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ADAPTIVE PUSHOVER ANALYSIS

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ABSTRACT

Nonlinear static methods are simplified procedures in which the problem of evaluating the maximum expected response of a MDOF system for a specified level of earthquake motion is replaced by response evaluation of its equivalent SDOF system. The common features of these procedures are the use of pushover analysis to characterize the structural system. In pushover analysis both the force distribution and the target displacement are based on the assumptions that the response is controlled by the fundamental mode and that the mode shape remains unchanged after the structure yields. Therefore, the invariant force distributions does not account for the change of load patterns caused by the plastic hinge formation and changes in the stiffness of different structural elements. That could have some effects in the outcome of the method depending on different structural parameters. This paper introduces an adaptive pushover analysis method to improve the accuracy of the currently used pushover analysis in predicting the seismic-induced dynamic demands of the structures. Comparison of the common pushover analyses, adaptive pushover analyses and time-history analyses performed for a number of multiple-bay, short and high-rise steel structures, demonstrates the efficiency of the proposed method.

Keywords: nonlinear time history analysis; pushover analysis; adaptive pushover analysis, nonlinear static analysis, FEMA 356, load pattern

1. INTRODUCTION

In recent years, the seismic design provisions necessary for the construction of new buildings and rehabilitation of existing structures have been witnessing some rapid changes. Comprehensive research is now being conducted to evaluate the current seismic design methodology implemented in different codes and standards. The FEMA356 [1], ATC40 [2] and vision2000 [3] are the most prominent references that have presented a simplified nonlinear static analysis technique that could be used to estimate the dynamic demands imposed on the structures during an earthquake episode. Therefore, more attention is paid to the nonlinear procedures as they can provide a more accurate assessment of the demands

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induced in different structural elements under earthquake loading than any other linear method available. Nonlinear dynamic analysis of refined mathematical models of structures subjected to site specific earthquakes is the most accurate means to evaluate these demands [4]. However, such an approach, for the time being, is not practical for everyday design use. Currently, the most rational analysis and performance evaluation methods for practical applications seem to be the simplified inelastic procedures. Therefore nonlinear static methods are getting more recognition as simple yet efficient methods of estimating the seismic demands in the structures [5].

As a matter of fact, the newly introduced provisions for seismic design or retrofitting of the structures are to provide the professional engineers with some easy to use guidelines for their daily practice. That would eliminate the need for a complicated linear or nonlinear time-history analysis of the multi-story buildings in majority of the cases. However, the development of such provisions should be based on a reasonable approach to fulfill the requirements for the seismic design of the structures. The deficiencies of the previous codes, revealed by the recent major earthquakes, should also be considered in the development of any new seismic code. The nonlinear static analysis methods has been proposed, formulated and evaluated in somewhat different formats in recent studies. This method basically consists of having a structural model with nonlinear material properties displaced to a target displacement under monotonically increasing lateral loading. The output of such an analysis is the demand in different structural elements, which is compared with their related capacities. Thus, it represents a relatively simple alternative to estimate the nonlinear behavior of structures.

In this paper, first a modification is proposed to the regular pushover analysis in a way that the lateral load pattern would be determined using the structure's first mode shape and its effective modal mass. This is followed by another approach to change the lateral load pattern during an analysis as the plastic hinges are formed in the structure. The proposed adaptive method not only automates the pushover analysis, but also improves its performance in predicting the system's demand parameters. A numerical example is used to compare the accuracy of different load patterns and to demonstrate the efficiency of the proposed adaptive pushover analysis.

2. DIFFERENTIAL EQUATIONS OF MOTION

The differential equations of motion for a lumped mass shear building type system can be written in a matrix notation as the following:

$$M\ddot{U}(t) + C\dot{U}(t) + R(U) = -M\{1\}\ddot{u}_g(t) \tag{1}$$

where M and **C** are the mass and damping matrices respectively. The vector U(t) contains the relative displacement of different floors with respect to ground, while R(U) is the resistance vector. Also, $\ddot{u}_g(t)$ is the ground acceleration. An approximate deflection vector $\boldsymbol{\Theta}(x)$ is assumed to transform the above MDOF system into an equivalent Single-

Degree-of-Freedom (SDOF) system, where x is the roof's relative displacement with respect to its base. This approximate deflection vector corresponds to the deflected shape of the structure under the action of a statically applied lateral load with a specific load pattern $\xi(x)$. For convenience, the deflection pattern and the applied load pattern are normalized with respect to the roof displacement and the base shear force, respectively. As will be shown later, the variable load and deflection patterns are functions of the structure's nonlinear behavior that are being traced by the parameter x, the relative displacement of the roof with respect to the ground. Therefore, at any time t, the displacement of different floors of the structural model can be expressed as:

$$U(t) = \Theta(x)\ x(t) \tag{2}$$

and the resistance vector R is

$$R(x) = \xi(x)\ V(x) \tag{3}$$

where $V(x)$ is the base shear of the structure. In the proposed adaptive load pattern method(APO), $\xi(x)$ at any stage of pushover loading is calculated using the significant mode shapes, i.e., the first few mode shapes of the structure that include at least %90 of the total mass of the system. Therefore, for any roof displacement x, it is assumed that the load pattern $\xi(x)$ can be determined from the following equation:

$$\xi(x) = \sqrt{\sum_{i=1}^{n}\left[EMM_i(x)\ \Psi_i(x)\right]^2} \tag{4}$$

in which, n is the number of the required mode shapes, EMM_i (**x**) is the ith effective modal mass, and $\psi_i(x)$ is calculated from:

$$\Psi_i(x) = K(x)\ \varphi_i(x) \tag{5}$$

or

$$\Psi_i(x) = \omega_i^2(x)\ \ M\ \varphi_i(x) \tag{6}$$

where $K(x)$ is the stiffness matrix of the system. Also, $\varphi_i(x)$ is the ith mode shape that is normalized with respect to its largest element and $\omega_i(x)$ is the ith angular frequency. As it was mentioned before, the $K(x)$, $\varphi_i(x)$, $\omega_i(x)$, and $\psi_i(x)$ are all functions of **x,** and change as the plastic deformations occur in the structure. As it was already mentioned, the deflection pattern $\Theta(x)$ is the corresponding displacement of the structure under the applied load pattern $\xi(x)$. Substituting Eqs. (2) and (3) into Eq. (1) leads to the following approximate equation of motion:

$$M\,\boldsymbol{\Theta}(x)\,\ddot{x}(t) + C\,\boldsymbol{\Theta}(x)\,\dot{x}(t) + \boldsymbol{\xi}(x)\,V(x) = -M\{1\}\,\ddot{u}_g(t) \quad (7)$$

Assuming the damping matrix C to be an orthogonal matrix, and pre-multiplying the Eq. (7) by the transpose of the deflection pattern $\boldsymbol{\Theta}^T(x)$, will result in the following scalar equation:

$$M^*(x)\,\ddot{x}(t) + C^*(x)\,\dot{x}(t) + R^*(x) = -L^*(x)\,\ddot{u}_g(t) \quad (8)$$

in which

$$M^*(x) = \boldsymbol{\Theta}(x)^T\,M\,\boldsymbol{\Theta}(x) \quad (9)$$

$$C^*(x) = \boldsymbol{\Theta}(x)^T\,C\,\boldsymbol{\Theta}(x) \quad (10)$$

$$R^*(x) = \boldsymbol{\Theta}(x)^T\,\boldsymbol{\xi}(x)\,V(x) \quad (11)$$

and

$$L^*(x) = \boldsymbol{\Theta}(x)^T\,M\{1\}\,\ddot{u}_g \quad (12)$$

The quantities, $M^*(x)$, $C^*(x)$, $R^*(x)$ and $L^*(x)$ are defned as the equivalent SDOF's mass, damping coefficient, resistance function, and modal participation factor respectively. Both sides of Eq. (8) are divided by $M^*(x)$ and the ratio $C^*(x)/M^*(x)$ is substituted with the following identity:

$$C^*(x)/M^*(x) = \alpha + \beta k_0 \quad (13)$$

In the above Eq., k_0 is the initial value of the $K^*(x)/M^*(x)$ ratio. Also, α and β are the coefficients of the assumed Rayleigh damping matrix for the original structural system:

$$C = \alpha M + \beta K \quad (14)$$

Equation (8) can now be re-written as:

$$\ddot{x}(t) + (\alpha + \beta k_0)\,\dot{x}(t) + r(x) = -\iota\,\ddot{u}_g(t) \quad (15)$$

in which, $r(x) = R^*(x)/M^*(x)$ is the stiffness of the system and is approximated by an equivalent bilinear curve [6]. The constant parameter ι in Eq. (15) is considered to be the average value of $L^*(x)/M^*(x)$ over the domain {x=0, x=x_m}, where x_m is an initial guess of the predicted inelastic structural response(roof displacement) at the design level earthquake.

It can be approximated as 1% of the height of the structure[7].

It should be noticed that the definition of the resistance function of the equivalent SDOF model presented herein, does not correspond to either the base shear or the base overturning moment, as it is in other available models [8]. The resistance function, $R^*(x)$, used here is in fact the multiplication of the deflection pattern and the applied load vector. This definition is preferred to the base shear force and base overturning moment, since it involves both deflection distribution and load distribution of the system. In dynamic response, the inertial forces and the resistance of the structure depend on its deflected shape. However, this dependency is not considered in defining the resistance function in terms of base shear or base overturning moment [7].

On the other hand, a simple equivalent static procedure to calculate the target displacement is provided by FEMA356 in the following form [1]:

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e{}^2}{4\pi^2}.g \tag{16}$$

where δ_t is the target displacement, S_a is the spectral acceleration, C_0 is the modification factor of the equivalent mode participation factor. Also, C2 is equal to 1.0 for a special moment resisting building and C_3 is equal to 1.0 where $P-\Delta$ effects are not considered. The parameter C_1 is equal to

$$C_1 = 1.0 \quad \text{for} \quad T_e \geq T_S\,,$$

$$C_1 = \left[1.0 + (R-1)\frac{T_0}{T_e}\right] / R \quad \text{for} \ T_e < T_S \tag{17}$$

and

$$R = \frac{S_a}{V_y / W}.C_m \tag{18}$$

In these equations, T_S is the characteristic period of design spectrum curve, where constant acceleration is changed into constant velocity. R is the ratio of the elastic capacity to the yield capacity of the structure and C_m is the effective mass factor whose value depends on the number of stories and the fundamental period of the structure. The V_y and W parameters are the yield capacity and the seismic weight of the system respectively. The T_e in Eq. (16) can be calculated from $T_e = T_i . \sqrt{(k_i / k_e)}$.

3. NUMERICAL EXAMPLES

Three 10-, 15- and 20-story special moment resisting steel frames are considered to evaluate the performance of the proposed adaptive pushover analysis (APO) method. The results for target displacements, interstory drifts and plastic hinge rotations are compared with those of the regular pushover analysis and the dynamic time history analysis. These 2-D structural models are three bays wide, with all floors 3.6(m) high. The side bays are 6.0(m) wide, while the width of the middle bay is 7.5(m), The columns are rigidly attached to the base. The gravity loads include a dead load of 400 kg/m^2 and a live load of 100 kg/m^2 for the roof and a dead load of 500 kg/m^2 and live load of 250 kg/m^2 for all floors. Exterior wall panels are assumed to have a weight of 125 kg/m^2 and the tributary width of every frame is assumed to be 6.0m. The frames are designed according to the 1997 UBC [9] for a structure located on stiff soil (soil type S_B) in seismic zone 4, with R factor equal to 8.5. The steel members are designed according to LRFD97 [10]. The structural frames are analyzed statically and dynamically using the DRAIN-2DX nonlinear analysis program. All members meet the required compactness ratio for local buckling and the joint and member requirements for special moment resisting frames. Yielding is assumed to occur at concentrated plastic hinges at the end points of the elements. The axial force-bending moment interaction is considered in the columns according to FEMA365. Since, the lateral stiffness of the bare frames turned out to be very low, the moment of inertia of all members are increased uniformly, so that their fundamental periods to be close to those provided by the UBC97 ($T=.085H^{3/4}$). The structural frames are analyzed statically and dynamically using the DRAIN-2DX nonlinear analysis program [11]. Also, P-delta effects are not included in this study. The %5 damping ratios are considered for the first two modes. Moment-rotation relationships for the elements assume 3% strain hardening.

Five strong ground motion records, Northridge Castaic, Loma Prieta Corralitos, Loma Prieta Gilroy#1, Northridge Los Angeles, and Northridge Pacoima are considered for performing the dynamic time history analyses. These ground motions which are recorded on stiff soil are represented in Table 1. The average acceleration response spectrum of these records is close to the UBC97 ground motion spectrum for the soil type S_B. All the records are scaled to a PGA value of 0.4g. The pushover load patterns used in this study include uniform load pattern (UFM), the FEMA356 modal load pattern (MOD) and the already described adaptive load pattern (APO). The modal load pattern is the same as FEMA356 equivalent static load pattern if the first mode’s effective modal mass is more than 75% of the total system’s mass. Otherwise, it will be equal to the load pattern defined by the SRSS combination of story shears forces, using enough number of modes to include 90% of the total mass of the system.

The tri-linear approximations of the pushover curve are used in the dynamic procedure. In constructing the tri-linear curve, the first line should intersects the main curve at 0.6Fy as it is shown in Figure 1. The second line is tangent to the pushover curve at the target displacement. The third line is placed such that the area under the main curve and the approximate tri-linear curve to be the same[12].

4. VERIFICATION STUDY

As it was already mentioned, analyses are performed for three 10-, 15- and 20-story special moment resisting steel frames. The results of the modal analyses of these structural models are shown in Table 2. Tables 3 to 5, compare the target displacements (maximum roof displacements) obtained from the nonlinear dynamic time history analyses and static and dynamic pushover analyses (Eq. 15) for the structural models. The dynamic pushover analysis is performed on the equivalent SDOF systems of the structural models using the tri-linear approximation of their pushover curves.

Table 1. The characteristics of the earthquake records

No.	Station, Earthquake	Magnitude (M_L)	PGA (cm/sec^2)	PGV (cm/sec)	PGD (cm)
1	CASTAIC - O.R.R., NORTHRIDGE, 1994 (N.C)	6.6	504.22	52.63	2.41
2	CORRALITOS, LOMA PRIETA, 1989 (L.C)	7.0	617.70	55.20	10.88
3	GILROY #1, S.Y.S., LOMA PRIETA, 1989 (L.G)	7.0	426.61	31.91	6.38
4	LOS ANGELES- T&H, NORTHRIDGE, 1994(N.L)	6.6	180.11	20.02	2.74
5	PACOIMA-K.C., NORTHRIDGE, 1994 (N.P)	6.6	424.21	50.88	7.21

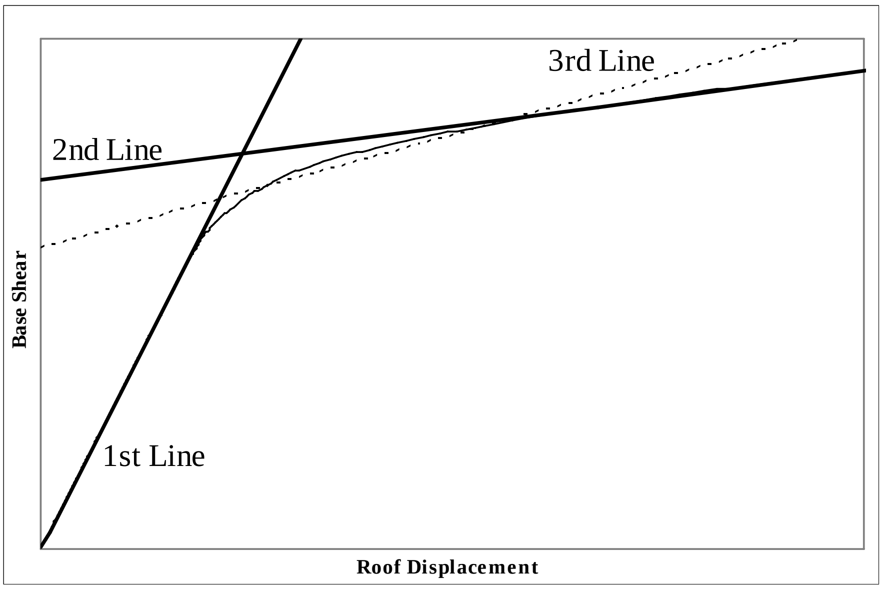

Figure 1. Tri-linear best fit to the pushover curves for dynamic analysis

Table 2. The modal properties of the structural models

Mode Number	10 Story Building		15 Story Building		20 Story Building	
	Period (sec)	E.M.M. (%)	Period (sec)	E.M.M. (%)	Period (sec)	E.M.M. (%)
1	1.471	77.581	1.917	74.807	2.339	73.773
2	0.543	10.636	0.702	11.561	0.861	12.021
3	0.341	3.877	0.420	4.239	0.512	4.154
4	0.248	2.339	0.302	2.553	0.362	2.446
5	0.193	1.617	0.237	1.335	0.276	1.430
6	0.154	1.178	0.194	1.054	0.221	1.091
7	0.126	0.922	0.159	0.975	0.182	0.790
8	0.108	0.628	0.137	0.521	0.156	0.560
9	0.093	0.580	0.119	0.658	0.139	0.462
10	0.077	0.643	0.106	0.491	0.121	0.468

E.M.M: Effective Modal Mass

The first five rows of Tables 3 to 5 are the results of dynamic analyses using five records. The 6th row is the average of the results, and the 7th row is the error of the average values with respect to the results of the exact time history analysis (THA). By the static FEMA356 procedure, it is meant to determine the modified target displacement from Eq. 16, using the effective period T_e obtained from the initial part of pushover diagram. The results indicate that in all load patterns, the dynamic pushover analysis with a tri-linear approximation have a better performance when compared to the static procedure, especially in 15 and 20 stories buildings. Modal load pattern (MOD) and uniform load pattern (UFM) seem to be complimentary and each may have a better performance in approximating the real response of the structures in some cases. But, the adaptive load pattern (APO) introduced here has more success in estimating different response parameters of all structural models considered here.

Table 3. The target displacements obtained by different methods for the 10-story structure

Earthquake Record	THA	UFM		MOD		APO
		FEMA STATIC	3- LINE DYNAMIC	FEMA STATIC	3- LINE DYNAMIC	
CASTAIC	0.120		0.142		0.146	0.124
CORRALITOS	0.175		0.171		0.162	0.131
GILROY #1	0.116		0.120		0.120	0.105
LOS ANGELES	0.146		0.180		0.201	0.156
PACOIMA-K.C	0.201		0.231		0.201	0.212
Average	0.1515	0.162	0.169	0.178	0.166	0.145
Error(%)		7.10	11.38	17.48	9.65	4.05

Table 4. Target displacements obtained by different methods for the 15-story tructure

EQ Earthquake Record	THA	UFM		MOD		APO
		FEMA STATIC	3- LINE DYNAMIC	FEMA STATIC	3- LINE DYNAMIC	
CASTAIC	0.260		0.219		0.265	0.254
CORRALITOS	0.187		0.165		0.157	0.161
GILROY #1	0.139		0.144		0.152	0.154
LOS ANGELES	0.230		0.291		0.292	0.273
PACOIMA-K.C	0.191		0.170		0.173	0.170
Avearge	0.201	0.225	0.198	0.246	0.208	0.2025
Error(%)		11.51	1.78	22.08	3.34	0.54

Table 5. Target displacements obtained by different methods for the 20-story structure

Earthquake Record	THA	UFM		MOD		APO
		FEMA STATIC	3- LINE DYNAMIC	FEMA STATIC	3- LINE DYNAMIC	
CASTAIC	0.236		0.215		0.247	0.193
CORRALITOS	0.203		0.120		0.152	0.143
GILROY #1	0.153		0.127		0.144	0.131
LOS ANGELES	0.227		0.242		0.249	0.296
PACOIMA-K.C	0.261		0.259		0.278	0.298
Average	0.216	0.289	0.193	0.314	0.214	0.212
Error(%)		33.80	10.89	45.16	1.00	1.75

Figures 2 to 4 compare the story drifts obtained from the uniform load pattern (UFM), FEMA356 modal load pattern (MOD), and the proposed adaptive load pattern (APO) pushover analyses together with the benchmark solutions based on nonlinear time history analyses (THA). The drifts are measured at the target displacement. Comparison of the results shows a better performance for APO method with respect to the other two methods. However, UFM provides less accurate results when compared to MOD.

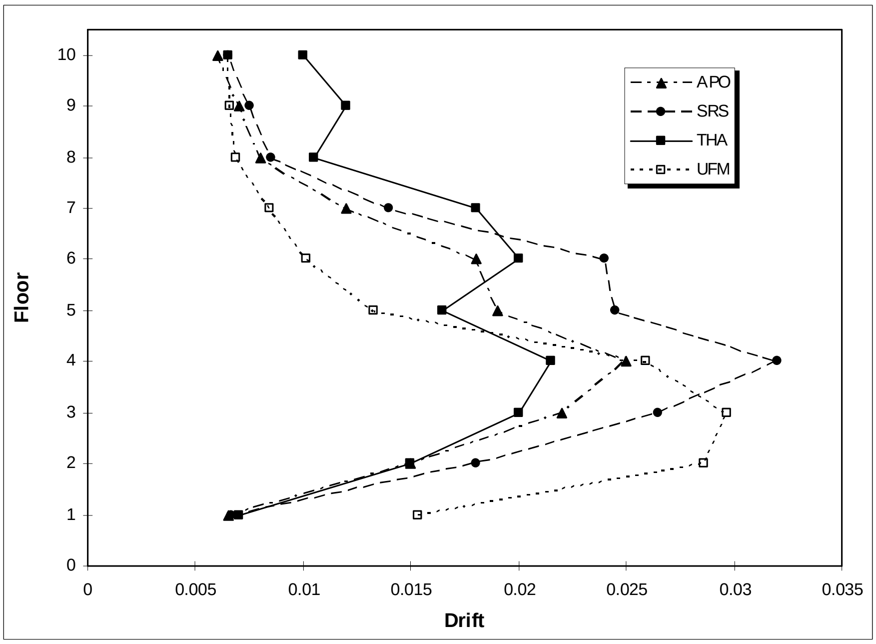

Figure 2. Story drifts obtained by different methods for the 10 story structural model

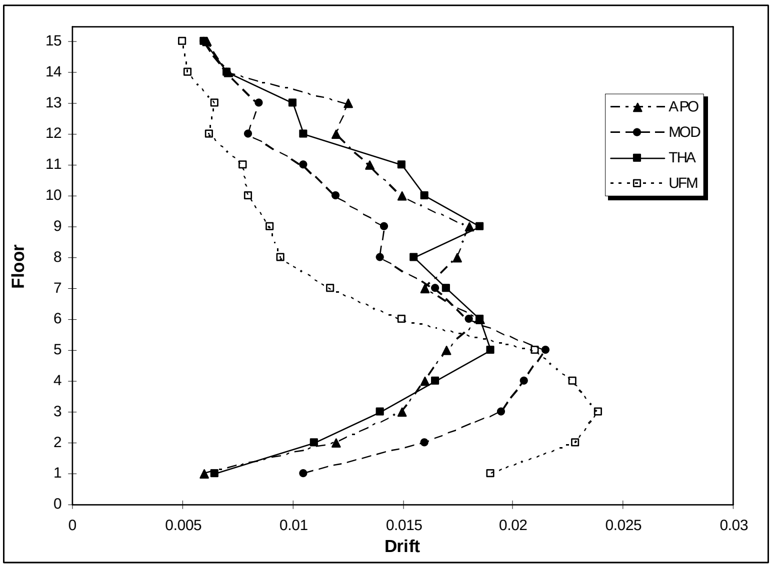

Figure 3. Story drifts obtained by different methods for the 15-story structural model

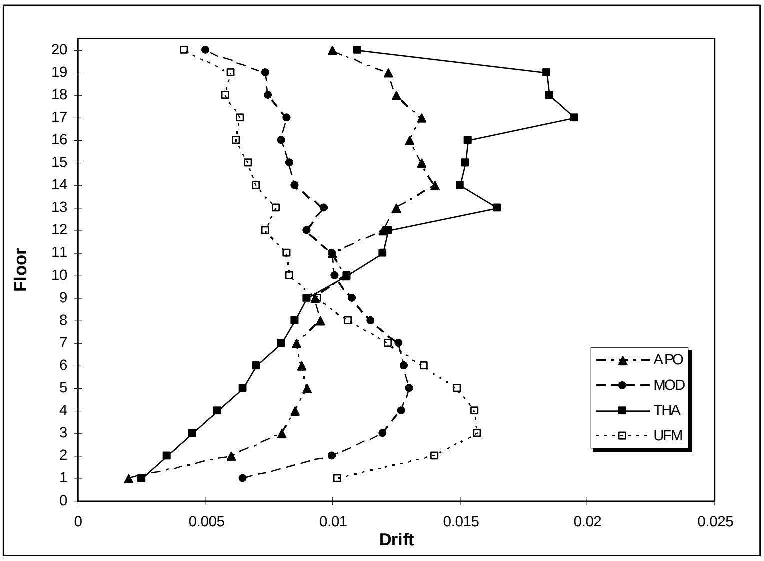

Figure 4. Story drifts obtained by different methods for the 20-story structural model

Figures 5 to 10 compare the plastic hinges rotations of the beams and columns obtained from pushover analysis with different load patterns with those obtained from nonlinear time history analysis for 10-, 15- and 20- story buildings. The rotations are also measured at the target displacements. Again, the APO method has predicted the results more accurately than MOD and UFM, while the UFM method led to the worst output, especially for taller building. In general, the plastic hinge rotations are very sensitive to the presence of the higher modes. This can cause the output of the pushover analyses to be non-reliable even when APO load pattern is used. Therefore, as the effect of higher modes in the structural models increase, one could expect that the accuracy of the pushover results for the plastic hinge rotations to be decreased.

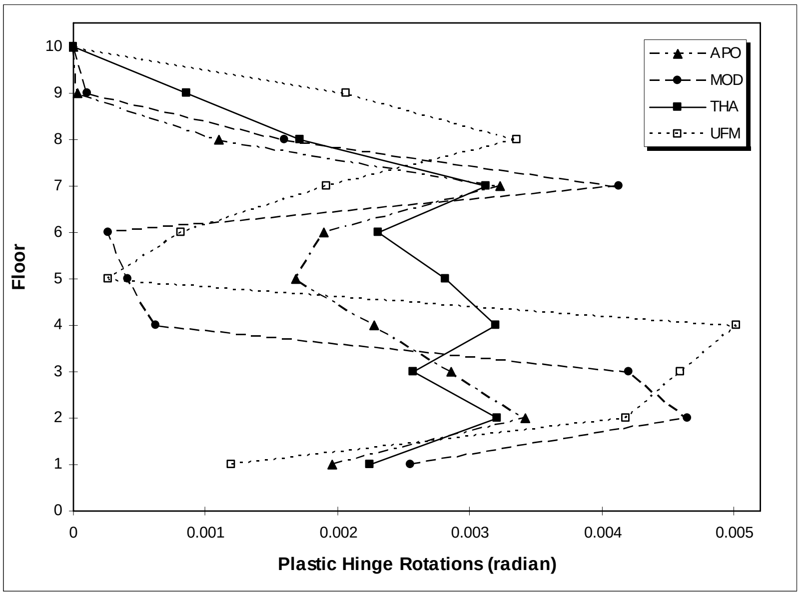

Figure 5. Maximum beam plastic hinge rotations for the 10-story structural model

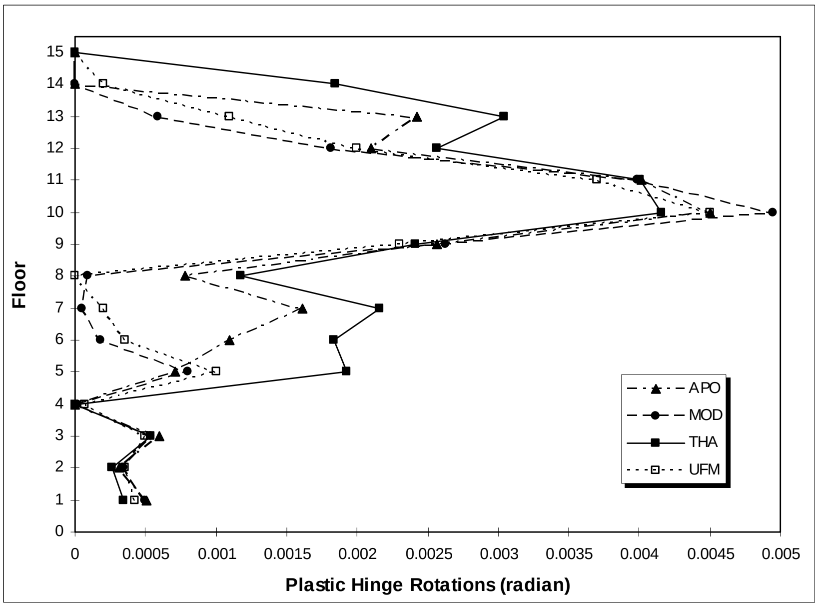

Figure 6. Maximum beam plastic hinge rotations for the 15-story structural model

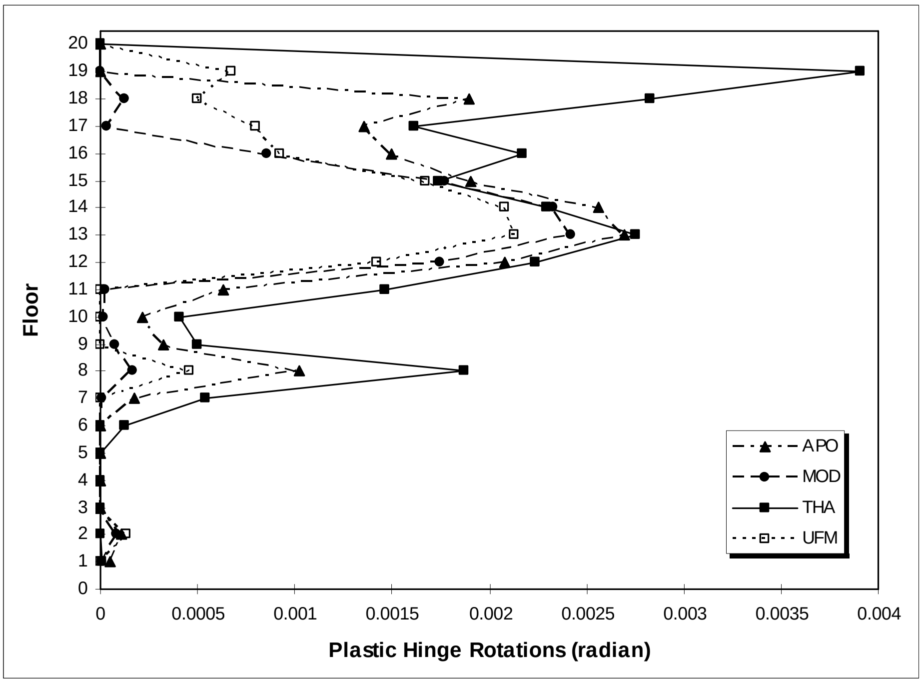

Figure 7. Maximum beam plastic hinge rotations for the 20-story structural model

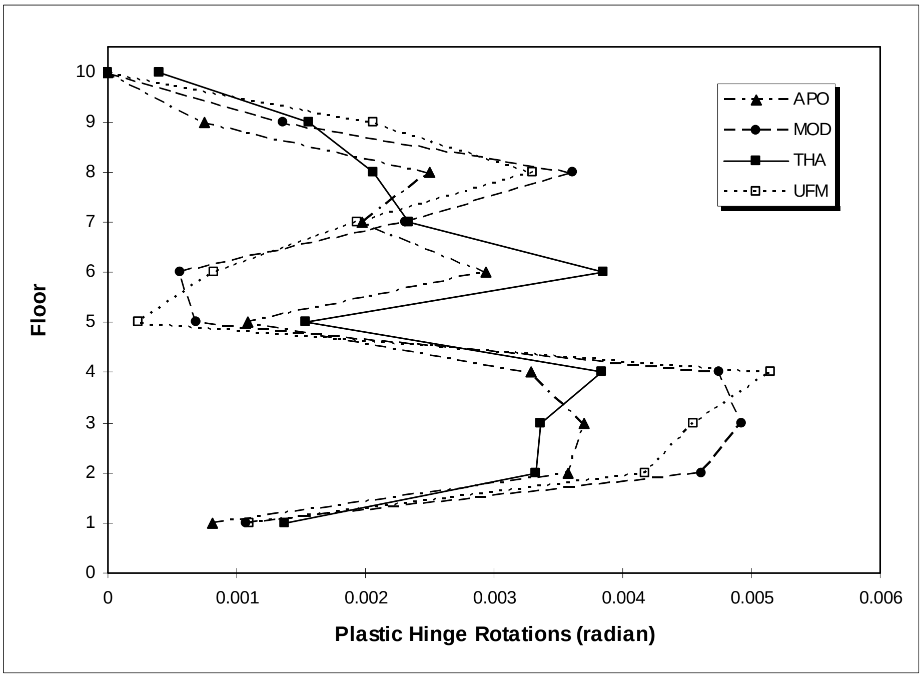

Figure 8. Maximum column plastic hinge rotations for the 10-story structure

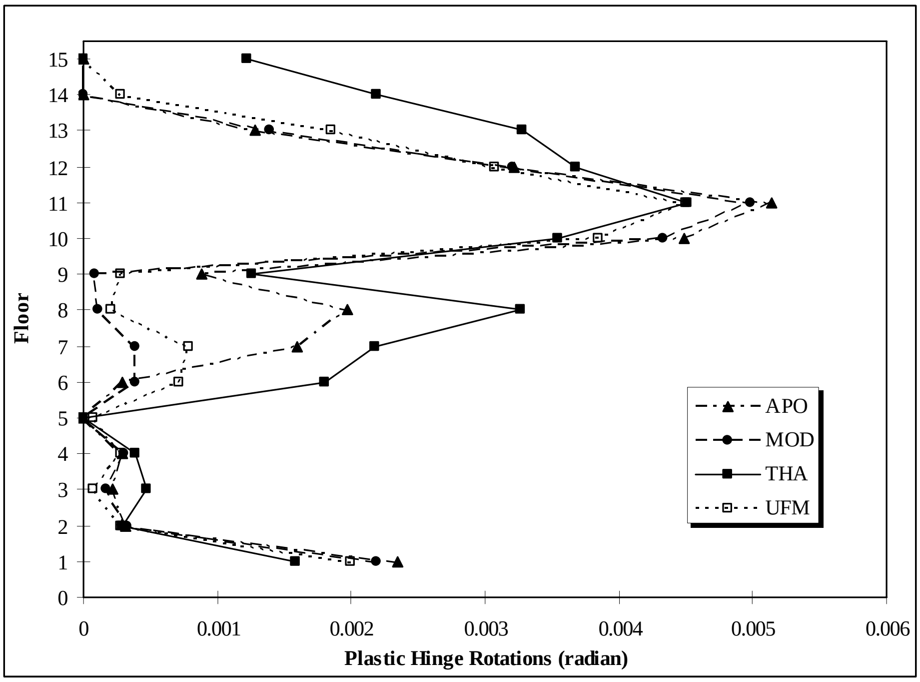

Figure 9. Maximum column plastic hinge rotations for the 15-story structure

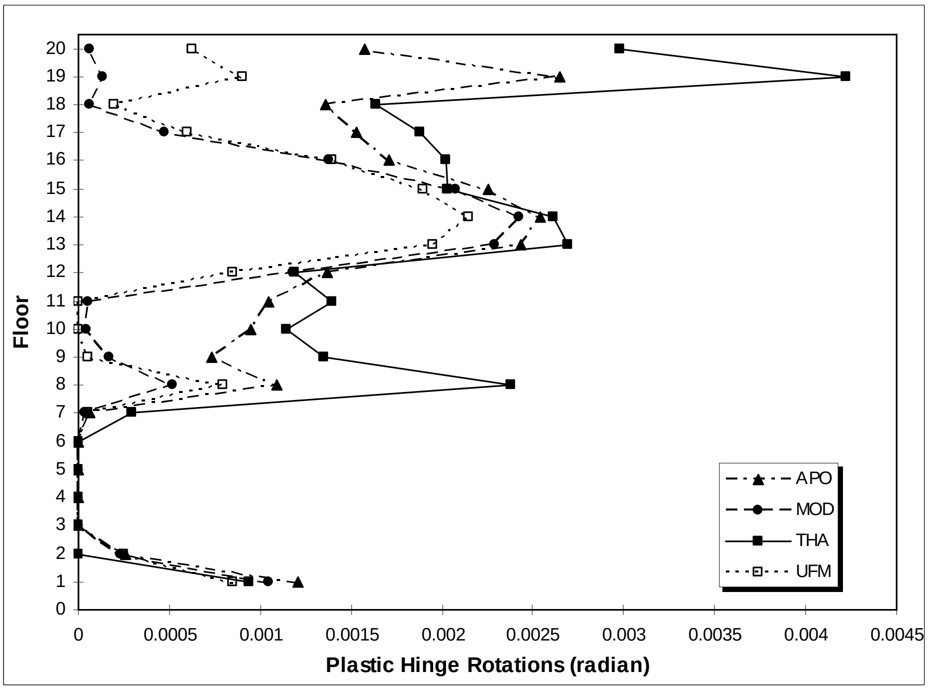

Figure 10. Maximum column plastic hinge rotations for the 20-story structure

5. CONCLUSIONS

An adaptive pushover method is introduced to improve the accuracy of the currently used pushover analysis in predicting the seismic-induced demands of the structures. The adaptive load pattern introduced here has more success in accurately estimating different response parameters of the structural models considered in this work. Comparison of the common pushover analyses, adaptive pushover analyses and time-history analyses performed for a number of multiple-bay, short and high-rise steel structures, demonstrates the efficiency of the adaptive load pattern method. The obtained results indicate that the proposed method provides more accurate results for the story drifts and the plastic hinge rotations in high-rise building structures compared to the classic pushover approach. However, even this improvement to the pushover analysis cannot replace the time history analysis for the final analysis of a high-rise structure.

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