end joints of member using (39) to calculate the unbalanced forces at the end joints of member.

$$m^{(i)} = \tilde{L} M^{(i)} \tag{39}$$

Where $M^{(i)}$ is the moment vector at two ends of the element and $m^{(i)}$ contains the moments at two joints. Modified values of shear forces of the element at joints are computed from (40).

$$V_p^{(i)} = (m_A^{(i)} + m_B^{(i)}) / L \tag{40}$$

Where $V_p^{(i)}$ is the shear force at joint ‘p’.

3. NONLINEAR SENSITIVITY ANALYSIS

Modified Newton-Raphson iteration for obtaining the incremental displacement at a load step l and iteration i can be written from Bathe [29]:

$$K_T^{(l)} \Delta d^{(i)} = P^{(l)} - F^{(i-1)}; \;\; d^{(l)} = \sum_i \Delta d^{(i)}; \;\; D^{(l)} = \sum_l d^{(l)} \tag{41}$$

Where $K_T^{(l)}$ is the global tangent stiffness matrix ; $\Delta d^{(i)}$ is the incremental displacement vector ; $P^{(l)}$ is the external load vector; $F^{(i-1)}$ is the internal load vector; $d^{(l)}$ is the incremental displacement vector; and $D^{(l)}$ is the total displacement vector. Differentiating (41) with respect to design variable x_j gives:

$$\Delta d^{(i)} \, dK_T^{(l)} \big/ dx_j + K_T^{(l)} \, d\Delta d^{(i)} \big/ dx_j = dP^{(l)} \big/ dx_j - dF^{(i-1)} \big/ dx_j \tag{42}$$

In this study, this equation is used as the basic formulation for obtaining the incremental displacement sensitivities. Assuming that all terms in (42) except $d\Delta d^{(i)} \big/ dx_j$ are known, it will be rearranged to find:

$$\mathrm{d\Delta d^{(i)}} \big/ \mathrm{dx_j} = \left[\mathrm{K_T^{(l)}}\right]^{-1} \left(\mathrm{dP^{(l)}} \big/ \mathrm{dx_j} - \mathrm{dF^{(i-1)}} \big/ \mathrm{dx_j} - \mathrm{\Delta d^{(i)}} \, \mathrm{dK_T^{(l)}} \big/ \mathrm{dx_j}\right) \tag{43-a}$$

Summing up the sensitivity of displacement at all iterations, sensitivity of incremental displacement vector for corresponding load step can be computed from:

$$\mathrm{dd^{(l)}} \big/ \mathrm{dx_j} = \sum_{\mathrm{i}} \mathrm{d\Delta d^{(i)}} \big/ \mathrm{dx_j} \tag{43-b}$$

Similarly the total sensitivity of displacement vector can be obtained by summing up the sensitivities in all load steps as follows:

$$dD^{(l)} / dx_j = \sum_i dd^{(l)} / dx_j \tag{43-c}$$

In the coming sections, the way each sensitivity item in (43-a) should be obtained, is explained.

3.1 Sensitivity of tangent stiffness matrix

The tangent stiffness matrix of the structure at each load step is obtained by properly assembling of the tangent stiffness matrix of elements:

$$K_T^{(l)} = \sum_{e=1}^{ne} K_e \tag{44}$$

Where ne is the number of structural elements and K_e is the tangent stiffness matrix of the element accounting for material and geometric nonlinearity. Differentiating (44) with respect to any generalized variable x_j sensitivity of the tangent stiffness matrix can be obtained from:

$$dK_T^{(l)} / dx_j = \sum_{e=1}^{ne} dK_e / dx_j \tag{45}$$

Sensitivity of tangent stiffness matrix of the element is obtained by differentiating (28) and using transformation (30).

$$dK_e / dx_j = T_e^T \left(dK_{te} / dx_j \right) T_e + T_e^T \left(dK_g / dx_j \right) T_e \tag{46}$$

In above equation, the first and second terms on the right are sensitivities of material and geometric nonlinearity, respectively. Noting that K_{g0} in (29) is a constant matrix, the derivative of K_g in (46), can be obtained from:

$$dK_g / dx_j = K_{g0} \left(dN / dx_j \right) \tag{47}$$

Where dN / dx_j in (47) is the sensitivity of axial force of the element and can be determined from section 3.3. It is noted that for beams, values of axial forces are usually negligible. Hence, K_g and its sensitivity can be ignored for beam elements. Considering that K_{a0} and R_E in (46), do not dependent on design variables, sensitivity of K_{te}, can be obtained by differentiating (16), (17) and (18):

$$dK_{te} / dx_j = K_{a0} \left(dA / dx_j \right) + R_E \left(dK_S / dx_j \right) R_E^T \tag{48}$$

It is noted that the first and second terms in the right hand side of (48) must be properly assembled. The first term represents the axial stiffness while the second one represents flexural stiffness. Assuming rigid diaphragm for floors, there will be no change in axial forces of beams. Accordingly the first term in the right hand side is zero. For column elements by defining $A = bh$ sensitivity of A with respect to b is h, and with respect to h is b. It is zero with respect to other variables. The derivative of K_S is determined by differentiating of (19):

$$dK_S/dx_j = \left(d\tilde{L}/dx_j\right)K'_S\tilde{L}^{-1} + \tilde{L}\left(dK'_S/dx_j\right)\tilde{L}^{-1} + \tilde{L}K'_S\left(d\tilde{L}^{-1}/dx_j\right) \tag{49}$$

Where:

$$\frac{d\tilde{L}}{dx_j} = -\frac{1}{1-\lambda_A-\lambda_B}\left\{\left(\frac{d\lambda_A}{dx_j}+\frac{d\lambda_B}{dx_j}\right)\tilde{L}+\begin{bmatrix} d\lambda_B/dx_j & -d\lambda_A/dx_j \\ -d\lambda_B/dx_j & d\lambda_A/dx_j \end{bmatrix}\right\}; \qquad \frac{d\tilde{L}^{-1}}{dx_j} = -\begin{bmatrix} d\lambda_A/dx_j & d\lambda_A/dx_j \\ d\lambda_B/dx_j & d\lambda_B/dx_j \end{bmatrix} \tag{50}$$

In (50), sensitivity of λ_A with respect to h_A is $1/2L$; also sensitivity of λ_B with respect to h_B is $1/2L$. For a typical beam h_A and h_B have been shown in Figure (3) Similar definitions can be made for columns between two floor beams. In Eq. (49) sensitivities of matrix K'_S to any design variable can be obtained by differentiation of its elements as follows:

$$\frac{dK_{S11}}{dx_j} = \frac{12f_bL'}{D_e}\left(\frac{dEI_0}{dx_j}EI_AEI_B + \frac{dEI_A}{dx_j}EI_0EI_B + \frac{dEI_B}{dx_j}EI_AEI_0\right) + \frac{12EI_0EI_AEI_B}{D_e}\left(\frac{df_b}{dx_j}L' + \frac{dL'}{dx_j}f_b - \frac{dD_e}{dx_j}\frac{L'f_b}{D_e}\right) \tag{51-a}$$

$$\frac{dK_{S12}}{dx_j} = \frac{dK_{S11}}{dx_j}\frac{f_c}{f_b} + \frac{df_c}{dx_j}\frac{K_{S11}}{f_b} - \frac{df_b}{dx_j}\frac{f_cK_{S11}}{f_b^2} \tag{51-b}$$

$$\frac{dK_{S22}}{dx_j} = \frac{dK_{S11}}{dx_j}\frac{f_a}{f_b} + \frac{df_a}{dx_j}\frac{K_{S11}}{f_b} - \frac{df_b}{dx_j}\frac{f_aK_{S11}}{f_b^2} \tag{51-c}$$

Where:

$$\begin{aligned} df_a/dx_j &= S'_2\left(6\alpha_A - 4\alpha_A^2 + \alpha_A^3\right) + 3S_3\alpha_B^2\left(d\alpha_B/dx_j\right) \\ &\quad + S_2\left(6 - 8\alpha_A + 3\alpha_A^2\right)\left(d\alpha_A/dx_j\right) + S'_3\alpha_B^3 + 4S'_1 \end{aligned} \tag{52-a}$$

$$\begin{aligned} df_b/dx_j &= S'_3\left(6\alpha_B - 4\alpha_B^2 + \alpha_B^3\right) + 3S_2\alpha_A^2\left(d\alpha_A/dx_j\right) \\ &\quad + S_3\left(6 - 8\alpha_B + 3\alpha_B^2\right)\left(d\alpha_B/dx_j\right) + S'_2\alpha_A^3 + 4S'_1 \end{aligned} \tag{52-b}$$

$$\begin{aligned} \mathrm{df_c}/\mathrm{dx_j} = &-\mathrm{S}_2'\left(2\alpha_A^2-\alpha_A^3\right)-\mathrm{S}_2\left(4\alpha_A-3\alpha_A^2\right)\left(\mathrm{d}\alpha_A/\mathrm{dx_j}\right) \\ &-\mathrm{S}_3'\left(2\alpha_B^2-\alpha_B^3\right)-\mathrm{S}_3\left(4\alpha_B-3\alpha_B^2\right)\left(\mathrm{d}\alpha_B/\mathrm{dx_j}\right)-2\mathrm{S}_1' \end{aligned} \tag{52-c}$$

$$D_e/dx_j = \left(f_b\, df_a/dx_j + f_a\, df_b/dx_j - 2 f_c\, df_c/dx_j\right)L'^2 + 2\left(f_a f_b - f_c^2\right)L'\, dL'/dx_j \tag{52-d}$$

$$dL'/dx_j = -\left(d\lambda_a/dx_j + d\lambda_b/dx_j\right)L \tag{52-e}$$

Where:

$$\left.\begin{aligned} \mathrm{S}_1' &= \mathrm{EI_B\, dEI_A/dx_j + EI_A\, dEI_B/dx_j} \\ \mathrm{S}_2' &= \left(\mathrm{dEI_0/dx_j - dEI_A/dx_j}\right)\mathrm{EI_B} + \left(\mathrm{EI_0 - EI_A}\right)\mathrm{dEI_B/dx_j} \\ \mathrm{S}_3' &= \left(\mathrm{dEI_0/dx_j - dEI_B/dx_j}\right)\mathrm{EI_A} + \left(\mathrm{EI_0 - EI_B}\right)\mathrm{dEI_A/dx_j} \end{aligned}\right\} \tag{53}$$

In these equations, the derivative of EI_0 with respect to any variable X can be obtained by differentiating of (24) with respect to X. This leads to (54).

$$\frac{\mathrm{dEI_0}}{\mathrm{dx_j}} = \frac{2}{\mathrm{EI_{A0}+EI_{B0}}}\left(\frac{\mathrm{dEI_{A0}}}{\mathrm{dx_j}}\mathrm{EI_{B0}} + \frac{\mathrm{dEI_{B0}}}{\mathrm{dx_j}}\mathrm{EI_{A0}}\right) - \frac{\mathrm{2EI_{A0}EI_{B0}}}{\mathrm{(EI_{A0}+EI_{B0})^2}}\left(\frac{\mathrm{dEI_{A0}}}{\mathrm{dx_j}} + \frac{\mathrm{dEI_{B0}}}{\mathrm{dx_j}}\right) \tag{54}$$

For an element with two end moments greater than cracking moment and single curvature the sensitivity of $\mathrm{EI_0}$ with respect to X should be modified by (55). Sensitivity of EI_{A0} and EI_{B0} will be obtained from (56) considering the case of $M_p \le M_{crp}$.

$$\frac{dEI_0}{dx_j} = \frac{2}{EI_A+EI_B}\left(\frac{dEI_A}{dx_j}EI_B + \frac{dEI_B}{dx_j}EI_A\right) - \frac{2EI_AEI_B}{(EI_A+EI_B)^2}\left(\frac{dEI_A}{dx_j} + \frac{dEI_B}{dx_j}\right) \tag{55}$$

In Eq. (51) to (55), to use the sensitivity EI_A and EI_B, it is sufficient to differentiate (15) w.r.t. X as follows:

$$dEI_p/dx_j = \left(1/\phi_{crp}\right)\left(dM_{crp}/dx_j\right) - \left(M_{crp}/\phi_{crp}^2\right)\left(d\phi_{crp}/dx_j\right) \tag{56-a}$$

$$\frac{dEI_p}{dx_j} = \frac{1}{\phi_{yp}-\phi_{crp}}\left(\frac{dM_{yp}}{dx_j} - \frac{dM_{crp}}{dx_j}\right) - \frac{M_{yp}-M_{crp}}{(\phi_{yp}-\phi_{crp})^2}\left(\frac{d\phi_{yp}}{dx_j} - \frac{d\phi_{crp}}{dx_j}\right) \tag{56-b}$$

$$\frac{dEI_p}{dx_j} = \frac{1}{\phi_{up}-\phi_{yp}}\left(\frac{dM_{up}}{dx_j} - \frac{dM_{yp}}{dx_j}\right) - \frac{M_{up}-M_{yp}}{(\phi_{up}-\phi_{yp})^2}\left(\frac{d\phi_{up}}{dx_j} - \frac{d\phi_{yp}}{dx_j}\right) \tag{56-c}$$

Where (56-a), (56-b) and (56-c) are used for zones of 1, 2 and 3 of $M-\phi$ curve, respectively.

Since moment-curvature relations are functions of design variables their sensitivities should be determined from Eqs.(1 to 6). These derivatives have been reported in Appendix. According to section 2.2, the sensitivity of α_A and α_B that have been used in (52), are zero provided that that moments are less than cracking moments or the element undergoes single curvature with end moments greater than cracking moment. For other cases the sensitivities can be determined by differentiating of (23):

$$\frac{d\alpha_p}{dx_j}=\frac{1}{|M_A-M_B|}\left[\left(\frac{dM_p}{dx_j}\frac{M_p}{|M_p|}-\frac{dM_{crp}}{dx_j}\right)-\frac{|M_p|-M_{crp}}{(M_A-M_B)}\left(\frac{dM_A}{dx_j}-\frac{dM_B}{dx_j}\right)\right] \tag{57}$$

Where dM_p/dx_j is the sensitivity of moment at end 'p' of the element; its value will be presented in section 3.3. In (57), if α_p >1, its sensitivity is zero and if $\alpha_p < \alpha_{pm}$, its sensitivity is equal to sensitivity of α_{pm}. When $\alpha_A+\alpha_B \succ 1$, The sensitivity of EI_0, α_A and α_B must be modified by differentiation of (26):

$$\begin{aligned}\frac{d\alpha'_A}{dx_j}=&\left(f_{Ax}-f_{Bx}+\frac{f_{Bx}-f_{0x}}{\alpha_B}-\frac{d\alpha_B}{dx_j}\frac{f_B-f_0}{\alpha_B^2}\right)\Big/\left(\frac{f_A-f_0}{\alpha_A}+\frac{f_B-f_0}{\alpha_B}\right)-\\&\left(\frac{f_{Ax}-f_{0x}}{\alpha_A}-\frac{d\alpha_A}{dx_j}\frac{f_A-f_0}{\alpha_A^2}+\frac{f_{Bx}-f_{0x}}{\alpha_B}-\frac{d\alpha_B}{dx_j}\frac{f_B-f_0}{\alpha_B^2}\right)\left(f_A-f_B+\frac{f_B-f_0}{\alpha_B}\right)\Big/\left(\frac{f_A-f_0}{\alpha_A}+\frac{f_B-f_0}{\alpha_B}\right)^2\end{aligned} \tag{58-a}$$

$$\frac{d\alpha'_B}{dx_j}=-\frac{d\alpha'_A}{dx_j} \tag{58-b}$$

$$\frac{df'_0}{dx_j}=f_{Ax}-\frac{f_{Ax}-f_{0x}}{\alpha_A}\alpha'_A-(f_A-f_0)\left(\frac{1}{\alpha_A}\frac{d\alpha'_A}{dx_j}-\frac{\alpha'_A}{\alpha_A^2}\frac{d\alpha_A}{dx_j}\right) \tag{58-c}$$

Where:

$$f_{Ax}=-\left(dEI_A/dx_j\right)\left(1/EI_A^2\right);\ f_{Bx}=-\left(dEI_B/dx_j\right)\left(1/EI_B^2\right);\ f_{0x}=-\left(dEI_0/dx_j\right)\left(1/EI_0^2\right) \tag{59}$$

3.2 Sensitivity of external loads

Since in the pushover analysis, load pattern depends on the natural period of the structure and the natural period depends on structural stiffness and building mass, any change in the structure stiffness causes a change in the external loads. Since gravity loads are constant, the sensitivity of external load vector at any load step l comprises only sensitivity of lateral loads.

$$dP^{(l)} / dx_j = dP_E^{(l-1)} / dx_j + d\Delta P_E / dx_j \tag{60}$$

Where the second term in right hand side is the sensitivity of incremental pushover loads and is obtained from

$$d\Delta P_E / dx_j = \Delta V_b \, dC_v / dx_j \tag{61}$$

Where the components of dC_v / dx_j vector are found from following equation:

$$\frac{dc_{v,s}}{dx_j} = \left(W_s H_s^k \Big/ \sum_{s=1}^{ns} W_s H_s^k \right) \left(l_n H_s - \sum_{s=1}^{ns} W_s H_s^k \left(l_n H_s \right) \Big/ \sum_{s=1}^{ns} W_s H_s^k \right) dk / dx_j \tag{62}$$

In above equation, for $T \le 0.5$ and $T \ge 2.5$, the sensitivity of k is zero. For other values of T, derivative of (33) gives:

$$dk / dx_j = 0.5 dT / dx_j \tag{63}$$

Where dT / dx_j is the sensitivity of the fundamental period of the structure. It can be found from Moharrami and Alavinasab [30]:

$$dT / dx_j = -\left(T^3 / 8\pi^2 \right) \phi^T \left(dK / dx_j \right) \phi \tag{64}$$

Where ϕ is the first mode shape of the structure.

3.3 Sensitivity of internal forces

Sensitivity of the incremental internal forces of any typical element at load step l and Newton-Raphson iteration i can be determined from differentiating of (34):

$$d\Delta F_e^{(l,i)} / dx_j = d_e^{(i-1)} \left(dK_e^{(l)} / dx_j \right) + K_e^{(l)} \left(dd_e^{(i-1)} / dx_j \right) \tag{65}$$

The sensitivity of internal forces of the element is updated by (66).

$$dF_e^{(l,i)} / dx_j = dF_e^{(l-1)} / dx_j + d\Delta F_e^{(l,i)} / dx_j \tag{66}$$

After obtaining the sensitivity of moments at the two end joints of RC member, it will have to transferred to the design end of the element form the following equation:

$$d\Delta M^{(i)} / dx_j = \left(d\tilde{L}^{-1} / dx_j \right) \Delta m^{(i)} + \tilde{L}^{-1} \left(d\Delta m^{(i)} / dx_j \right) \tag{67}$$

The sensitivity of moments should be modified according to the change in curvature of the element. The curvature sensitivities of the element are calculated by differentiating of (37):

$$d\phi_p^{(i)}/dx_j = d\phi_p^{(i-1)}/dx_j + \left(d\Delta M_p^{(i)}/dx_j\right)/EI_p^{(l)} - \left(dEI_p^{(l)}/dx_j\right)\Delta M_p^{(i)}/\left(EI_p^{(l)}\right)^2 \tag{68}$$

Modified values of moment sensitivities of the element are computed by differentiating of (38):

$$dM_p^{(i)}/dx_j = \left(dEI_p^{(l)}/dx_j\right)\phi_p^{(i)} + EI_p^{(l)}\left(d\phi_p^{(i)}/dx_j\right) \tag{69-a}$$

$$dM_p^{(i)}/dx_j = dM_{crp}^{(l)}/dx_j + \left(dEI_p^{(l)}/dx_j\right)\left(\phi_p^{(i)} - \phi_{crp}^{(l)}\right) + EI_p^{(l)}\left(d\phi_p^{(i)}/dx_j - d\phi_{crp}^{(l)}/dx_j\right) \tag{69-b}$$

$$dM_p^{(i)}/dx_j = dM_{yp}^{(l)}/dx_j + \left(dEI_p^{(l)}/dx_j\right)\left(\phi_p^{(i)} - \phi_{yp}^{(l)}\right) + EI_p^{(l)}\left(d\phi_p^{(i)}/dx_j - d\phi_{yp}^{(l)}/dx_j\right) \tag{69-c}$$

$$dM_p^{(i)}/dx_j = dM_{up}^{(l)}/dx_j \tag{69-d}$$

Where (69-a), (69-b), (69-c) and (69-d) are used for zones of 1, 2, 3 and 4 of $M-\phi$ curve, respectively.

By having modified moment sensitivities at ends of the element by (69), moment sensitivities at joints of the element are determined by differentiating of (39):

$$dm^{(i)}/dx_j = \left(d\tilde{L}/dx_j\right)M^{(i)} + \tilde{L}\left(dM^{(i)}/dx_j\right) \tag{70}$$

Modified values of shear force sensitivities at joints of the element are computed by differentiating of (40):

$$dV_p^{(i)}/dx_j = (dm_A^{(i)}/dx_j + dm_B^{(i)}/dx_j)/L \tag{71}$$

3.4 Summary of Sensitivity analysis procedure

The nonlinear sensitivity analysis of RC frames under pushover analysis that was developed in this study, can be carried out in following steps:

1. Determine the moment-curvature relations from (1) to (6) and Compute their derivatives from appendix.
2. Compute the first mode shape, first period and its sensitivity from (64) to (69)
3. Calculate the matrixes $\tilde{L}$ and $\tilde{L}^{-1}$ and their derivatives for each element from (20) and (50)
4. Compute the lateral load distribution factors and their sensitivities by (32) and (62)
5. Set load step index $l=1$ and apply the gravity loads only.
6. Compute the lateral loads and their sensitivities from (31) and (60)
7. Compute the flexural stiffness and yield penetration parameters and their

sensitivities for each element from (15), (23) to (27) and (54) to (59)

8. Compute the tangent stiffness matrix and its sensitivity for each element by (30) and (46)
9. Assemble the global tangent stiffness matrix and its matrix of sensitivity by (44) and (45)
10. Set Newton-Raphson iteration $i = 1$
11. Solve (41) for displacements and find their incremental sensitivities from (43)
12. Calculate the internal forces and their sensitivities from (34) to (40) and (70) to (76)
13. Set $i = i + 1$, if unbalance forces are zero, stop the sensitivity analysis at load step l and go to step 14; otherwise go to step 11
14. Set $l = l + 1$, if the roof lateral displacement is greater than target displacement, stop the sensitivity analysis and go to step 15; otherwise go to step 6.
15. Compute the total incremental sensitivities coefficients

For more details about the algorithm of nonlinear sensitivity analysis refer to Habibi (may be 2006).

4. NUMERICAL EXAMPLE

A three-story, two-bay planar frame of Figure 4 is used to illustrate the method of proposed analytical nonlinear sensitivity analysis. The concrete is assumed to have a cylinder strength of 30 Mpa, a modulus of rupture of 3.45 Mpa, a modulus of elasticity of 27,400 Mpa, a strain of 0.002 at maximum strength and an ultimate strain of 0.003. The steel has a yield strength of 300 Mpa and a modulus of elasticity of 200,000 Mpa. A uniformly distributed gravity load of 12 KN/m is applied on the beams of each story. Reinforcements have the cover to the steel centroid of 50 mm. It is assumed that columns and beams have rectangular cross sections. Fourteen design variables that have been reported in Table. 1 have been defined in this frame. In order to allow the structure enter to the inelastic behavior, Target displacement of 2% has been chosen as a stop criterion for load steps of sensitivity analysis. Convergence tolerance of 0.1% was considered as a stop criterion for Newton-Raphson iterations. The pushover (capacity) curve of this frame is shown in Figure 5 and target point has specified on it.

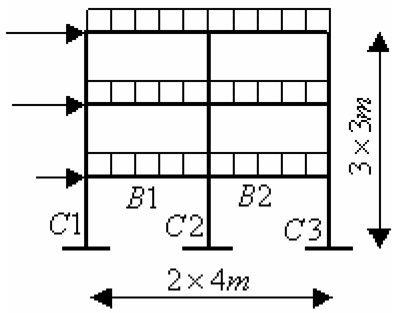

Figure 4. A three-story, two-bay RC frame

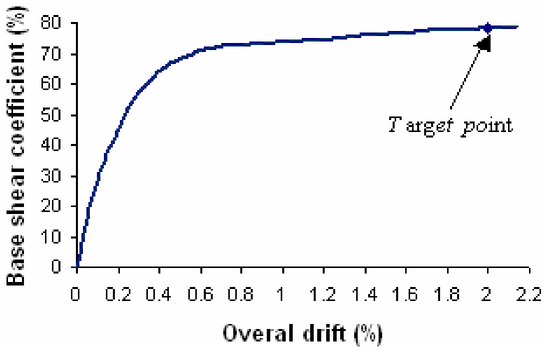

Figure 5. Capacity curve of the three-story RCMRF

Nonlinear sensitivity analysis is performed on this frame by the proposed Analytical Method (AM) and compared to the results from central Finite Difference Method (FDM). In this example, as a sample, sensitivity analysis of the overall drift that may indirectly represent the sensitivity of internal forces to change in some design variables is considered. The results have been briefed in Table. 2 for two nonlinearity cases namely: (i) material nonlinearity only NC1, and (ii) both geometric and material nonlinearity NC2. For interpreting the results, for example consider the design variable No.1. The overall drift sensitivity shows that its value will be decreased by 0.2688% and –0.2318% for a unit change in design variable No.1, for nonlinearity cases (i) and (ii), respectively. Linear sensitivity analysis for the same variable indicates the sensitivity of –0.0113% which is much less than its actual value. Values of nonlinear sensitivity coefficients predict that if value of width of columns C1 and C3 increase from 350mm to, 360mm; overall drift of 2% decreases to 1.9973% and 1.9977% for NC1 and NC2, respectively. These predictions are very important in optimal performance-based design process. In Table 2, percentage of error of FDM compared to AM has evaluated from:

$$ERROR = 100(S_{FDM} - S_{AM}) / S_{AM} \tag{72}$$

In Eq. (72), S_{FDM} and S_{AM} are the sensitivity coefficients obtained from FDM and AM respectively. In FDM method, the design variables perturbation has been considered to be 1%, 0.1% and 0.01%. Comparing the AM results with FDM results, it is observed that by reducing perturbation values results of FDM almost all design variables converge to AM results; that is, the AM results are in good agreement with FDM results in very small perturbation cases.

The results of sensitivity analysis by FDM shows that assuming perturbation of 1% in design variables this method in most cases generates inaccurate results; while this is not the case in the linear sensitivity analysis.

Comparing results for two cases NC1 and NC2 shows that sensitivity coefficients of frame when both geometric and material nonlinearity are accounted for are smaller than the case where only the material nonlinearity is considered.

Table 1. Design variables

Number of design variable	**Description**	**Value**
1	Width of columns C1 and C3	350 mm
2	Height of columns C1 and C3	350 mm
3	Width of columns C2	450 mm
4	Height of columns C2	450 mm
5	Reinforcement area of columns C1 and C3 on each face of column	942.5 mm^2
6	Reinforcement area of columns C2 on each face of column	1570.8 mm^2
7	Width of beams	250 mm
8	Height of beams	350 mm
9	Lower reinforcement area for beams B1 at left hand and for beams B2 at right hand of beam (first and second stories)	1256.6 mm^2
10	Upper reinforcement area for beams B1 at left hand and for beams B2 at right hand of beam (first and second stories)	942.5 mm^2
11	Lower reinforcement area for beams B1 at right hand and for beams B2 at left hand of beam (first and second stories)	1256.6 mm^2
12	Upper reinforcement area for beams B1 at right hand and for beams B2 at left hand of beam (first and second stories)	1570.8 mm^2
13	Lower reinforcement area for third story beams	628.3 mm^2
14	Upper reinforcement area for third story beams	628.3 mm^2

Table 2. Sensitivity coefficients of overall drift at target point (%)

Method		**Sensitivity with respect to design variable**						
		1	**4**	**5**	**7**	**8**	**9**	**12**
AM	NC1	-0.2688	-10.8864	-3284.1	-1.1499	-46.3542	-2333.1	-2183.6
	NC2	-0.2318	-10.5285	-3190.1	-0.7619	-45.0750	-2296.4	-2151.2
FDM 1%	NC1	-0.4757	-10.6962	-3171.2	-1.7722	-61.3068	-2140.8	-2548.7
	NC2	-0.2184	-12.2850	-3417.3	-1.9828	-61.5887	-1950.0	-2776.3
FDM 0.1%	NC1	-0.2707	-10.8195	-3284.0	-1.1491	-50.2593	-2332.8	-2183.5
	NC2	-0.2363	-12.1370	-3190.4	-0.7609	-38.4454	-2295.2	-2151.9
FDM 0.01%	NC1	-0.2688	-10.8859	-3284.1	-1.1499	-46.3487	-2332.9	-2183.3
	NC2	-0.2331	-10.5285	-3190.3	-0.7622	-45.1176	-2296.4	-2151.2
ERROR 1%	NC1	76.95	-1.75	-3.44	54.12	32.26	-8.24	16.72
	NC2	-5.80	16.68	7.11	160.24	36.64	-15.09	29.06
ERROR 0.1%	NC1	0.69	-0.61	-0.004	-0.07	8.42	-0.01	-0.004
	NC2	1.94	15.28	0.004	-0.13	-14.71	-0.05	0.04
ERROR 0.01%	NC1	-0.03	-0.005	-0.001	-0.001	-0.01	-0.006	-0.01
	NC2	0.53	-0.001	-0.001	0.03	0.09	0.0003	0.0002

5. CONCLUSION

In the present study a procedure for exact sensitivity analysis of RCMRF using pushover analysis was proposed. The procedure accounts for both material and geometric nonlinearity in the context of modified Newton-Raphson analysis technique. The proposed method does not face to the difficulties that the FDM method confronts.

The proposed sensitivity analysis involves in much less computational effort compared to FDM. This achievement provides a powerful tool in optimal design of nonlinear structures and is an essential requirement in successful optimal performance-based seismic design of RCMRF.

Sensitivity analysis of a three-story RC frame illustrated the capability and effectiveness of the derived formulations. Results of the case study for a nonlinear structure indicate that sensitivity calculation via FDM method may end up to inaccurate sensitivity results especially if the value of perturbation is high.

It was shown that when both material and geometric nonlinearity is accounted for, the sensitivity of a structural behavior, such as overall drift, is less than that when only material nonlinearity is accounted for.

REFERENCES

1. Krawinkler, H., Seneviratna G.D.P.K., Pros and Cons of a Pushover Analysis of Seismic performance evaluation. *Engineering Structures*, **20**(1998) 452-464.
2. FEMA-273, NEHRP Guideline for the Seismic Rehabilitation of Buildings, *Federal Emergency Management Agency*, Washington, DC, 1997.
3. ATC-40, Seismic Evaluation and Retrofit of Concrete Buildings, *Applied Technology Council*, California Seismic Safety Commission, 1997.
4. Moharami, H., Design Optimization of Reinforced Concrete Building Frameworks, *Ph.D. Thesis*, University of Waterloo, Ontario, Canada, 1993.
5. Ryu, Y.S., Haririan, M., Wu, C.C., Arora J.S, Structural Design Sensitivity Analysis of nonlinear response, *Computers and Structures*, **21**(1985) 245-255.
6. Choi, K.K., Santos, J.L.T., Design Sensitivity Analysis of Nonlinear Structural Systems Part I: Theory, *International Journal for Numerical Methods in Engineering*, **24**(1987) 2039-2055.
7. Gopalakrishna, H.S., Greimann, F., Newton-Raphson Procedure for the Sensitivity Analysis of Nonlinear Structural Behavior, *Computers and Structures*, **30**(1988) 1263-1273.
8. Santos, J.L.T., Choi, K.K., Shape Design Sensitivity Analysis of Nonlinear Structural Systems, *Structural and Multidisciplinary Optimization*, **4**(1992) 23-35.
9. Ohsaki, M., Arora, J.S., Design Sensitivity Analysis of Elasto-Plastic Structures, *International Journal for Numerical Methods in Engineering*, **37**(1994) 737-762.
10. Lee, T.H., Arora, J.S., A Computational Method for Design Sensitivity Analysis of Elastoplastic Structures, *Computer Methods in Applied Mechanics and Engineering*, **122**(1995) 27-50.
11. Barthold, F.J., Stein, E., A Continuum Mechanical-Based Formulation of the Variational Sensitivity Analysis In Structural Optimization Part I: Analysis, *Structural and Multidisciplinary Optimization*, **11**(1996) 29-42.
12. Szewczyk, Z.P., Ahmed, K.N., A Hybrid Numerical/Neurocomputing Strategy for Sensitivity Analysis of Nonlinear Structures, *Computers and Structures*, **65**(1997) 869-880.
13. Yamazaki, K., Sensitivity Analysis of Nonlinear Material and its Application to Shape Optimization, *AIAA Journal*, **36**(1998) 1113-1115.
14. Bugeda, G., Gil, L., Onate, E., Structural Shape Sensitivity Analysis for Nonlinear Material Models with Strain Softening, *Structural and Multidisciplinary Optimization*, **17**(1999) 162-171.
15. Schwarz, S., Ramm, E., Sensitivity Analysis and Optimization for Nonlinear Structural

Response, *Engineering Computations*, **18**(2001) 610.
16. Gong, Y., Xu, L., Grierson, D.E., Sensitivity Analysis of Steel Moment Frames Accounting for Geometric and Material Nonlinearity, *Computers and Structures*, **84**(2006) 462-475.
17. Saiidi, M., Sozen, M.A., Simple Nonlinear Seismic Analysis of R/C Structures, *Journal of Structural Division*, ASCE, **107**(1981) 937-952.
18. Bracci, J.M., Kunnath, S.K., Reinhorn, A.M., Seismic Performance and retrofit Evaluation of Reinforced Concrete Structures, *Journal of Structural Engineering*, ASCE **123**(1997) 3-10.
19. Kilar, V., Fajfar, P., Simple Pushover Analysis of Asymmetric Buildings, *Earthquake Engineering and Structural Dynamics*, **26**(1997) 233-249.
20. Gupta, A., Krawinkler, H., Behavior of ductile SMRFs at Various Seismic Hazard Levels, *Journal of Structural Engineering*, ASCE, **126**(2000) 98-107.
21. Mwafy, A.M., Elnashai, A.S., Static Pushover Versus Dynamic Collapse Analysis of RC Buildings, *Engineering Structures*, **23**(2001) 407-424.
22. Hassan, R., Xu, L., Grierson, D.E., Pushover Analysis for Performance-based Seismic design, *Journal of Computers and Structures*, **80**(2002) 2483-2493.
23. Chopra, A.K., Goel, R.K., A Modal Pushover Analysis Procedure for Estimating Seismic Demands for Buildings, *Earthquake Engineering and Structural Dynamics*, **31**(2002) 561-582.
24. Park, R., Paulay, T., Reinforced Concrete Structures, John Wiley and Sons, Inc., New York, 1974.
25. Park, Y.J., Ang, A.H.S., Mechanistic Seismic Damage Model for Reinforced Concrete. Journal of Structural Engineering, ASCE, **111**(1985) 722-739.
26. Park, Y.J., Reinhorn, A.M., Kunnath, S.K. IDARC: Inelastic damage analysis of frame shear-wall Structures, *Technical Report NCEER-87-0008*, National Center for Earthquake Engineering Research, State University of New York at Buffalo, 1987.
27. Habibi, A.R., Methodology of Optimization and Automated Performance-based Seismic design of 2D RC Frameworks, *Reports No.1 and No.2 of Ph.D. Thesis*, Tarbiat Modares University, Tehran, Iran, 2006.
28. Przemieniecki, J.S., Theory of Matrix Structural Analysis, McGraw Hill, Inc, 1968.
29. Bathe, K.J., Finite Element Procedures, Prentice Hall, 1996.
30. Moharrami, H., Alavinasab, S.A., Design Optimization of Seismic Resistant Steel Frames, *Proceeding of the Eighth International Conference on Civil and Structural Engineering Computing*, Eisenstadt, Vienna, Austria, 2001.

APPENDIX

Derivatives of moment-curvature characteristics
a) Cracking state

$$dM_{cr} / dx_j = f_r I_x / (h - y) - f_r I (h_x - y_x) / (h - y)^2$$

$$d\phi_{cr} / dx_j = -f_r\left(h_x - y_x\right) / E_c\left(h - y\right)^2$$

b) Yielding state

$$\begin{aligned} dM_y / dx_j = {} & 0.5f_c\left(b_{tx}\left(h-c\right)^2 + 2b_t h_x\left(h-c\right)\right)\left[(2-\eta)p + \left(\eta - 2\beta\right)\alpha p'\right] + 0.5f_c b_t\left(h-c\right)^2\left[-\eta_x p + (2\right. \\ & \left. -\eta)p_x + \left(\eta_x - 2\beta_x\right)\alpha p' + \left(\eta - 2\beta\right)(\alpha_x p' + \alpha p'_x)\right] \end{aligned}$$

$$d\phi_y / dx_j = -1.05\varepsilon_y\left[-k_x\left(h-c\right) + \left(1-k\right)h_x\right] / \left[\left(1-k\right)^2\left(h-c\right)^2\right]\}$$

c) Ultimate state

$$dM_u / dx_j = -0.15 p_x M_y + \left(1.24 - 0.15p\right) dM_y / dx_j$$

$$d\phi_u / dx_j = -1.05\beta_1\varepsilon_u\left[0.5(2RR_x + S_x)(R^2 + S)^{-0.5} - R_x\right] \Big/ \left[(R^2 + S)^{0.5} - R\right]^2$$

Where:

$$\begin{aligned} R_x &= \left[(\rho'_x\varepsilon_u E_s - \rho_x f_y)(h-c) + (\rho'\varepsilon_u E_s - \rho f_y)h_x\right] / (1.7f_c) \\ S_x &= \varepsilon_u E_s\beta_1 c\left[\rho'_x(h-c) + \rho' h_x\right] / (0.85f_c) \end{aligned}$$

In above equations, extra parameters that have not been defined in previous sections are obtained from following equations:

$$\begin{aligned} I_x = {} & b_{tx}t^3/12 + b_t t^2 t_x/12 + b_{tx}t\left(y - 0.5t\right)^2 + b_t t_x(y - 0.5t)^2 + 2b_t t(y - 0.5t)(dy/dx_j - 0.5t_x) + b_{bx}(h-t)^3/12 \\ & + b_b(h-t)^2(h_x - t_x)/4 + \left[b_{bx}\left(h-t\right) + b_b\left(h_x - t_x\right)\right]\left(0.5h + 0.5t - y\right)^2 + 2b_b\left(h-t\right)\left(0.5h + 0.5t - y\right)(0.5h_x + 0.5t_x \\ & - dy/dx_j) + (n_{sc} - 1)\left[A_{sx}(h - c - y)^2 + A'_{sx}\left(y - c\right)^2\right] + \left(n_{sc} - 1\right)\left[2A_s\left(h - c - y\right)\left(h_x - y_x\right) + 2A'_s\left(y - c\right)dy/dx_j\right] \end{aligned}$$

$$k_x = \begin{cases} \left.\begin{aligned} & 0.5\left[\left(p + p'\right)^2/4\alpha^2 + \left(p + \beta p'\right)/\alpha\right]^{-0.5} \times \left[2\left(p + p'\right)\left(p_x + p'_x\right)/4\alpha^2 - (p + p')^2\alpha_x/2\alpha^3\right. \\ & \left. + \left(p_x + \beta_x p' + \beta p'_x\right)/\alpha - \left(p + \beta p'\right)\alpha_x/\alpha^2\right] - \left(p_x + p'_x\right)/2\alpha + \left(p + p'\right)\alpha_x/2\alpha^2 \end{aligned}\right\} k \le \dfrac{t}{h-c} \\[2ex] \left.\begin{aligned} & 0.5\left\{\left[n_{sc}(\rho + \rho') + t\left(b_t/b_b - 1\right)/(h-c)\right]^2 + 2n_{sc}\left(\rho + \rho'\beta\right) + t^2\left(b_t/b_b - 1\right)\right. \\ & \left./(h-c)^2\right\}^{-0.5} \times \left\{2\left[n_{sc}\left(\rho + \rho'\right) + t(b_t/b_b - 1)/(h-c)\right]\left[n_{sc}\left(\rho_x + \rho'_x\right) + t(b_{tx}/b_b\right.\right. \\ & \left.\left. - b_t b_{bx}/b_b^2)/(h-c) + \left(b_t/b_b - 1\right)\left(t_x/(h-c) - th_x/(h-c)^2\right)\right] + 2n_{sc}(\rho_x + \rho'_x\beta\rho'\beta_x) \\ & \left. + t^2\left(b_{tx}/b_b - b_t b_{bx}/b_b^2\right)/(h-c)^2 + \left(b_t/b_b - 1\right)\left(2tt_x/(h-c)^2 - 2t^2h_x/(h--c)^3\right)\right\} \\ & - n_{sc}\left(\rho_x + \rho'_x\right) - t(b_{tx}/b_b - b_t b_{bx}/b_b^2)/(h-c) - \left(b_t/b_b - 1\right)\left[t_x/(h-c) - th_x/(h-c)^2\right] \end{aligned}\right\} k \succ \dfrac{t}{h-c} \end{cases}$$

$$\eta_x = 0.525[\phi_y(h-c) - \varepsilon_y]^{-0.3}[(h-c)d\phi_y/dx_j + \phi_y h_x] \Big/ [(1 + \varepsilon_y/\varepsilon_0)\varepsilon_0^{0.7}]$$

$$\alpha_x = -\beta_x \left\lfloor \phi_y (h-c) - \varepsilon_y \right\rfloor / \varepsilon_y + (1-\beta) \times \left\lfloor (h-c) d\phi_y / dx_j + \phi_y h_x \right\rfloor / \varepsilon_y - \beta_x$$

$$\rho_x = A_{sx} / b_b (h-c) - A_s \left[b_{bx} (h-c) + b_b h_x \right] / b_b^2 (h-c)^2$$

$$\rho'_x = A'_{sx} / b_b (h-c) - A'_s \left[b_{bx} (h-c) + b_b h_x \right] / b_b^2 (h-c)^2$$

$$p_x = \rho_x f_y / f_c ; \; p'_x = \rho'_x f_y / f_c$$

$$\beta_x = -c h_x / (h-c)^2$$

$$dy / dx_j = (y_{1x} y_2 - y_{2x} y_1) / y_1^2$$

$$\begin{aligned} y_1 &= 0.5 b_t t^2 + 0.5 b_b \left(h^2 - t^2 \right) + \left(n_{sc} - 1 \right) \left(A_s (h-c) + A'_s c \right) \\ y_2 &= b_t t + b_b \left(h - t \right) + \left(n_{sc} - 1 \right) \left(A_s + A'_s \right) \end{aligned}$$

$$\begin{aligned} y_{1x} &= 0.5 b_{tx} t^2 + b_t t t_x + 0.5 b_{bx} \left(h^2 - t^2 \right) + b_b \times \left(h h_x - t t_x \right) + \left(n_{sc} - 1 \right) \left[A_{sx} (h-c) + A_s h_x + A'_{sx} c \right] \\ y_{2x} &= b_{tx} t + b_t t_x + b_{bx} \left(h - t \right) + b_b h_x + \left(n_{sc} - 1 \right) \left(A_{sx} + A'_{sx} \right) \end{aligned}$$

In equations presented in this appendix, values of b_{tx}, b_{bx}, t_x and h_x are equal to 1 for $x_j = b_t, b_b, t$ and h, respectively. Values of these parameters for other design variables are equal to zero. In equation related to α_x, if value of α is greater than 1, value α_x must be considered zero.