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EFFECT OF BEAM SPACING IN THE HARMONY SEARCH BASED OPTIMUM DESIGN OF GRILLAGES

F. Erdal[*a] and M.P. Saka[b]
[a]*Department of Civil Engineering, Akdeniz University, Antalya, Turkey*
[b]*Department of Engineering Sciences, Middle East Technical University, Ankara, Turkey*

Abstract

The spacing between the longitudinal and transverse beams of a grillage system has an important effect in the minimum weight design of these systems. In this study this effect is investigated using an optimum design algorithm which is based on recently developed harmony search algorithm. The optimum design problem of a grillage system is formulated implementing LRFD-AISC (Load and Resistance Factor Design-American Institute of Steel Construction) limitations. It is decided that W-Sections are to be adopted for the longitudinal and transverse beams of the grillage system. 169 W-Sections given in LRFD code are collected in a pool and the optimum design algorithm is expected to select the appropriate sections from this pool so that the weight of the grillage is the minimum and the design limitations implemented from the design code are satisfied. The solution of this discrete programming problem is determined by using the harmony search algorithm. This algorithm simulates jazz improvisation into a numerical optimization technique. Design example is presented to demonstrate the effect of beam spacing in the optimum design of grillage systems.

Keywords: Grillage optimization; discrete optimum design; beam spacing, stochastic search technique; harmony search algorithm

1. Introduction

Grillage systems are widely used in structures to cover large areas, in bridge decks and in ship hulls. They consist of longitudinal and transverse beams which constitute an orthogonal system. It is generally up to the designer to select the spacing between these beams until unless some restrictions are imposed due to architectural reasons. It is apparent that selection of large or small spacing between the longitudinal and transverse beams yields adoption of large or small steel sections for these beams. However, while the large spacing reduces the number beams to construct the grillage smaller spacing increase the number of beams to be used in the system. Hence there exists an optimum spacing in both directions which

[*] E-mail address of the corresponding author: mpsaka@metu.edu.tr (M.P. Saka)

provides a grillage system with the minimum weight. In [1] genetic algorithm is used to determine the optimum spacing in grillage systems. In this study number of beams in longitudinal and transverse direction is treated as design variables in addition to the steel sections to be selected for the beams of both directions. The design algorithm developed determines optimum number of beams in both directions as well as universal beam section designations required for these beams. Furthermore the effect of warping is also studied in the optimum spacing design of grillage systems.

In this study, the optimum design algorithm for grillage systems presented in [2] is used to demonstrate the effect of beam spacing in the optimum design of grillage systems. This technique is based on harmony search algorithm [3-8] which is a recent addition to stochastic search techniques of combinatorial optimization [9]. Harmony search approach is based on the musical performance process that takes place when a musician searches for a better state of harmony. Jazz improvisation seeks to find musically pleasing harmony similar to the optimum design process which seeks to find optimum solution. The pitch of each musical instrument determines the aesthetic quality, just as the objective function value is determined by the set of values assigned to each decision variable. From the optimum structural design point of view the objective is to determine the appropriate steel sections for each group of a structure from the available steel sections set such that with these harmonies set of sections the response of the structure is within the limitations imposed by the design code and it has the minimum weight. This is similar to finding appropriate notes for a musical so that pleasing harmony can be achieved for the esthetic quality. In recent applications harmony search algorithm is successfully utilized to determine the optimum solutions of different structural design problems [2, 10 and 11].

2. Optimum Design Problem to LRFD-AISC

The optimum design problem of a typical grillage system shown in Figure 1 where the behavioral and performance limitations are implemented from LRFD-AISC [12] and the design variables which are selected as the sequence number of a W section given in the W-section list of LRFD-AISC can be expressed as follows.

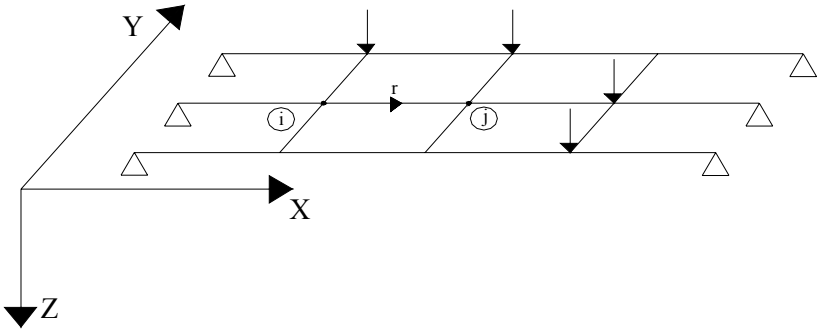
$$\min \quad W = \sum_{k=1}^{ng} m_k \sum_{i=1}^{n_k} l_i \qquad 1(a)$$

Subject to

$$\delta_j / \delta_{ju} \le 1 \quad , j = 1,2,.....,p \qquad 1(b)$$

$$M_{ur} /(\phi_b M_{nr}) \le 1 \, , r = 1,2,....., nm \qquad 1(c)$$

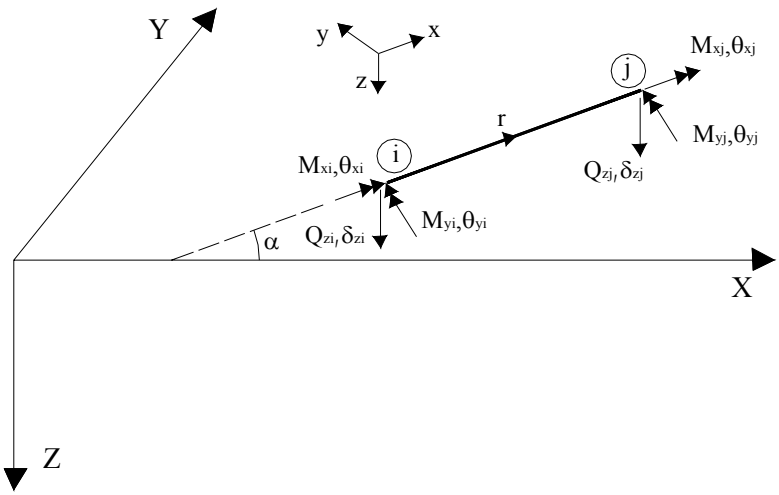
$$V_{ur} /(\phi_v V_{nr}) \le 1 \quad , r = 1,2,....., nm \qquad 1(d)$$



a) Steel grillage system

$$\{D_i\} = \begin{Bmatrix} \theta_{xi} \\ \theta_{yi} \\ \delta_{zi} \end{Bmatrix}, \quad \{P_i\} = \begin{Bmatrix} M_{xi} \\ M_{yi} \\ Q_{zi} \end{Bmatrix}$$

b) Displacements and forces at joint i



c) End forces and end displacements of a grillage member

Figure 1. Typical grillage structure

where m_k in Eq. 1(a) is the unit weight of the W-section selected from the list of LRFD-AISC W-sections list for the grillage element belonging to group k, n_k is the total number of members in group k, and ng is the total number of groups in the grillage system. l_i is the length of member *i*. δ_j in Eq. 1(b) is the displacement of joint j and δ_{ju} is its upper bound. The joint displacements are computed using the matrix displacement method for grillage systems. Eq. 1(c) represents the strength requirement for laterally supported beam in load and resistance factor design according to LRFD-F2. In this inequality $Ø_b$ is the resistance factor for flexure which is given as 0.9, M_{nr} is the nominal moment strength and M_{ur} is the

factored service load moment for member r.

Eq. 1(d) represents the shear strength requirement in load and resistance factor design according to LRFD-F2. In this inequality $Ø_v$ represents the resistance factor for shear given as 0.9, V_{nr} is the nominal strength in shear and V_{ur} is the factored service load shear for member r. The details of obtaining nominal moment strength and nominal shear strength of a W-section according to LRFD are given in the following.

2.1 Load and resistance factor design for laterally supported rolled beams

The computation of the nominal moment strength M_n of a laterally supported beam, it is necessary first to determine whether the beam is compact, non-compact or slender. In compact sections, local buckling of the compression flange and the web does not occur before the plastic hinge develops in the cross section. On the other hand in practically compact sections, the local buckling of compression flange or web may occur after the first yield is reacted at the outer fiber of the flanges. The computation of M_n is given in the following as defined in LRFD-AISC.

a) If $\lambda \le \lambda_p$ for both the compression flange and the web, then the section is compact and

$$M_n = M_p \text{ (Plastic moment capacity)} \qquad 2(a)$$

b) If $\lambda_p < \lambda \le \lambda_r$ for the compression flange or web, then the section is partially compact and

$$M_n = M_p - (M_p - M_r)\frac{\lambda - \lambda_r}{\lambda_r - \lambda_p} \qquad 2(b)$$

c) If $\lambda > \lambda_r$ for the compression flange or the web, then the section is slender and

$$M_n = M_{cr} = S_x F_{cr} \qquad 2(c)$$

Where $\lambda = b_f/(2t_f)$ for I-shaped member flanges and the thickness in which b_f and t_f are the width and the thickness of the flange, and $\lambda = h/t_w$ for beam web, in which $h = d - 2k$ plus allowance for undersize inside fillet at compression flange for rolled I-shaped sections. d is the depth of the section and k is the distance from outer face of flange to web toe of fillet. t_w is the web thickness. h/t_w values are readily available in W-section properties table. λ_p and λ_r are given in table LRFD-B5.1 of the code as

$$\left.\begin{aligned} \lambda_p &= 0.38\sqrt{\frac{E}{F_y}} \\ \lambda_r &= 0.83\sqrt{\frac{E}{F_y - F_r}} \end{aligned}\right\} \textit{for compression flange} \qquad 2(d)$$

$$\left.\begin{aligned}\lambda_p &= 3.76\sqrt{\frac{E}{F_y}}\\ \lambda_r &= 5.70\sqrt{\frac{E}{F_y}}\end{aligned}\right\}\ \textit{for the web} \qquad 2(e)$$

in which E is the modulus of elasticity and F_y is the yield stress of steel. F_r is the compressive residual stress in flange which is given as 69 MPa for rolled shapes in the code. It is apparent that M_n is computed for the flange and for the web separately by using corresponding λ values. The smallest among all is taken as the nominal moment strength of the W section under consideration.

2.2 Load and resistance factor design for shear in rolled beams

Nominal shear strength of a rolled compact and non-compact W section is computed as follows as given in LRFD-AISC-F2.2

$$\text{For}\quad \frac{h}{t_w} \le 2.45\sqrt{\frac{E}{F_{yw}}},\ V_n = 0.6F_{yw}A_w \qquad 2(f)$$

$$\text{For}\ 2.45\sqrt{\frac{E}{F_{yw}}} < \frac{h}{t_w} \le 3.07\sqrt{\frac{E}{F_{yw}}},\ V_n = 0.6F_{yw}A_w\left(\frac{2.45\sqrt{\frac{E}{F_{yw}}}}{\frac{h}{t_w}}\right) \qquad 2(g)$$

$$\text{For}\ 3.07\sqrt{\frac{E}{F_{yw}}} < \frac{h}{t_w} \le 260,\ V_n = A_w\frac{4.52Et_w^{\ 2}}{h^2} \qquad 2(h)$$

where E is the modulus of elasticity and F_{yw} is the yield stress of web steel. V_n is computed from one of the expressions of 2(f)-(h) depending upon the value of h/t_w of the ***W*** section under consideration.

3. Harmony Search Method

The solution of the optimum design problem described from Eq. 1(a) to Eq. 1(d) is obtained by harmony search algorithm. The method consists of five basic steps as listed below.

Step 1. Harmony search parameters are initialized.

A possible value range for each design variable of the optimum design problem is specified. A pool is constructed by collecting these values together from which the algorithm selects values for the design variables. Furthermore the number of solution vectors in harmony memory (HMS) that is the size of the harmony memory matrix, harmony

considering rate (HMCR), pitch adjusting rate (PAR) and the maximum number of searches are also selected in this step.

Step 2. Harmony memory matrix (HM) is initialized.

Harmony memory matrix is initialized. Each row of harmony memory matrix contains the values of design variables which are randomly selected feasible solutions from the design pool for that particular design variable. Hence, this matrix has n columns where N is the total number of design variables and HMS rows which is selected in the first step. HMS is similar to the total number of individuals in the population matrix of the genetic algorithm. The harmony memory matrix has the following form:

$$[\mathrm{H}] = \begin{bmatrix} x_{1,1} & x_{2,1} & .. & .. & x_{n-1,1} & x_{n,1} \\ x_{1,2} & x_{2,2} & .. & .. & x_{n-1,2} & x_{n,2} \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ x_{1,hms-1} & x_{2,hms-1} & .. & .. & x_{n-1,hms-1} & x_{n,hms-1} \\ x_{1,hms} & x_{2,hms} & .. & .. & x_{n-1,hms} & x_{n,hms} \end{bmatrix} \tag{3}$$

$x_{i,j}$ is the value of the ith design variable in the jth randomly selected feasible or near feasible solution. $x_{i,j}$ represents the sequence number of a steel section in the design pool. These candidate designs are sorted such that the objective function value corresponding to the first solution vector is the minimum. In other words, the feasible solutions in the harmony memory matrix are sorted in descending order according to their objective function value. It is worthwhile to mention that not only the feasible designs that are those which satisfy the constraints 1(a)-1(d) are inserted into the harmony memory matrix. Those designs having a small infeasibility are also included in the harmony memory matrix as explained in the next step.

Step 3. New harmony memory matrix is improvised.

In generating a new harmony matrix the new value of the ith design variable can be chosen from any discrete value within the range of ith column of the harmony memory matrix with the probability of *HMCR* which varies between 0 and 1. In other words, the new value of x_i can be one of the discrete values of the vector $\{x_{i,1}, x_{i,2}, ..., x_{i,hms}\}^T$ with the probability of *HMCR*. The same is applied to all other design variables. In the random selection, the new value of the ith design variable can also be chosen randomly from the entire pool with the probability of *1-HMCR*. That is

$$x_i^{new} = \begin{cases} x_i \in \{x_{i,1}, x_{i,2}, ...x_{i,hms}\}^T \text{ with probability HMCR} \\ x_i \in \{x_1, x_2, ..., x_{ns}\}^T \text{ with probability } (1 - \text{HMCR}) \end{cases} \tag{4}$$

Where ns is the total number of values for the design variables in the pool. If the new value of the design variable is selected among those of the harmony memory matrix, this value is then checked whether it should be pitch-adjusted. This operation uses pitch adjustment parameter PAR that sets the rate of adjustment for the pitch chosen from the harmony memory matrix as follows:

$$Is \;\; x_i^{new} \; to \; be \; pitch-adjusted \; ? \left\{ \begin{array}{ll} Yes & with \; probability \; of \; PAR \\ No & with \; probability \; of \; (1-PAR) \end{array} \right\} \quad (5)$$

Supposing that the new pitch-adjustment decision for x_i^{new} came out to be *yes* from the test and if the value selected for x_i^{new} from the harmony memory is the kth element in the general discrete set, then the neighboring value k+1 or k–1 is taken for new x_i^{new}. This operation prevents stagnation and improves the harmony memory for diversity with a greater change of reaching the global optimum.

Once the new harmony vector x_i^{new} is obtained using the above-mentioned rules, it is then checked whether it violates problem constraints. If the new harmony vector is severely infeasible, it is discarded. If it is slightly infeasible, there are two ways to follow. One is to include them in the harmony memory matrix by imposing a penalty on their objective function value. In this way the violated harmony vector which may be infeasible slightly in one or more constraints, is used as a base in the pitch adjustment operation to provide a new harmony vector that may be feasible. The other way is to use larger error values such as 0.08 initially for the acceptability of the new design vectors and reduce this value gradually during the design cycles and use finally an error value of 0.001 towards the end of the iterations. This adaptive error strategy is found quite effective in handling the design constraints in large design problems.

Step 4. Harmony memory matrix is updated.

After selecting the new values for each design variable the objective function value is calculated for the new harmony vector. If this value is better than the worst harmony vector in the harmony matrix, it is then included in the matrix while the worst one is taken out of the matrix. The harmony memory matrix is then sorted in descending order by the objective function value.

Step 5. Steps 3 and 4 are repeated until the termination criterion which is the pre-selected maximum number of cycles is reached. This number is selected large enough such that within this number of design cycles no further improvement is observed in the objective function.

4. Optimum Design Algorithm

The optimum design algorithm is based on the harmony search method steps of which are

given above. The discrete set from which the design algorithm selects the sectional designations for grillage members is considered to be the complete set of 169 W-sections which start from W200x15mm to W690×240mm as given in LRFD-AISC [12]. The design variables are the sequence numbers of W-sections that are to be selected for member groups in the grillage system. These sequence numbers are integer numbers which can take any value between 1 and 169. Harmony search method then randomly selects integer number for each member group within the above bounds. Once these numbers are decided, then the sectional designation and cross sectional properties of that section becomes available for the algorithm. The grillage system is then analyzed with these sections under the external loads and the response of the system is obtained. If the design constraints given in 1(b, c and d) are satisfied this set of sections are placed in the harmony memory vector, if not the selection is discarded. This process is continued until the harmony memory matrix is filled with design vectors. The rest of the procedure is applied as explained in Section 3.

5. Design Example

The optimum design algorithm presented in the previous sections is used to demonstrate the effect of beam spacing in the optimum design of grillages. For this purpose, 12m×12m square area is considered. The design problem is to set up a grillage system that is suppose to carry 15kN/m^2 uniformly distributed load total of which is 2160kN. The grillage system that can be used to cover the area will have 12m long the longitudinal beams and 12m long the transverse beams. Five different grillage systems are considered having 3m, 2.4m, 2m, 1.5m and 1m beam spacing. The total external load is distributed to joints of the grillage system as a point load value of which is calculated according to beam spacing. A36 mild steel is selected for the design which has the yield stress of 250MPa, the modulus of elasticity of 205kN/mm^2 and shear modulus of 81kN/mm^2 respectively. The members of grillage structures are collected in two groups. The longitudinal beams are considered to belong to group 1 and transverse beams are taken as group 2. Harmony search parameters; Harmony memory size (HMS) is taken as 50, Harmony memory considering rate (HMCR) is selected as 0.9 while pitch adjusting rate (PAR) is considered as 0.5 on the basis of the empirical findings by Geem and Lee [5] in the design of all grillage systems.

The grillage system with 3m beam spacing is shown in Figure 1 where the system has 24 members. The total external loading 2160kN is distributed to the joints as 240kN point load. The vertical displacements of joints 4,5,6 and 7 are restricted to 25 mm.

The optimum sectional designations obtained by design method presented under the external loading for the 24-member grillage system is shown in Figure 2 are given in Table 1.

Table 1. Optimum design for 24-member grillage system with 3m beam spacing

Optimum W-Section Designations		**δ_{max} (mm)**	**Maximum Strength Ratio**	**Minimum Weight (kg)**
Group 1	**Group 2**			
W610x174	W610x262	15.6	0.99	15712

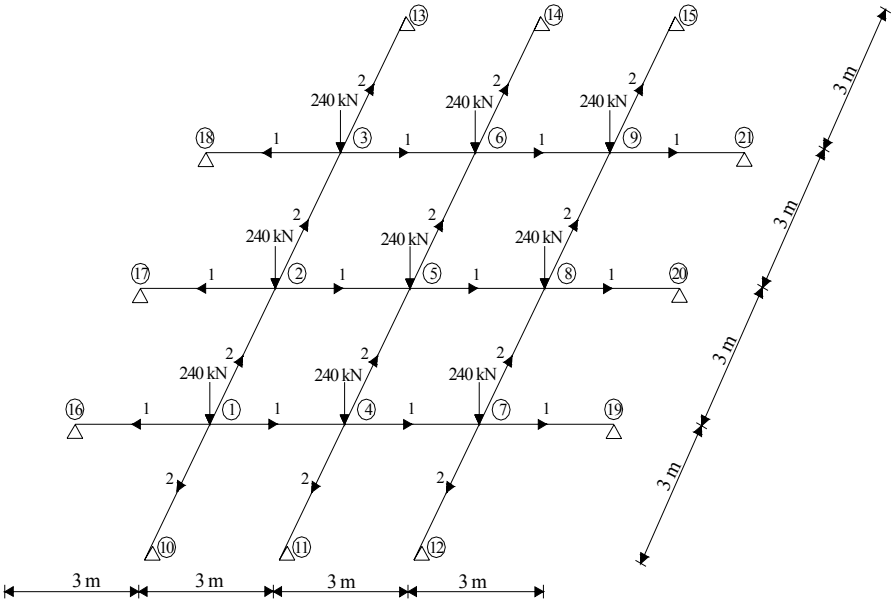


Figure 2. A 24-member grillage system

The grillage system shown in Figure 3 is obtained by considering 2.4m beam spacing in both directions. This grillage system has 40 members which are collected in two groups. The total external load 2160kN is distributed to the joints as 135kN point load. The vertical displacements of joints 6, 7, 10 and 11 are restricted to 25 mm.

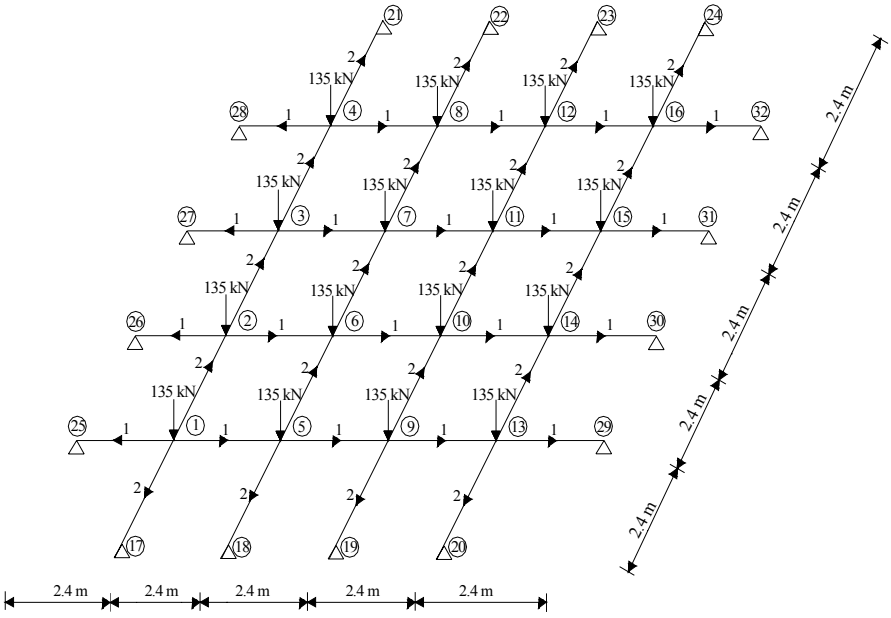


Figure 3. A 40-member grillage system

The optimum sectional designations obtained for the 40-member grillage system is given in Table 2. It is apparent that when the beam spacing is reduced from 3m to 2.4m the minimum weight of the grillage system also reduces from 15712kg to 15488kg.

Table 2. Optimum design for 40-member grillage system with 2.4m beam spacing

Optimum W-Section Designations		**δ_{max} (mm)**	**Maximum Strength Ratio**	**Minimum Weight (kg)**
Group 1	**Group 2**			
W200x15	W610x307	24.9	0.49	15488

The grillage system shown in Figure 4 is obtained by considering 2m beam spacing in both directions. This grillage system has 60 members which are collected in two groups. The total external load 2160kN is distributed to the joints as 86.4kN point load. The vertical displacements of joints 8, 13, 14 and 18 are restricted to 25 mm.

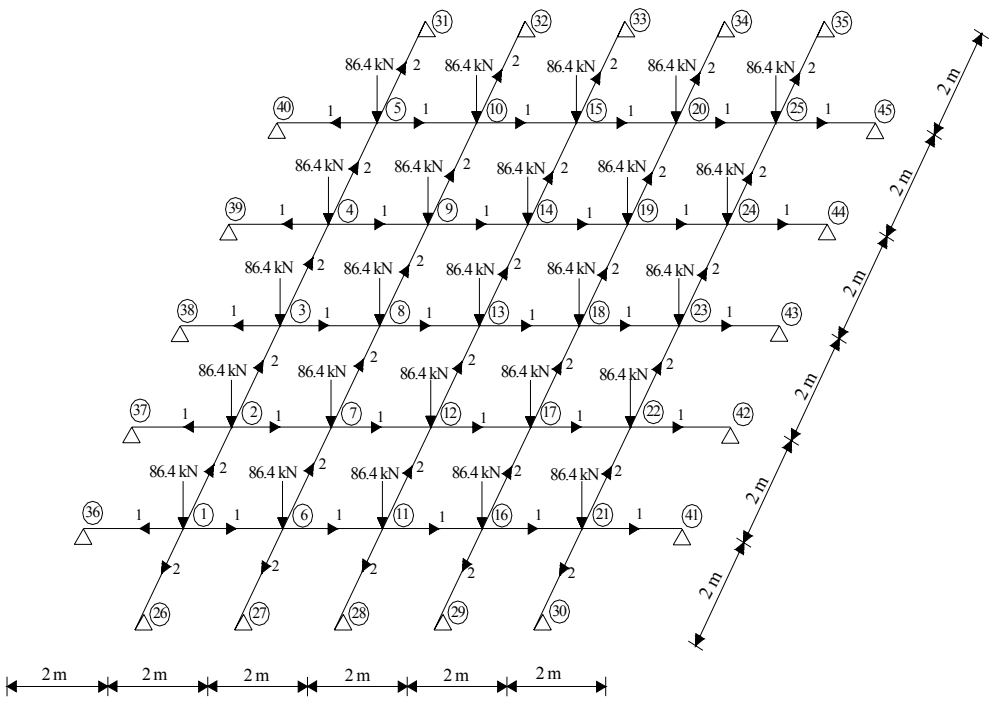


Figure 4. A 60-member grillage system

The optimum sectional designations obtained for the 60-member grillage system is given in Table 3. It is apparent that when the beam spacing is reduced from 2.4m to 2m the minimum weight of the grillage system also reduces from 15488kg to 14384kg.

Table 3. Optimum design for 60-member grillage system with 2m beam spacing

Optimum W-Section Designations		**δ_{max} (mm)**	**Maximum Strength Ratio**	**Minimum Weight (kg)**
Group 1	**Group 2**			
W200x22.5	W690x217	25.0	0.48	14384

The grillage system shown in Figure 5 is obtained by considering 1.5m beam spacing in both directions. This grillage system has 112-members which are collected in two groups. The total external load 2160kN is distributed to the joints as 44.08kN point load. The vertical displacements of joints 18, 19, 25 and 26 are restricted to 25 mm.

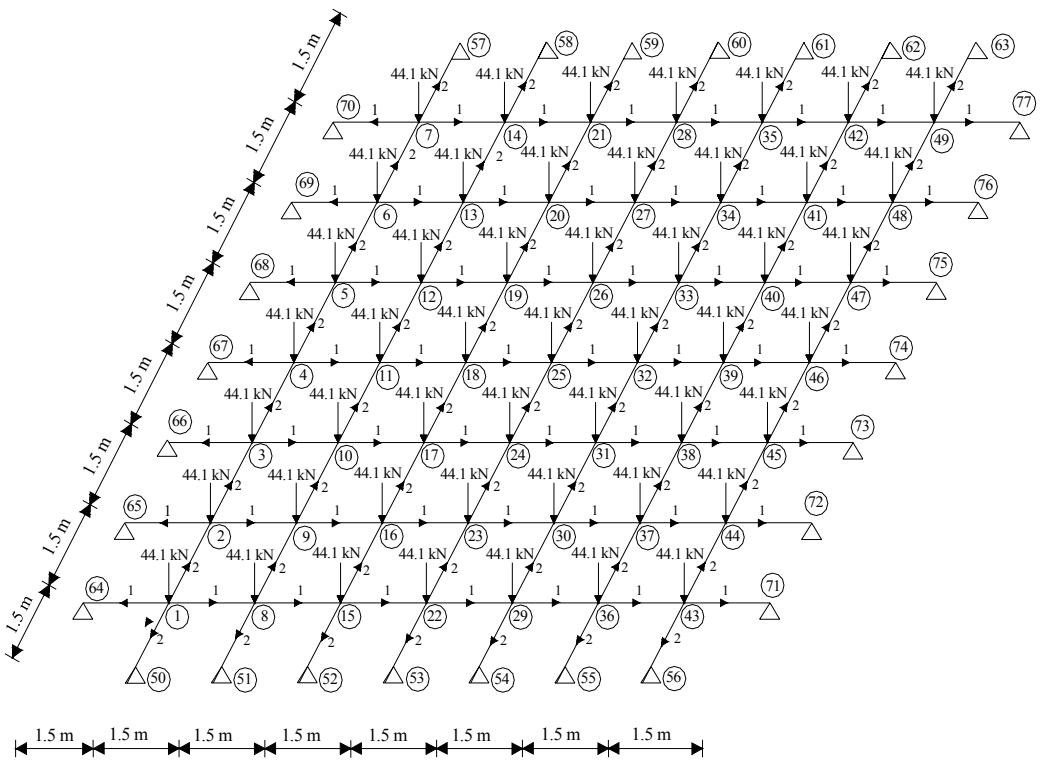


Figure 5. A 112-member grillage system

When the spacing between the longitudinal and transverse beams of the grillage system is decreased from 2m to 1.5m, the minimum weight of the grillage system is increased from 14384kg to 16198kg. The optimum sectional designations obtained by design method presented the under external loading for the 112-member grillage system is given in Table 4.

Table 4. Optimum design for 112-member grillage system with 1.5m beam spacing

Optimum W-Section Designations		δ_{max} (mm)	Maximum Strength Ratio	Minimum Weight (kg)
Group 1	**Group 2**			
W690x170	W200x22.5	24.1	0.45	16198

Finally the beam spacing is reduced to 1.5m beam spacing in both directions. In this case a grillage system is obtained that has 264-members which are collected in two groups. The total external load 2160kN is distributed to the joints such that each joint has 17.85kN point load. The vertical displacements of joints 50, 51, 61 and 62 are restricted to 25 mm. The optimum sectional designations obtained for both groups of the 264-member grillage system by the design method presented is given in Table 5.

Table 5. Optimum design for 264-member grillage system with 1m beam spacing

Optimum W-Section Designations		δ_{max} (mm)	Maximum Strength Ratio	Minimum Weight (kg)
Group 1	**Group 2**			
W690x125	W200x19.3	21.1	0.39	19100

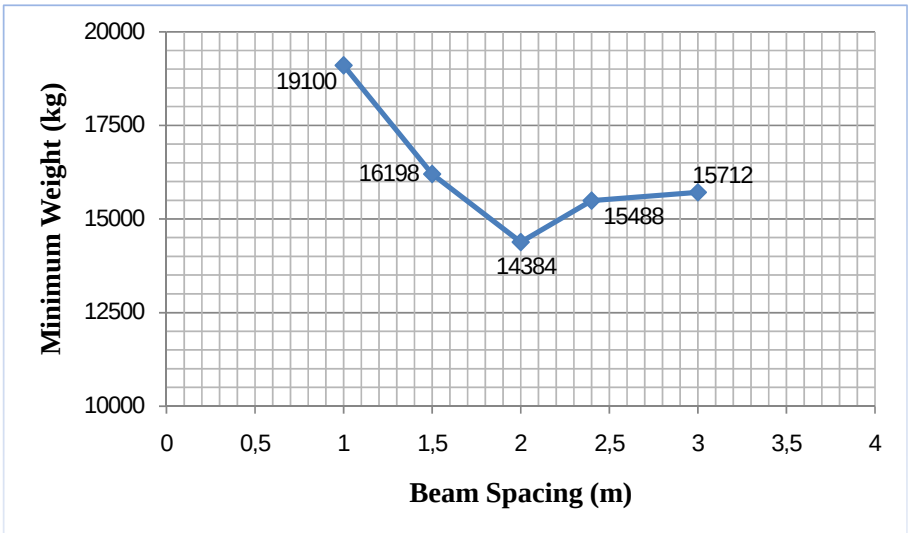


Figure 5. Variation of weight versus beam spacing

The minimum weight of the grillage system changes with beam spacing. There is a decrease from 3-m beam spacing to 2-m beam spacing, then the minimum weight increases. The variation of the minimum weight with the beam spacing is shown in Figure 5. In this study, the beam spacing is selected as numbers that are practically preferred. It is apparent from the figure that 2-m spacing is the optimum spacing among the values considered. It should be pointed out that in the design of grillage systems beam spacing should be taken as design variable in addition to steel section designations to be selected for the member groups.

6. Conclusions

It is shown that the harmony search method which is one of the recent additions to metaheuristic algorithms can successfully be used in the optimum design of grillage systems. Harmony search method has three parameters that are required to be determined prior to its use in determining the optimum solution. These parameters are problem dependent and some trials are necessary to determine their appropriate values for the problem under consideration. It is also shown that beam spacing in the optimum design of grillage systems has an effect on the minimum weight and it is more appropriate to consider this parameter as a design variable if a better design is looked for. It is also interesting to notice that while for the larger values of beam spacing the optimum design problem is strength dominant, for the smaller values of beam spacing the problem becomes displacement dominant.

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