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ACTIVE VIBRATION CONTROL OF STRUCTURES AGAINST EARTHQUAKES USING MODERN CONTROL THEORY

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ABSRACT

Structural control against earthquakes is becoming increasingly important. The linear-quadratic optimal control algorithm is proposed here to design active control system for buildings against earthquake excitations. Full-state feedback system has been adopted. Active Tuned Mass Damper is used as the control mechanism. The efficiency of the designed system has been verified against El-Centro earthquake. Active control gives 35% more reduction in vibration of the structure than passive control. Mass of the damper has appreciable effects on response parameters than its stiffness. A flexible damper proves to be more effective, but at the cost of actuating forces required. Performance of system is found to be optimum, when mass-damper is tuned to fundamental frequency of the structure. Stability of the structure is also enhanced by Active Control system. A SDOF building is presented to illustrate the study.

Keywords: active control, active tuned mass damper, structural dynamics, quadratic optimal control theory

1. INTRODUCTION

Active Control is the most advanced technique to perform vibration control. It makes the passive system automatic in the manner that it supplies the forces enough to control the response of a structure depending upon the extent of external forces and state of the structure during vibrations. The direction and magnitude of the controlling forces to be applied to the structure are estimated by an algorithm. One such algorithm has been developed and studied here with application example.

Tuned mass dampers (TMDs), first demonstrated by McNamara [1] in 1977 and then by Wiesner [2] (1979), are widely used for vibration control of civil engineering structures.

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TMDs are generally prone to tuning error, which Active Tuned Mass Dampers (ATMD) [3, 4] have been found to withstand. Various control algorithms have been developed [5, 6, 7] to be adopted for active control system in the past. Few other theories were presented by Soong and Pitarresu [8] and Hammerstorm [9] and thereby applied to vibration absorbers by Watanabe and Yoshida [10].

A procedure for design of ATMD for vibration control of tall buildings subjected to wind loads was given by Ankireddi and Yang [11] in 1996. Brown *et al* [12] presented a simple algorithm for multiobjective linear quadratic Gaussian control using Pareto optimal trade-off curves. A multistorey building was considered for illustration achieving control by active tendon control. Yang *et al* [13] investigated the feasibility of instantaneous active control theory for controlling the vibration of civil engineering structures. The results of this algorithm are compared with those obtained by the algorithm presented here.

The active control to tall buildings has been applied recently. ATMDs have been successfully designed and installed in full-scale structures [7]. Using active control and a TMD with mass 2% of the building, a reduction of 40% in the building sway has been achieved in the Citicorp Center of New York City [17]. The ATMD system has been installed in the 11-storey Sendagaya INTES building in Tokyo in 1991 and in the 160m tall, 34-storey Hankyu building located in Osaka, Japan in 1992 [15]. In the later, heliport at the rooftop is utilized as the moving mass of the AMD, which weighs 480 tons (about 3.5% of the mass of the tower).

In the present paper, the active control system has been designed for a building using the proposed modern quadratic optimal theory and the optimum parameters of active tuned mass damper (ATMD) are decided based upon the performance of the building against unit-step excitation and El-Centro earthquake. The theory is also applied to an eight-storey building and efficiency of the system has been compared with that proposed by Yang [13]. The structure is modeled for simulating its response against ground acceleration using MATLAB 5.3 [16].

2. THE ACTIVE CONTROL

Basic configuration of an active structural control system is shown in Figure 1 [17]. It primarily consists of (i) Sensors located in the structure to measure either external excitations and structural response variables - displacement, velocity and acceleration, (ii) Devices to process the measured information and compute necessary forces needed based on a given control algorithm and (iii) Actuators, which are usually powered by external energy sources, to produce and apply the required forces in desired direction.

2.1 Structural Dynamics and Control Theory

A building modeled is n-degrees of freedom system as shown in Figure 2. The equation of dynamic motion of the building can be written as [18]:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{D}\boldsymbol{f}_{\mathbf{act}}(t) + \mathbf{E}\boldsymbol{f}_{\mathbf{e}}(t) \tag{1}$$

Here, **M**, **C** and **K** are mass, damping and stiffness matrices of the structure. Matrix **D** identifies the points of application of controlling forces. **E** has all its elements zeros except the first one, which is unity for earthquake excitations to act at bottom level only. $\boldsymbol{f_{act}}$ is the control force exerted by the actuator and $\boldsymbol{f_e}$ is the earthquake force acting at ground level.

2.2 Modification of System Matrices

After active control is applied, the property matrices of the structure are modified. The controlling force is defined as a function of state of system and external excitations. Thus, $\boldsymbol{f_{act}}$ can be expressed as [19],

$$\boldsymbol{f}_{\mathbf{act}} = \mathbf{C}_1\dot{\mathbf{x}}(\mathrm{t}) + \mathbf{K}_1\mathbf{x} + \mathbf{E}_1\boldsymbol{f}_{\mathbf{e}} \tag{2}$$

From the equations (1) and (2),

$$\mathbf{M}\ddot{\mathbf{x}}(\mathrm{t}) + (\mathbf{C} - \mathbf{D}\mathbf{C}_1)\dot{\mathbf{x}}(\mathrm{t}) + (\mathbf{K} - \mathbf{D}\mathbf{K}_1)\mathbf{x} = (\mathbf{E} + \mathbf{D}\mathbf{E}_1)\boldsymbol{f}_{\mathbf{e}} \tag{3.1}$$

which can be written as,

$$\mathbf{M}\ddot{\mathbf{x}}(\mathrm{t}) + \mathbf{C}_a\dot{\mathbf{x}}(\mathrm{t}) + \mathbf{K}_a\mathbf{x} = \mathbf{E}_a\boldsymbol{f}_{\mathbf{e}} \tag{3.2}$$

where, **C** and **K** matrices have been modified to $\mathbf{C_a}$ and $\mathbf{K_a}$ respectively, revealing that the active control modifies the structural parameters to obtain a favorable and expected response.

2.3 State Space Modeling

The original differential equations are reduced to lower order equations in matrix form by state space modeling, as shown below:

The matrix z of order 2(n+1) ×1 may be written as,

$$z = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} \text{ and } \dot{z} = \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} \tag{4}$$

Combining the equations (1) and (4),

$$\dot{z} = Az + Bf_{act} + Hf_e \tag{5}$$

Where, $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{H}$ are the system matrix, control vector and location matrix respectively.

The equation (5) yields actively controlled response, if modified matrices $\mathbf{C_a}$, $\mathbf{K_a}$ and $\mathbf{E_a}$ are used to obtain $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{H}$. The response of the building is expressed as:

$$x = C_R z + D_R f_e \tag{6}$$

$\boldsymbol{C_R}$ and $\boldsymbol{D_R}$ are the output vector of order 1×2(n+1) and the direct transition matrix of same order as $\boldsymbol{f_e}$(t). Equation (5) is dynamical state equation and Eq. (6) is output equation. The storey drift of the building being the desired response parameter, $\boldsymbol{C_R}$ has to have its *n*th element unity and $\boldsymbol{D_R}$ zero since earthquake has no direct application to top storey of the building.

2.4 Modern Quadratic Optimal Control Theory

In linear quadratic optimal theory [20], state of the system, z is determined at any instant by means of sensors located in the structure. The objective is to determine the controlling force $\boldsymbol{f_{act}}$ which has been defined as a linear function of the state of the system z. in Eq. (2).

For a closed-loop mode of active control, $\boldsymbol{f_{act}}$ is expressed as:

$$\boldsymbol{f_{act}}(\mathrm{t}) = \boldsymbol{G}^{\mathrm{T}} z(\mathrm{t}) \tag{7}$$

2.5 Derivation of the Gain Matrix, G^T

$\boldsymbol{G}^{\mathrm{T}}$ is obtained by minimizing the performance index $\boldsymbol{J}$ (Liapunov's cost-function) [21]:

$$\boldsymbol{J} = \int_0^{\mathrm{T_i}} \phi(z, \boldsymbol{f_{act}}, \mathrm{t})\mathrm{dt} \tag{8}$$

(0, $\mathrm{T_i}$) is an appropriate time interval over which, response of the actively controlled structure is to be found out. The concept behind this approach is that of minimization of sum of potential energy and kinetic energy of the system as a whole. In quadratic optimal control theory [21], ϕ was proposed to be:

$$\phi(z, \boldsymbol{f_{act}}, \mathrm{t}) = \frac{1}{2}(z^{\mathrm{T}}\boldsymbol{Q}z + \boldsymbol{f_{act}}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{f_{act}}) \tag{9}$$

The matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ are weighting matrices, whose magnitudes are assigned according to the relative importance, attached to the state variables and to the control forces in the minimization procedure. Assigning large values to $\boldsymbol{Q}$ indicates that response reduction is given priority over the control forces required. The opposite is true when the elements of $\boldsymbol{R}$ are large in comparison with those of $\boldsymbol{Q}$. Hence, by varying the relative magnitudes of $\boldsymbol{Q}$ and $\boldsymbol{R}$, one can synthesize the controllers to achieve a proper trade off between control effectiveness and control energy consumption.

For $\mathrm{T_i} \to \infty$, it follows from modern control theory that the optimal linear feedback control law for $\boldsymbol{f_{act}}$(t) is given by,

$$\boldsymbol{f_{act}}(\mathrm{t}) = -\frac{1}{2}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{R}^{-1}z(\mathrm{t}) \tag{10}$$

z(t) is known through measurement using sensors, $\boldsymbol{B}$ is already described; $\boldsymbol{P}$ is an unknown real symmetric matrix and is to be determined from the equation,

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}=-\boldsymbol{Q} \tag{11}$$

Further, $\boldsymbol{P}$ has been also found to satisfy the Matrix-Ricatti equation,

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+2\boldsymbol{Q}-\frac{1}{2}\boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}=\boldsymbol{O} \tag{12}$$

Where, $\boldsymbol{O}$ is the null matrix. In the present paper, equations (11) and (12) have been solved to obtain $\boldsymbol{P}$.

From the Eqs. (7) and (10) one gets,

$$\boldsymbol{G}^{\mathrm{T}}=-\frac{1}{2}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{R}^{-1} \tag{13}$$

The feedback gain matrix $\boldsymbol{G}^{\mathrm{T}}$ is computed using $\boldsymbol{P}$ and known $\boldsymbol{B}$ and $\boldsymbol{R}$ matrices. Then, the control force $\boldsymbol{f}_{act}$ is obtained, which is optimal in the sense that it is the best function of z(t) for minimizing the performance index function defined by equation (8).

2.6 Evaluation of Response of the Structure

Having obtained $\boldsymbol{G}^{\mathrm{T}}$ of the controller, the equations (5) and (7) are solved to obtain reduced response. Equation (5) can be modified to

$$\dot{z}(\mathrm{t})=(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{G}^{\mathrm{T}})z(\mathrm{t})+\boldsymbol{H}\boldsymbol{f}_{e} \tag{14}$$

Response of the structure is calculated from equations (6) and (14) using MATLAB 5.3 software [19].

2.7 Stability of the Designed Control System

In the quadratic optimal control theory, Liapunov's cost function is intended to get minimized as time approaches infinity which means, the control system designed by optimal control theory should achieve a steady state after certain time and be stable. In the present paper, the stability of the designed control system is verified by Bode's method [21].

2.8 Application Example

To illustrate the application of modern quadratic optimal control theory, a single-storey building is considered for active control design, Figure 3. Three response quantities of interest- percentage reduction in peak displacement of the top floor, settling time (T_s) of the building and magnitude of controlling forces ($\boldsymbol{f}_{\mathbf{act}}$) to be generated by the actuator are observed for a wide range of certain ATMD parameters. Stability of the controlled system is also verified by Bode's method.

The ATMD parameters considered for studying the variation in response are:

(i) Mass ratio, $\boldsymbol{\alpha}$ (Ratio of mass of ATMD to that of the building)

(ii) ATMD's stiffness, $\boldsymbol{k}_d$

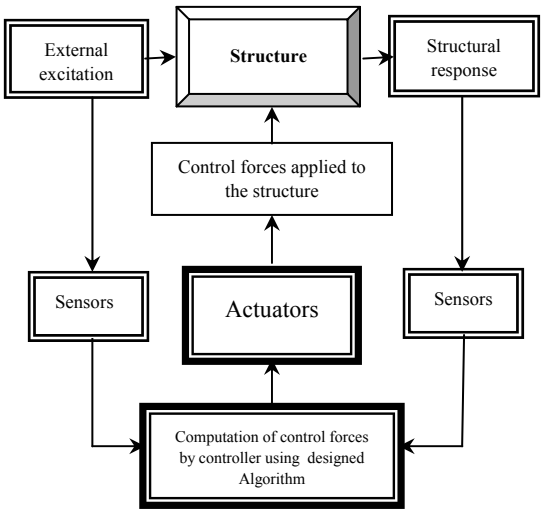

Figure 1. Schematic diagram of active control (Soon [15])

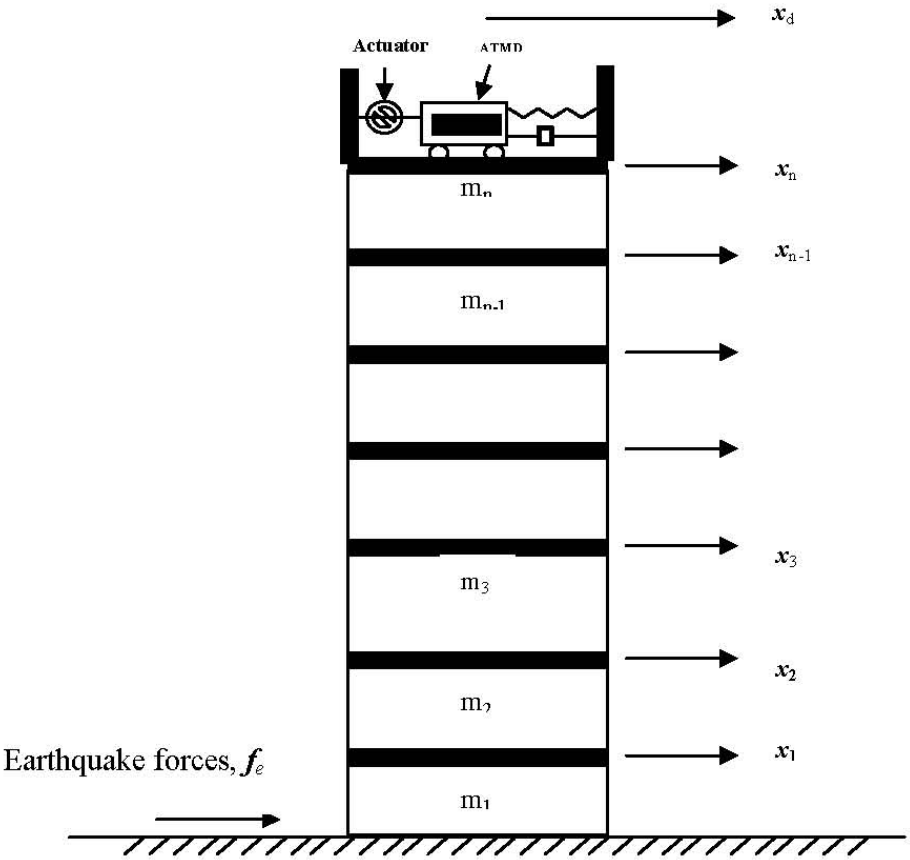

Figure 2. Shear Frame model of an n-storey building with ATMD

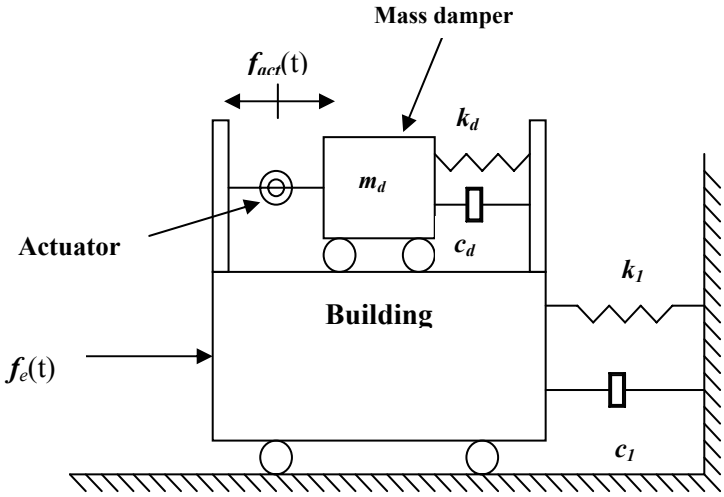

Figure 3. Idealized system for the single-storey building with ATMD on the top

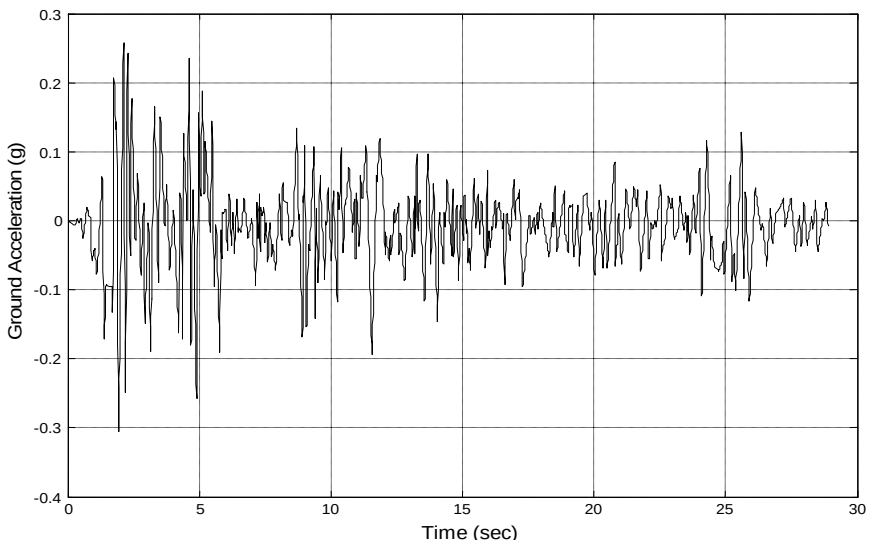

Figure 4. Accelerogram-ns component of El-Centro earthquake (May 18, 1940) (PGA=0.33g)

The optimum combination of the ATMD parameters for both the buildings has been decided separately from the response data collected. The effectiveness of the designed system has further been visualized by plotting the response of the buildings against El-Centro earthquake.

The structural parameters of this building are mass 100 tons, lateral stiffness 100kN/m and damping 100 t/sec. Mass of the ATMD has been varied from 0.1% to 5% of mass of the building and its stiffness from 6 to 30 kN/m, keeping its damping coefficient constant and equal to 1.0 t/sec. The weighting matrices are selected to be [19]:

$$\boldsymbol{Q} = \begin{bmatrix} 900 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } \boldsymbol{R} = [0.01].$$

3. ANALYSIS OF RESULTS

A number of plots showing variation in response quantities of the structures are shown here, followed by a discussion of the results.

3.1 Effect of Mass Ratio (α) on Response Quantities

The response quantities i.e. percentage response reduction (*RR*), settling time (T_s) and magnitude of controlling forces ($\boldsymbol{f_{act}}$) are found to vary appreciably with $\boldsymbol{\alpha}$ as described below:

(a) Effect of mass ratio, *α* on *RR* (Figures 5 and 6)

As the mass of damper is increased, an appreciable increase in *RR* is observed, but after certain value of $\boldsymbol{\alpha}$, reduction in response ceases. *RR* is seen to become almost invariable at higher masses. This can be attributed to the inertia effect of heavy mass dampers on the response of the building.

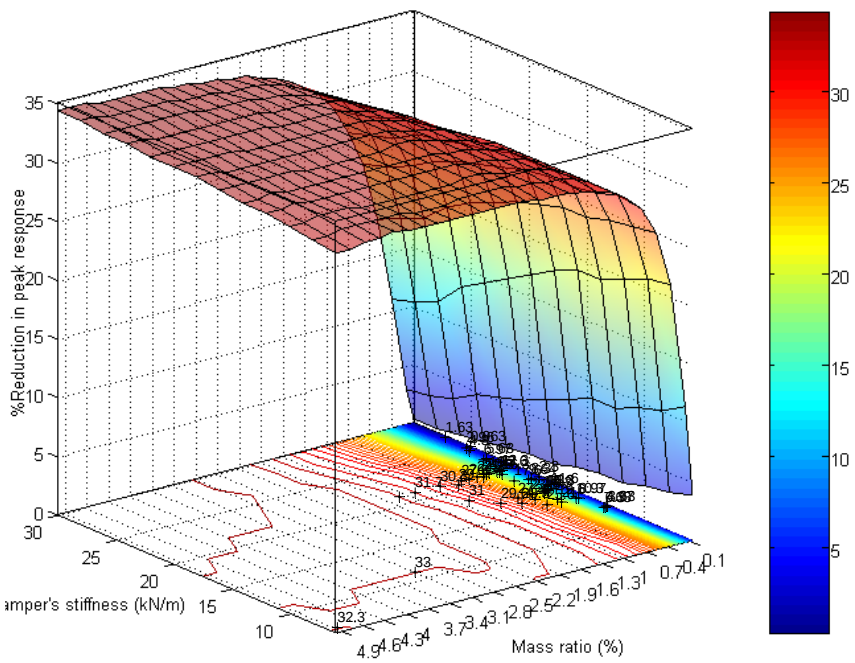

Figure 5. 3D surface plot showing effect of damper's mass and its stiffness on response reduction (single-storey building)

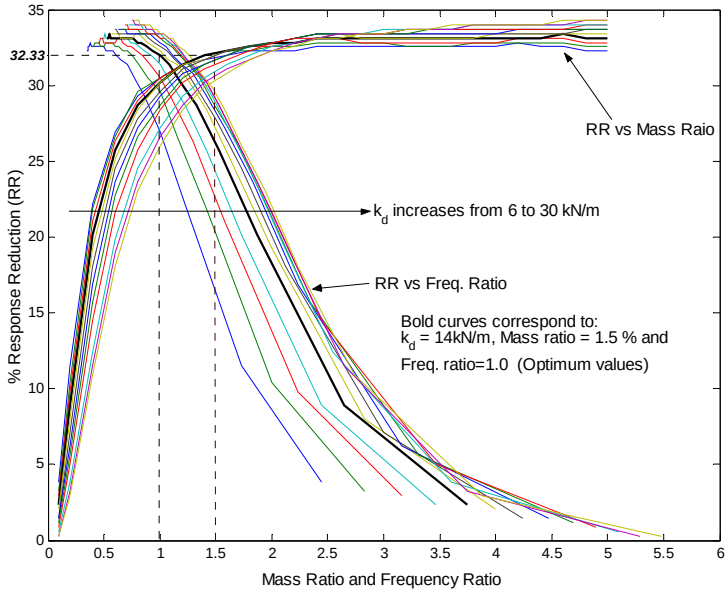

Figure 6. Variation in response reduction of SDOF building with Frequency Ratio (β) and mass ratio ($\boldsymbol{\alpha}$) for different values of $\boldsymbol{k_d}$

(b) Effect of α on f_{act} (Figure 7)

Here a decrease in amount of controlling forces, $\boldsymbol{f_{act}}$ is observed with increase in mass ratio, which is abrupt for low values of $\boldsymbol{\alpha}$, but very little for $\boldsymbol{\alpha}$ beyond 1.8%. As the cost of ATMD depends largely upon the amount of forces to be generated by the actuator, it is feasible to choose the case for which $\boldsymbol{f_{act}}$ required is less.

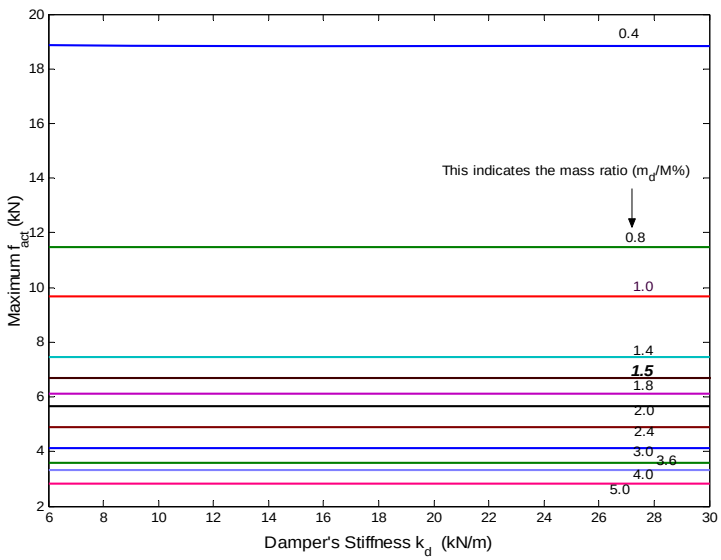

Figure 7. Plot between $\boldsymbol{f_{act}}$ and $\boldsymbol{k_d}$ for different values of mass ratio ($\boldsymbol{\alpha}$)

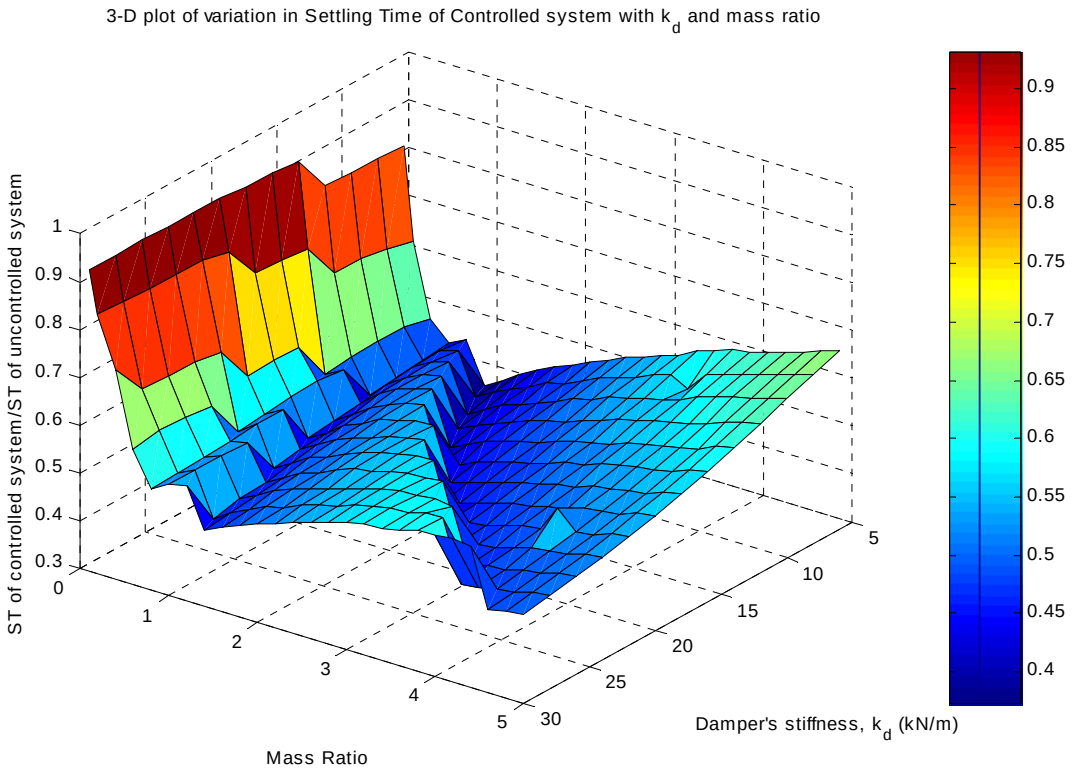

Figure 8. 3D plot of variation in settling time of controlled system with $\boldsymbol{k_d}$ and mass ratio (α)

(c) Effect of α on Settling time, T_s of the Controlled structure (Figure 8)

It is desired that the structure should return to its mean position in a little time, after it is disturbed. Figure 8 has been plotted to study the effect of mass ratio on settling time. It is evident that as the mass of damper is increased, settling time of actively controlled system approaches settling time of uncontrolled structure. Thus the settling time puts restriction over adoption of larger mass for more reduction, as more settling time means exposure of the structure to reversal of stresses for a longer duration.

3.2 Effect of ATMD Stiffness, k_d on Response Quantities

The stiffness of damper, $\boldsymbol{k_d}$ has been varied over a wide range to study its effect on response quantities, which are discussed below.

(a) Effect of k_d on *RR* (Figures 5 and 6)

It is clear from Figure 5 that stiffness of the damper shows little influence on the *RR*. When the mass damper has its frequency close to the fundamental frequency of the main structure, response reduction (Figure 6) is near maximum. At this stage, the mass damper is said to be Tuned Mass Damper (TMD). *RR* is more for less values of $\boldsymbol{k_d}$, indicating that flexible mass-dampers would be more effective.

(b) Effect of k_d on f_{act} (Figure 7)

From the Figure 7, it is clear that k_d affects the amount of actuating forces required to a very small extent. Actuating force required is less for higher values of damper's stiffness.

Based on the analysis above, the optimum mass-ratio and stiffness of ATMD have been decided to be 1.5% and 14 kN/m respectively. Response of the building with optimum ATMD placed on its top, to unit-step excitation and El-Centro earthquake are plotted using MATLAB and shown in Figs. 9 and 10 respectively.

(c) Effect of k_d on Settling time, T_s of the controlled structure (Figure 8)

It is observed from the plots given in Figure 8 for SDOF system that the settling time of the building does not exceed that of the uncontrolled structure. Settling time is large for smaller values of k_d, when mass ratio was taking the values less than 0.8 % or more than 3%. In between, k_d showed little effect on settling time. The suitable zone for design in Figure 8 seems to be for k_d between 10 to 15 kN/m and mass ratio between 1% to 2% of the building mass.

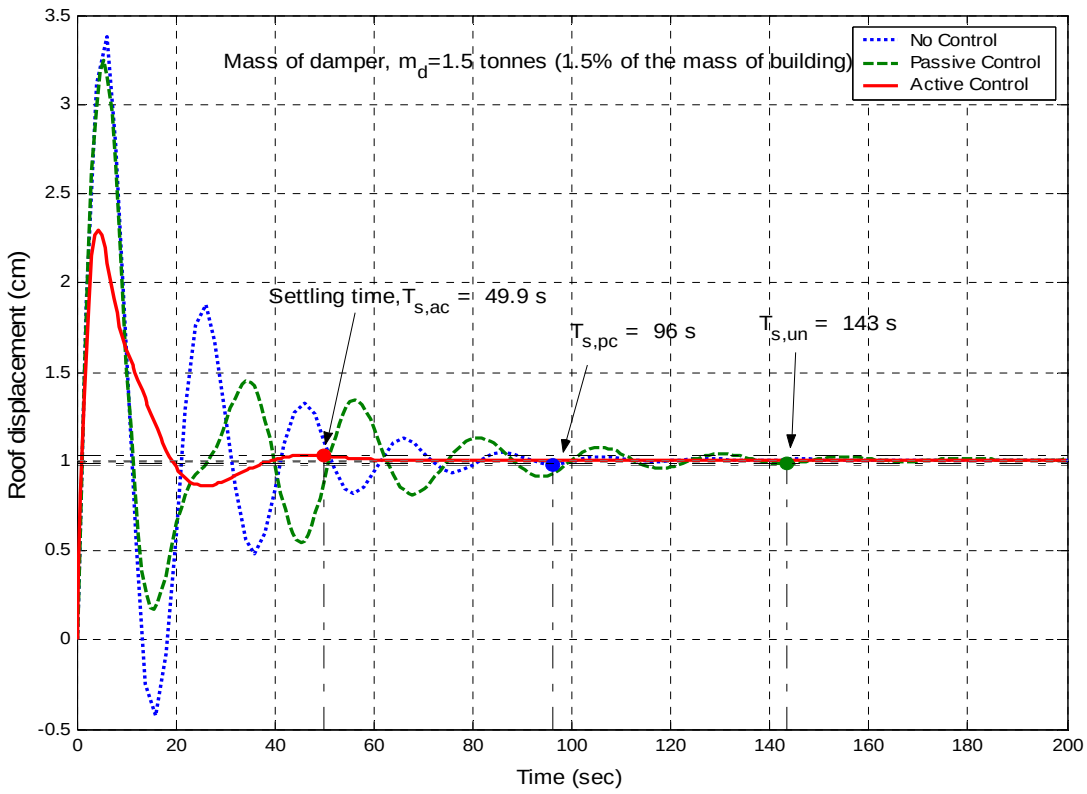

Figure 9. Variation of roof displacement (y_1) with time against unit step excitation for k_d = 14 kN/m and α = 1.5%

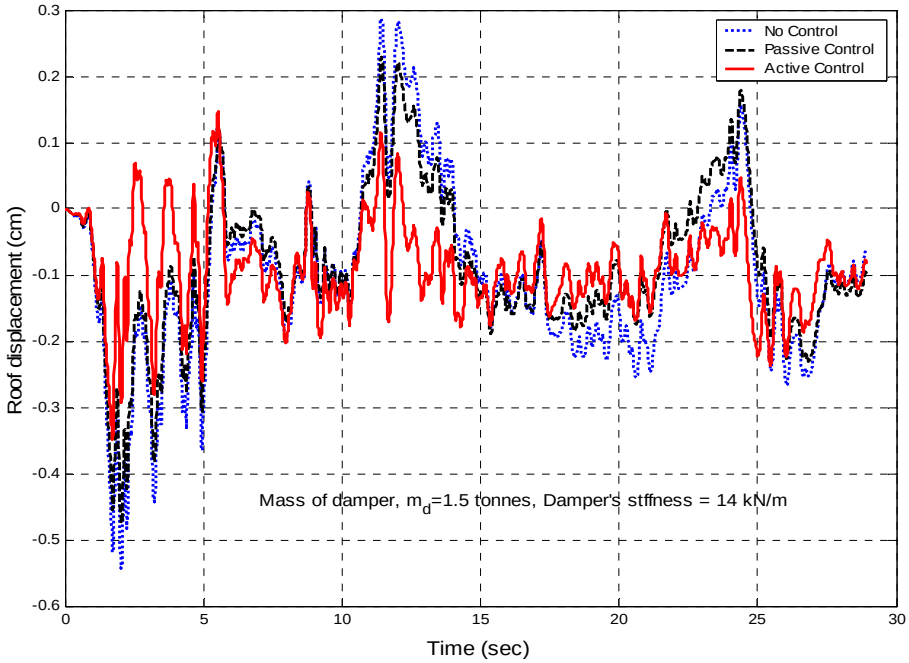

Figure 10. Variation in roof displacement with time against El-Centro earthquake (α = 1.5%; k_d = 14kN/m)

3.3 Comparison of Response Reduction with Yang et al [13]

Table 1 shows the response reduction achieved herein and that obtained by Yang *et al* [13] for an eight-storey building using instantaneous control theory. Each storey mass has been taken to be 345.6 tons, lateral stiffness 3.404×10^5 kN/m and damping 2.937×10^3 t/sec [13]. The building was subjected to an arbitrary ground motion with PGA of 0.8 m/s^2. The percentage response reduction achieved has been chosen as the parameter to be compared. This comparison reveals that ATMD designed with modern control theory (quadratic optimal control theory) gives better response reduction (71.71%) than that obtained by classical instantaneous control (62.44%). Even more response reduction (73.62%) is observed with optimum ATMD parameters.

3.4 Stability Analysis of the Controlled Structure

The stability of the structure for all the three cases has been interpreted from Bode's plots [21]. Certain evidences from Bode's plots are discussed below.

The Bode's plot for the single-storey building is given in Figure 11. The plot shows that the gain margin and phase margin are both positive for uncontrolled, passively and actively controlled systems indicating that the system is stable in all the three cases. But, for no control and passive control conditions, kinks in the curve mean the response of the structure is oscillatory in some range of frequency. These kinks are flattened out after the active control is applied. This can be attributed to the increase in the value of damping ratio (ξ) of the system owing to introduction of ATMD.

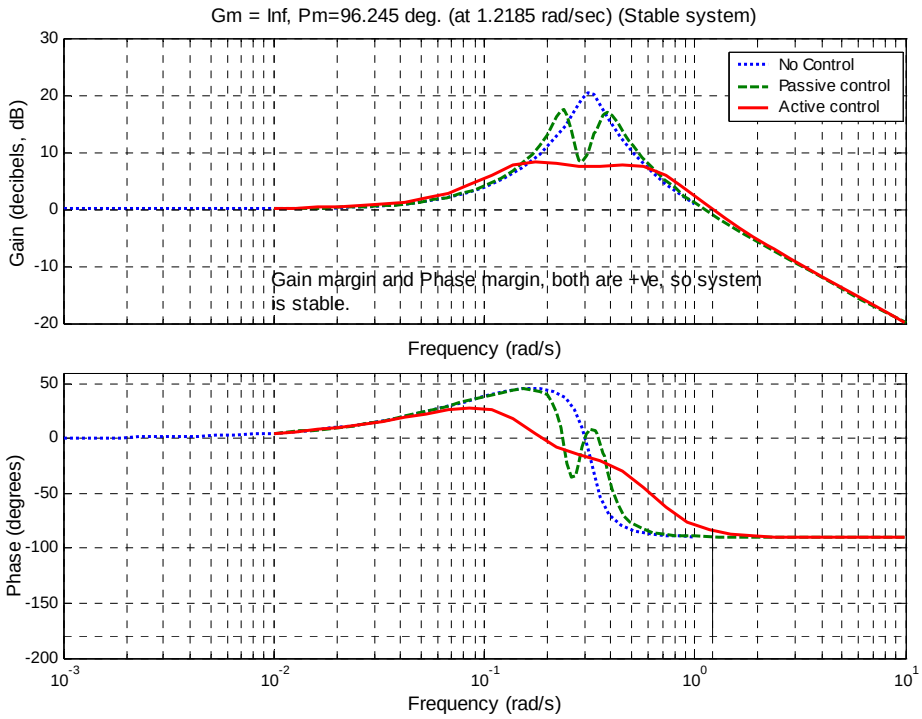

Figure 11. Bode's plots for the designed active control system and uncontrolled structure to check its stability (α = 1.5%; k_d = 14 kN/m)

Table 1. Comparison of results for 8-storey building with those obtained by Yang *et al* [13]

Parameters / **Control Algorithm**	m_d **(tons)**	k_d **(kN/m)**	**Subjected to (PGA)**	**Peak roof displacement (Uncontrolled) (cm)**	**Peak roof displacement (Controlled) (cm)**	**% response reduction**
Yang *et al*	29.630	957.2	Arbitrary ground motion (0.8 m/s^2)	4.10	1.54	62.44
Present study	29.630	957.2	El-Centro earthquake (3.33 m/s^2)	32.3	9.14	71.71
Present study (Optimum)	27.648	600.0	El-Centro earthquake (3.33 m/s^2)	32.3	8.52	73.62

4. CONCLUSIONS

The following conclusions are drawn from the present study:

(1) The active control system provides nearly 35% more response reduction than passive system.

(2) As far as the extent of response reduction and magnitude of external forces required are concerned, the trends of results revealed that the mass of ATMD, $\boldsymbol{m_d}$ should be large. But, simultaneously its value is restricted due to increase in settling time of the controlled system. Moreover, extent of response reduction is halted at large values of mass of the damper, indicating that use of larger mass will be redundant. So, the mass of ATMD is a tradeoff between response reduction and settling time of the controlled structure.

(3) The stiffness of ATMD, $\boldsymbol{k_d}$ influences the percentage response reduction to a more extent than it does to magnitude of actuating forces. Lesser values of stiffness show more response reduction. This study gives the impression that, a flexible damper is better than a stiffer one, but minimum value of $\boldsymbol{k_d}$ is limited by the increase in settling time of the structure.

(4) As soon as the damper's natural frequency approaches that of the main structure, response reduction attains more or less its steady state value, which means the damper is tuned at this stage (ATMD) and performance of active control system is optimum when the damper is tuned to the natural frequency of the main structure.

(5) The mass of ATMD has remarkable effects on the response quantities than its stiffness does.

(6) The active control system makes the system remarkably stable as revealed by Bode's diagrams. The damping properties of the structure are remarkably enhanced after the active control is applied as is clear from stability analysis. The system responds faster and amplitude of vibrations is damped out smoothly and quickly.

(7) The modern quadratic optimal control algorithm shows more response reduction even with same ATMD parameters as used by Yang *et al* [13] while applying instantaneous optimal control theory. Further reduction in response is observed with lesser values of the mass and stiffness of ATMD as designed by proposed optimal control theory.

The paper has presented state-space modeling of building dynamics and development of state feedback optimal control. Extensive analysis of the building model has been carried out using application of modern optimal control. This study results in some guidelines for selecting ATMD parameters for optimal performance.

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NOTATION

A	System matrix of the building without control
A_n	New system matrix of the building after control is applied
B	Control vector identifying the locations at which the structure is subjected to actuating forces
C	Damping matrix of structure of order n × n
C_1	Control gain matrix that proportionate velocity feedback to F_{act}
c_1	Damping coefficient of the single storey building
C_a	Modified damping matrix of the controlled structure
c_d	Damping of the mass damper
C_R	Output vector of the order 1×2(n+1), which is a row matrix having its certain elements unity corresponding to desired response parameters only, rest of the elements being zeros
D	Location matrix for $F_{act}(t)$ of order n × m
D_R	Direct transition matrix of same order as f(t), which is used to include the direct effect of disturbances on the output, if any
E	Location matrix for $f(t)$ of order n × r
E_1	Control gain matrix that proportionate disturbances to F_{act}
E_a	Modified matrix that defines the points under the effect of earthquake forces on the building
$f_e(t)$	transient force acting on the building due to earthquake
$f_{act}(t)$	actuating force provided by the actuator to control the vibrations of the building, after the control is applied
G^T	A column vector containing gain coefficients corresponding to every element of state of the system
H	Location matrix in State Space for earthquake forces of order 2(n+1)×r, which is analogous to E in the basic dynamic equation
I	Unit matrix of the order (n+1)×(n+1)
J	Performance index (Liapunov's cost function)
K	Stiffness matrix of structure of order n × n
k_1	Lateral stiffness of the single-storey building
K_a	Modified stiffness of the structure after the control is applied
k_d	Lateral stiffness of the mass damper
M	Mass matrix of the structure of order n × n
m	Number of degrees of freedom along which controlling forces act
m_1	Mass of the single-storey building
m_d	Mass of the damper
m_n	Mass of each storey unit in a multistorey building
n	Number of storeys in the building
N	Number of eigenvalues of [M] with positive real parts
O	Null matrix of the order (n+1)×(n+1)
P	Ricatti matrix
Q	2(n+1)×2(n+1) positive semi-definite matrix, whose elements affect the

	response reduction achieved
r	Number of degrees of freedom along which earthquake forces act on the building
R	m×m positive definite matrix, whose elements affect the amount of control forces generated
$T_{S,AC}$	Settling time of the actively controlled structure
$T_{S,UN}$	Settling time of the uncontrolled structure
x(t)	Displacement vector of order n × 1
$\dot{\boldsymbol{x}}(t)$	Velocity vector of order n × 1
$\ddot{\boldsymbol{x}}(t)$	Acceleration vector of order n × 1
x_1	Lateral displacement of the mass m_1 in single-storey building
$x_{1,ac}$	Peak response of the building top with active control applied
$x_{1,un}$	Peak response of the building top without control
$x_d(t)$	Displacement of the mass damper, m_d with respect to the ground
$x_i(t)$	Displacement of the ith floor of the building with respect to the ground (i = 1, 2, …, n)
z	State vector containing the displacement vector and velocity vector concatenated
$\dot{z}$	Vector containing the velocity vector and acceleration vector concatenated
α	Ratio of damper's mass to that of the building expressed as percentage
β	Ratio of natural frequency of the damper to that of the building
ϕ	Energy function that is quadratic in z and f_{act}
ω_b	Fundamental natural frequency of the building
ω_d	Natural frequency of the mass damper
ξ	Damping ratio of the structure
β	Ratio of the natural frequency of the actively controlled structure to that of uncontrolled structure