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STRUCTURAL MORPHOLOGY OF TENSEGRITY SYSTEMS

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Abstract

The coupling between form and forces, their structural morphology, is a key point for tensegrity systems. In the first part of this paper we describe the design process of the simplest tensegrity system which was achieved by Kenneth Snelson. Some other simple cells are presented and tensypolyhedra are defined as tensegrity systems which meet polyhedra geometry in a stable equilibrium state. A numerical model giving access to more complex systems, in terms of number of components and geometrical properties, is then evoked. The third part is devoted to linear assemblies of annular cells which can be folded. Some experimental models of the tensegrity ring which is the basic component of this "hollow rope" have been realized and are examined.

Keywords: Tensegrity; morphology; form finding; rings; hollow rope

1. Introduction

The coupling between forms and forces is one of the main topics of Structural Morphology. This coupling is very strong for systems in tensegrity state, currently called "tensegrity systems". Since some years the number of publications on tensegrity systems is increasing. The aim of this paper is to focus on the morphogenesis of tensegrity systems since earlier cells to present tensegrity rings studied by our research team. Among publications devoted to mechanical behaviour of tensegrity systems, the work carried out by Schenk [2] provides an interesting literature review.

2. From Simple to Complex Cells

2.1 Introduction

The problem of form finding is central in the study of tensegrity systems. Since the very beginning of their creation, by Snelson, and Emmerich, who realized the concept that has been enounced by Fuller, the definition of cells catches the interest of the designers. The

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following paragraphs illustrate the main steps between the simplest system, the so-called "simplex" and the last complex systems which are actually designed. This is a way from simplicity to complexity with a set of several models: physical models, form models based on polyhedra, force models mainly based either on force density or on dynamic relaxation.

2.2 The double x and the simplest cell

Among different explanations concerning the design of the first tensegrity cell with nine cables and three struts, the most convincing one, according to my own opinion, can be found in the patent delivered to Kenneth Snelson [3]

A key explanation is developed in this patent (see Figure 1). The basic idea is contained in X-shape which is an assembly of two struts and four cables the whole system being in self equilibrium. By cutting one of the four cables of the X-shape, the remaining system acts like an hydraulic jack along the direction of this cable (we called it the "strut effect" since it is equivalent to a strut under compression).

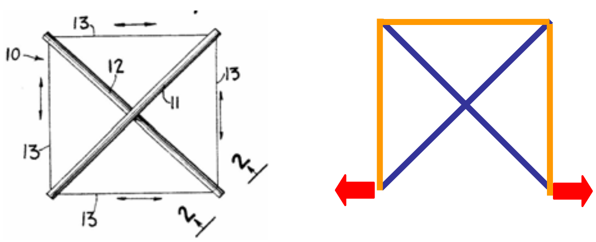


Figure 1. Strut effect along direction 1-2

This idea was used by Kenneth Snelson after a specific work on the assembly of components by mean of a rhombus of cables [1]: "one to another" and "one to the next" sculptures have opened the way to the "Double-X". In this third sculpture, we can see that Snelson assembled two "X-shapes" with a rhombus of cables in-between. Several other cables were added in order to prevent a motion of the "X-shapes" out of their own plane. The next step was to assemble three "X-shapes" together using again three rhombuses of cables. This assembly theoretically ends up with twelve cables, but three of them are common to two rhombuses: nine cables only remained. Each of the three "X-shape" played the role of a strut. This assembly was finally composed of nine cables and three struts and constituted the simplest tensegrity system which could be realized in three dimensional space. Some authors call it the "simplex" (Figure 2).

2.3 Simple systems

The first attempts to create new elementary cells were based on some simple characteristics:

- Use of single straight struts as compressed components
- Use of polygonal compressed components (chains of struts)
- Choice of only one set of cable length ("c")
- Choice of only one set of strut length ("s")

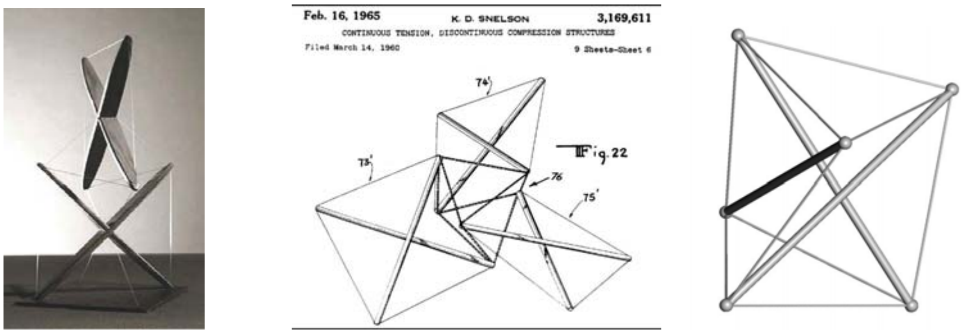


Figure 2. Double-X , Triple-X , Simplex

2.3.1 Prismatic cells

The simplex, evoked in the previous paragraph, can also be seen as the result of the transformation of a straight triangular prism. The equilibrated self stress geometry is defined by the relative rotation of the two triangular bases equal to 30° degrees (see Figure). Clockwise and anticlockwise solutions can be used.

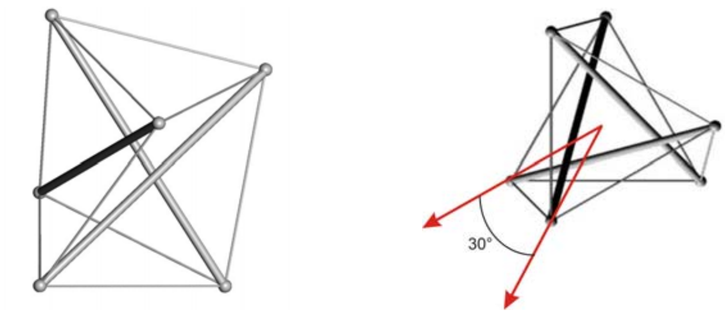


Figure 3. Equilibrium geometry

It can be demonstrated (see [4]) that, for p-prism, the relative rotation has to satisfy the following relation

$$\theta = \pm \frac{\pi \cdot (p - 2)}{2 \cdot p} \quad (1)$$

2.3.2 From polyhedra to tensypolyhedra

The so-called “form controlled method” [5] was mainly used by David Georges Emmerich. The problem is to know if there is a possibility to design a tensegrity cell by keeping the node coordinates in the geometry of a regular (or a semi regular) polyhedron. It is possible for some cases, and not for others.

When it is possible to insert struts inside the polyhedron and to establish a self stress state of equilibrium, we suggested to use the denomination “tensypolyhedron”. Olivier Foucher [6] realized a comprehensive study from which I extract two examples among polyhedra, which can not be classified as tensypolyhedra.

These two examples correspond to systems comprising six struts with eighteen cables for the truncated tetrahedron, and six struts with twenty four cables for the expanded octahedron.

a) Truncated tetrahedron

This semi regular polyhedron has four triangular faces and four hexagonal faces. It is impossible to obtain a tensegrity system in its initial geometry (see Figure 4(a)). The hexagonal faces are not planar, and it is visible on the corresponding physical model at its top hexagon (see Figure 4(b)).

(a) (b)

Figure 4. Truncated tetrahedron wit six struts inside (initial geometry and physical model)

This result has been validated by calculations made with a numerical model based on dynamic relaxation by Belkacem [7]. Il can also be checked on the specific software that we developed in our laboratory in order to identify the states of self stress ("Tensegrite 2000").

But it also useful to make a very simple remark: if we consider one of the nodes, let say A, it can be seen that a necessary condition of equilibrium is to have the corresponding strut in position as shown on Figure 5(a) (a simple symmetry consideration has to be done). But in this case the other end of the strut would not be on an other node; Figure 5(b) shows the situation and simultaneously the impossibility of equilibrium in the original shape.

b) Expanded octahedron (icosahedron)

The second example of a six struts system is related to the geometry of the regular polyhedron known as icosahedron. It is possible to compute the shape resulting from the insertion of the struts. The number of cables of this tensegrity system is equal to twenty four, and it is less than the number of edges of the icosahedron (thirty).

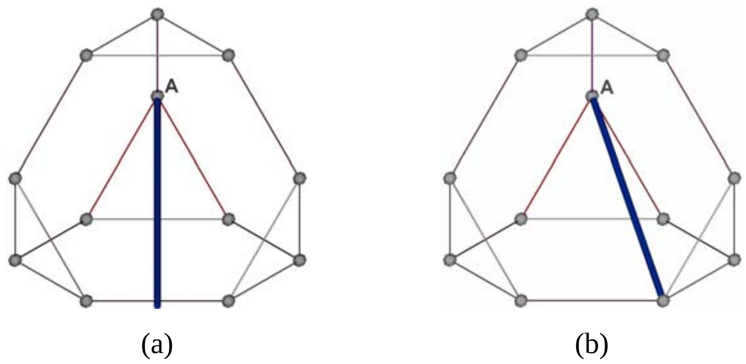


Figure 5. Truncated tetrahedron: research for equilibrium geometry

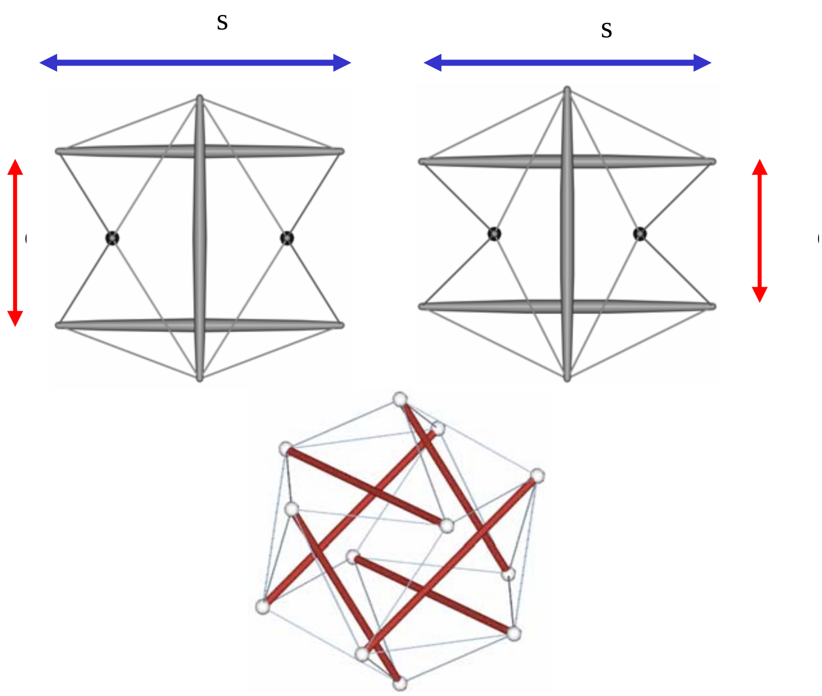
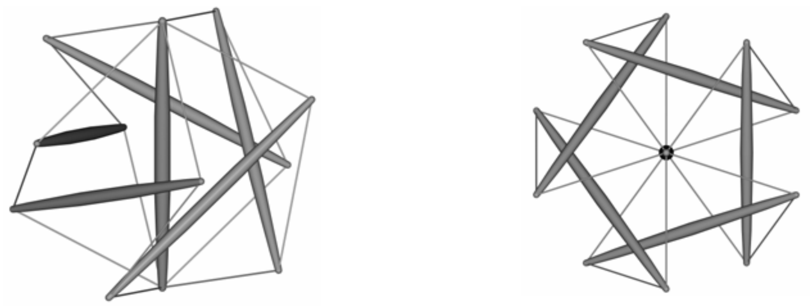


Figure 6. Comparison between icosahedron geometry and expanded octahedron geometry

The two geometries can be compared on basis of the ratio between the length of struts “s” and the distance between two parallel struts “d”. For the icosahedron this ratio is equal to approximately 1.618 (that is the “golden” ratio), for the associated tensegrity system it is equal to exactly 2. This resulting tensegrity system can be seen as the expansion of an octahedron, since there are at the end eight triangles of cables (the same as the number of triangular faces for an octahedron), and the three pairs of struts can be understood as the splitting of the three internal diagonals.

c) The Spinning icosahedron

Since it is not possible to design a regular icosahedron with six equal struts, we tried to build one with six struts, one of them being greater than the five others. The basis of this design is a prismatic pentagonal system; a central strut is placed on the vertical symmetry axis. This axis becomes a rotation axis. The lengths of the struts and of the cables are calculated in order to reach an equilibrium state which is characterized by the fact that the twelve nodes occupy the geometrical position of the apices of an icosahedron. The name is chosen by reference to this axis of rotation and to the icosahedron.



(a) (b)

Figure 7. Spinning icosahedron: perspective and in plane views

It can be noticed also that this system can be classified as a “Z” like tensegrity system according to the classification submitted by Anthony Pugh [8]. There are only two cables and one strut at each node, except for the central strut.

2.3.3 Complex compressed components: circuit like systems

Among all tensegrity systems, some are characterized by the specific topology of their compressed components. These components are no more single struts, but chains of struts. Two examples are presented.

a) Cuboctahedron

For this example the continuum of cables is exactly mapped on the edges of a Cuboctahedron, which is one of the semi regular polyhedra (also called Archimedean polyhedra). There are four triangular compressed components. Each of them constitutes a circuit of struts (a circuit is a particular case of chain). These triangles are intertwined and their equilibrium is ensured simultaneously by a hexagon of cables and the effect of the three other triangles for three of the apices of each hexagon. This is a case of tensypolyhedron (Figure 8).

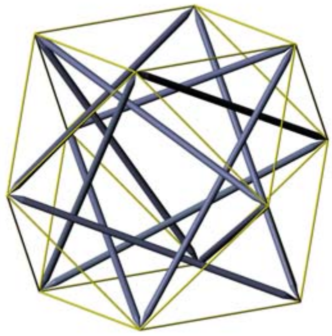


Figure 8. Cuboctahedron tensegrity system

b) Mono circuit tensypolyhedron
This second case is a very interesting one; the chain of fifteen struts is closed and creates a circuit which is the only compressed component. The continuum of tensioned components is a polyhedron with two pentagonal parallel faces, five quadrangular and ten triangular faces. We will develop a study on “tensegrity rings” in the following paragraphs, based on this specific cell.

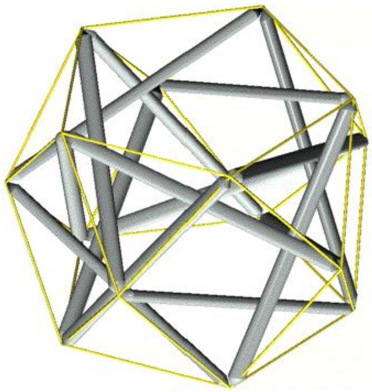


Figure 9. Mono circuit tensypolyhedron

3. Toward Complexity

3.1 Introduction
If the elementary cells were based on polyhedra, it became obvious that it could be interesting to design more complex systems, with many different lengths for cables and struts. Specifically, we had this need not for architectural structures, but for a specific problem in biology: the cytoskeleton of human cells can be analogically compared to tensegrity systems as far as their common mechanical behaviour is concerned. The first attempts were developed with force density method by Nicolas Vassart [4] and allowed to

work on multi parameter systems. But this method is not very well adapted for very complex systems since it is difficult to control the final shape. Therefore we began to work on physical models before developing a numerical method which gives some first interesting results.

3.2 Preliminary physical models

It is useful to begin with physical models, because it is the best way to understand the complexity of the design with all implied parameters. Conversely a virtual model is certainly easier to use in terms of the number of resulting solutions, but before modelling a process it is necessary to understand the different difficulties which can occur and to develop an adapted virtual model for taking these particularities into account. The first complex system was achieved some years ago and was called “cloud n°1”.

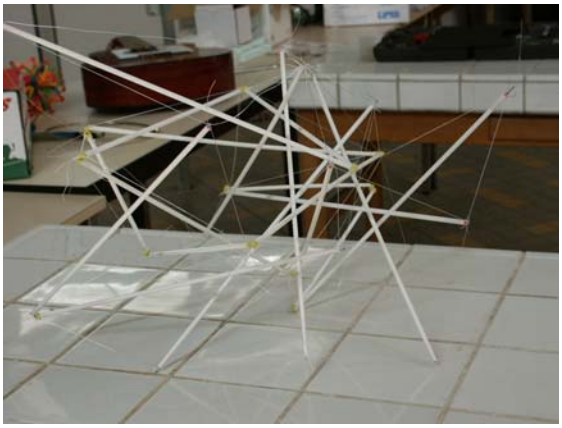


Figure 10. Cloud n°1

We developed then a more systematic process at the school architecture in Montpellier. Figure 11 is an illustration of the models which have been built during a workshop.

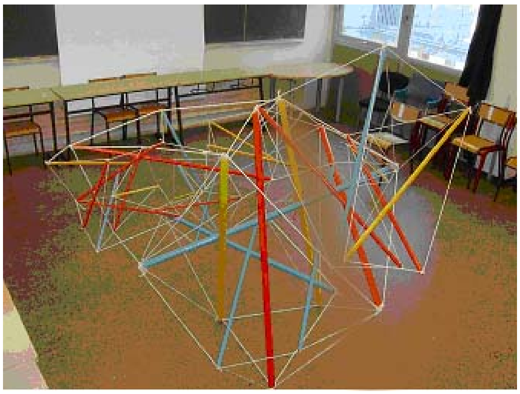


Figure 11. Cloud n°2

4. Numerical Models Toward Complex Systems

4.1 Introduction

It was necessary to model and to generalize the process through numerical methods. This work has been achieved by Zhang et al. [9].

The form-finding process that we use started from an initial specification of the geometry. At the same time, self stresses in some or all the components are also arbitrarily specified. Hence, excepted particular cases or lucky situations, the system can not be in equilibrium. A motion of the structure is then caused by the unbalanced internal forces. The displacements are computed by using the dynamic relaxation method that is based upon the calculation of a sequence of decreasing energy peaks and leads the system to reach the steady equilibrium state.

4.2 Contact check

During form-finding process, the minimum distance between two spatial line segments should be checked for avoiding contact. It is necessary especially when system geometry is complex and several algorithms for checking can be used [10]. If in final equilibrium state some elements touch each other (which means improper topology or geometry chosen by designer), then the topology or the geometry has to be modified until no contact is ensured. It can be done in a "slight way" by modifying stiffness values or more roughly by changing the topology.

4.3 Applications

4.3.1 "Stella octangula"

The used topology for this application corresponds with one of David Georges Emmerich's proposals and is represented in Figure 12 (see references [1] and [7]). The system is designed on the basis of a triangular anti-prism: struts lie on the triangular bracing faces along the bisecting direction, one of their ends is an apex of a layer triangular face and the other end is in the second parallel plane. There are 6 struts, 18 cables connected to 12 nodes and, for each strut one node is only connected to two cables: the corresponding equilibrium is thus realized into a plane. The length of all struts is roughly 19 and roughly 11 for all cables. (all values are a dimensional).

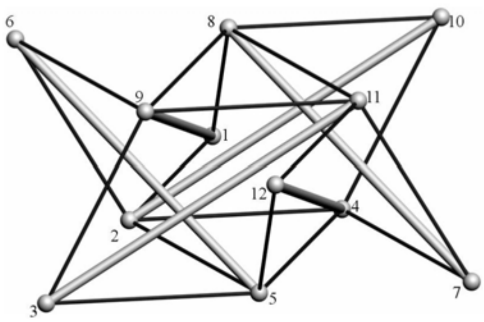


Figure 12. Stella octangula

We investigated the equilibrium geometry by dynamic relaxation method by prescribing initial stresses in struts and cable elements (–10 and 20 respectively). For struts the stiffness is EA = 1000 and for cables EA = 10; parameters $\Delta t = 1$ and $\lambda = 1$ (λ is a convergence parameter; the maximum outbalanced force of the system is 10^{-4}.

An equilibrium state is then obtained: the compressions in struts are roughly -33 and the tensions in cables roughly 19. Even though the process is started from an arbitrary initial self stress specification, in final equilibrium state the absolute values of the ratio between the normal force and the reference length (i.e. the force density coefficient [1]) in all elements are almost the same (the absolute value is approximately 1.79).

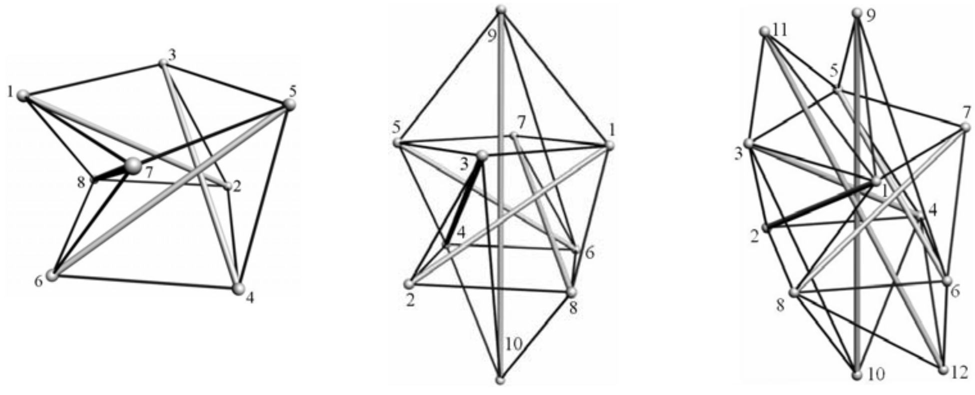
4.3.2 "Free form tensegrity"

No topology of the whole system is specified in advance for that example. The process is started from a simple system and, next, more and more struts and cables are added step by step. The computational sequence is summarized as follows: the process starts from a quadruplex (Figure 13(a), simple regular shape), and another vertical strut 9-10 is added (Figure 13(b)). To keep nodes 9 and 10 in equilibrium state, it is necessary to add six cables (three connected to node 9 and another three to node 10). Note that other possibilities exist for adding these new elements but we have chosen the simplest way. Following the same procedure, three other struts (11-12 ; 13-14 ; 15-16) and eighteen cables are added to the system step by step; the topologies are respectively shown in Figure 14(a), (b) and (c).

In the system represented in Figure 14(b), there are 8 struts and 36 cables connected to 16 nodes. Calculation parameters are EA = 1000 and for cables EA = 10; parameters $\Delta t = 1$ and $\lambda = 1$; the maximum outbalanced force of the system is still 10^{-4}.; initial tension and compression in all cables and struts are respectively 2 and –1.

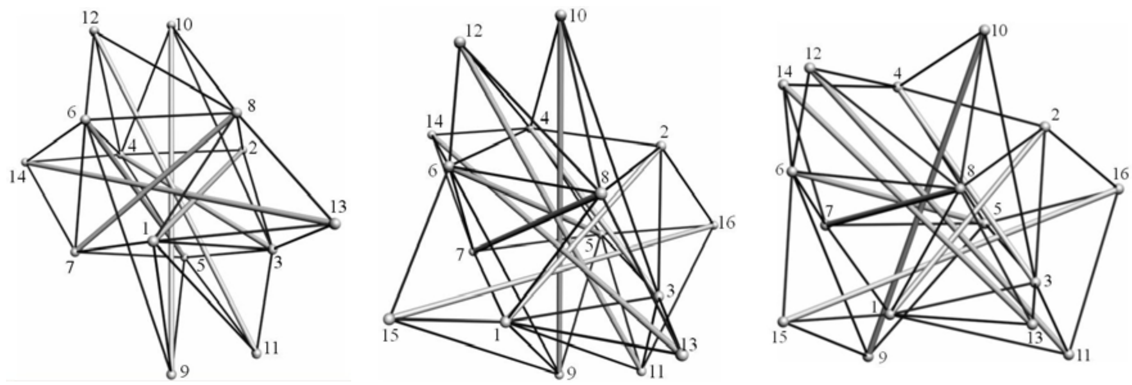
An equilibrium state is obtained by the dynamic relaxation method based on this given topology. The minimum distance between any two spatial elements is 0.481; the compression in struts is between –2.854 and –4.328, the tensions in cables between 0.346 and 3.453. The result shows that the tensions in element 4-6, 9-1, and 11-5 are respectively 0.640, 0.391 and 0.346. They are lower when compared with the values in other cables and by topology analysis it can be found that there are more than three cables connected to nodes 1, 4, 5, 6, 9, and 11. Since some of these cables can be regarded as redundant elements, they are removed from the system. This is the case for cables 4-6, 9-1 and 11-5. Keeping all other parameters the same as previously, form-finding process is restarted. Finally, a new geometry and equilibrated self stress state are obtained (Figure 14(c)). The compressions in struts range from –2.680 to –4.342; the tensions in cables are between 0.758 and 3.049 and the minimum distance between any two spatial elements is 0.611. There are 33 cables and 8 struts connected respectively to 16 nodes in the whole system.

In this example only two different lengths (19.9 and 32.9) for the eight struts are necessary at the starting configuration. During the form-finding process, one strut following another one is added to the system randomly. To keep this strut in stability, a certain number of cables are added to its ends. Many possibilities exist for such topology modifications and the designer can choose the more suitable solution.



(a) (b) (c)

Figure 13. From four struts to six struts



(a) (b) (c)

Figure 14. From seven struts to eight struts

It is a matter of fact that after many years of work on structural morphology of tensegrity systems, it is now possible to design free form systems. These cells can be used alone or in assemblies for architectural or other purposes. It will then be possible to use the structural principle of tensegrity systems with its advantages and disadvantages.

5. Linear Assemblies

5.1 Introduction

In his book devoted to a first approach of tensegrity, Anthony Pugh [8] showed three models which attracted my attention. A first one comprised four triangular compressed components inside a net of tensile ones. The overall geometry was organized according to a cuboctahedron, one of the semi regular polyhedra. The second model was very surprising since the struts constituted a single circuit with 15 nodes and 15 compressed components. For this model, the cables are the edges of a polyhedron with two pentagonal bases. The third one is a twenty-strut four-layer circuit pattern system. There are represented on Figure 15. This presentation concerns only the second cell.

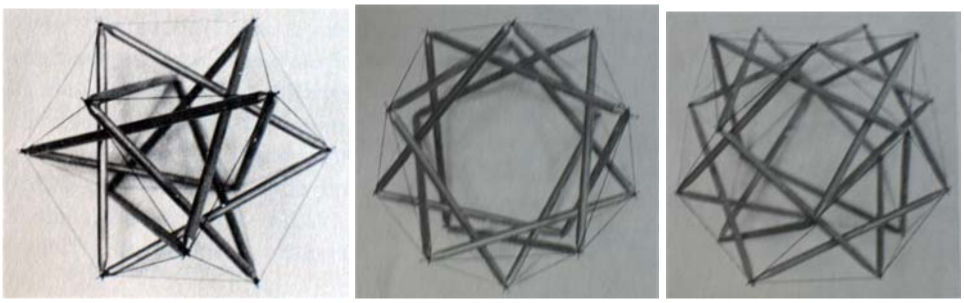


Figure 15. Three "circuit systems"

5.2 Structural composition principle

5.2.1 Basic idea and developments

When I decided to build a physical model of the fifteen-strut circuit pattern (Figure 16), I needed to use five vertical plastic “mounting” struts that I removed at the end of the process, but it became obvious that a general method, valid for many other cells could be developed, starting on a geometrical basis. It is necessary to have a geometrical description of the nodes position, and then a topological process can lead to different structural compositions according to a prescribed objective: single-circuit system, or mp-circuit system (m circuits of p struts). In the Figure 15, the second system is a mono-component system all the struts constitute a single circuit. The left hand side system comprises four 3-strut circuits, and the right hand side system comprises five 4-struts circuits.

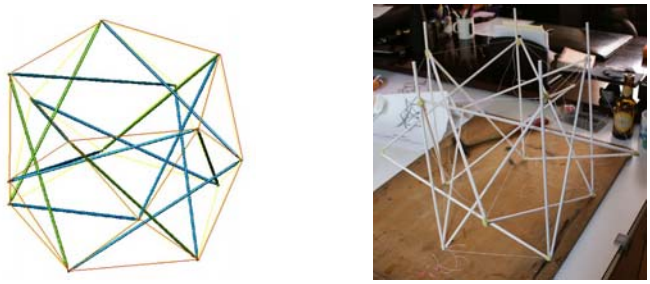
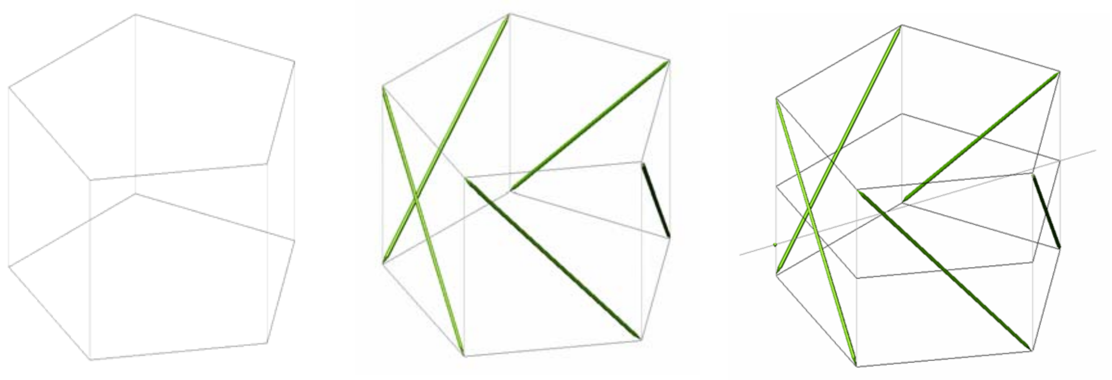


Figure 16. Module assembly

5.2.2 Fifteen-strut tensegrity ring

This idea is illustrated for the fifteen-strut circuit pattern system. The geometrical basis is a straight prism with pentagonal basis (Figure 17(a)).



(a) (b) (c)

Figure 17. First step: five lateral struts implementation

The vertical edges will be removed at the end of the process. In each of the lateral quadrangular faces one strut is implemented along a diagonal, respecting a five-order symmetry of rotation (Figure 17(b)). Additional nodes and struts are created according to the following rules: each new node lays on a bisector line of the pentagon, which is a cross section of the initial straight prism, at mid height (Figure 17(c)). Their position on this straight line can be variable, but these new nodes have to be outside of the prism. It could be chosen other geometrical positions for these nodes, but it is necessary to respect some regularity for these first cells. The resulting cell will be a regular one, with only length for the struts and one length for the cables. It is then necessary to link this new node with two others, by adding two struts.

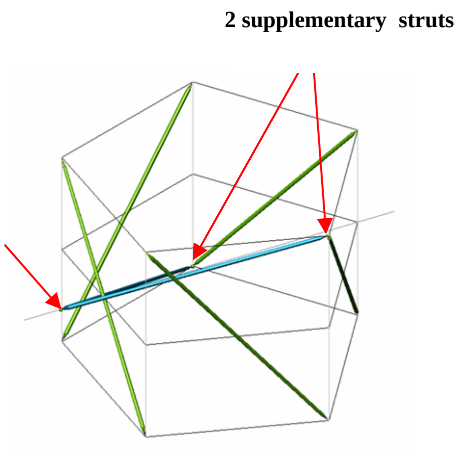


Figure 18. Addition of two supplementary struts

These struts have a common node ("**e**" on Figure 18), one of them is linked to a bottom node “**b**”, the other to a top node of the pentagonal prism “**t**”.

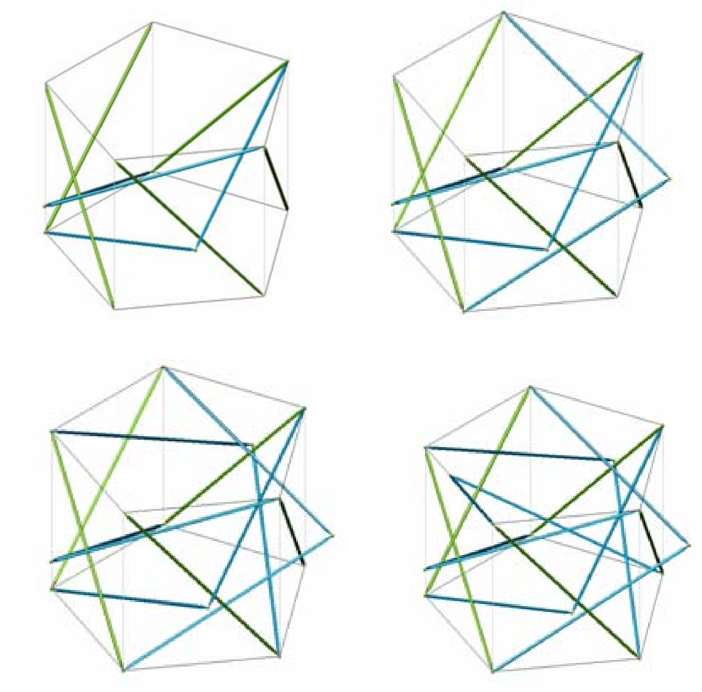


Figure 19. Addition of eight other struts

The addition of eight other struts is realized according to the same process to end up with a tensegrity cell with fifteen struts and thirty cables: five for each basis and four per external node (these cables are linked to the four angles of each lateral quadrangular face of the initial prism).

5.2.3 Tensegrity rings

Since the whole components, cables and struts are inside a hollow tube shape, these tensegrity cells are grouped under the denomination “tensegrity rings” (Figure 20).

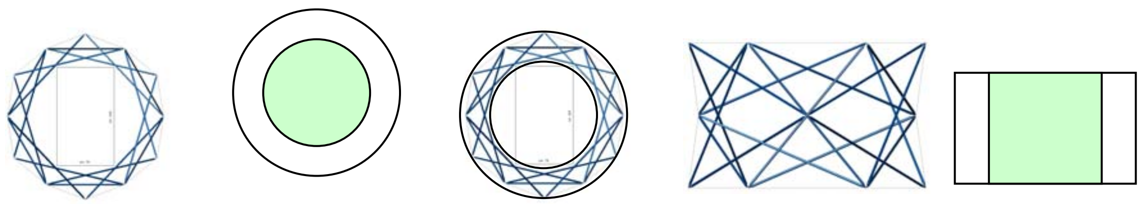


Figure 20. Tensegrity rings

It is simple to act on the geometrical parameters, namely the height “h” of the cell, the interior radius “r” and the exterior radius “R” in order to meet some criteria of architectural type. The overall geometry can also be described with the height, one of the radii and the thickness of the tube. At this stage only regular systems have been studied, but there is no doubt that other possibilities are opened in the field of irregular shapes.

5.3 Physical models

5.3.1 Context

It is always useful to build some physical models so as to check some parameters and procedures. Apart the initial plastic models, we built two sets of tensegrity rings during a first workshop at Istituto Universitario de Archittetura de Venetia (February 2006). Two geometries were experimented: hexagonal and pentagonal shapes. The size of the models is characterized by struts of one meter length.

5.3.2 Hexagonal tensegrity ring

The model presented on Figure 9 was satisfying according to the building process that we adopted with a first stage taking a straight prism as basis.

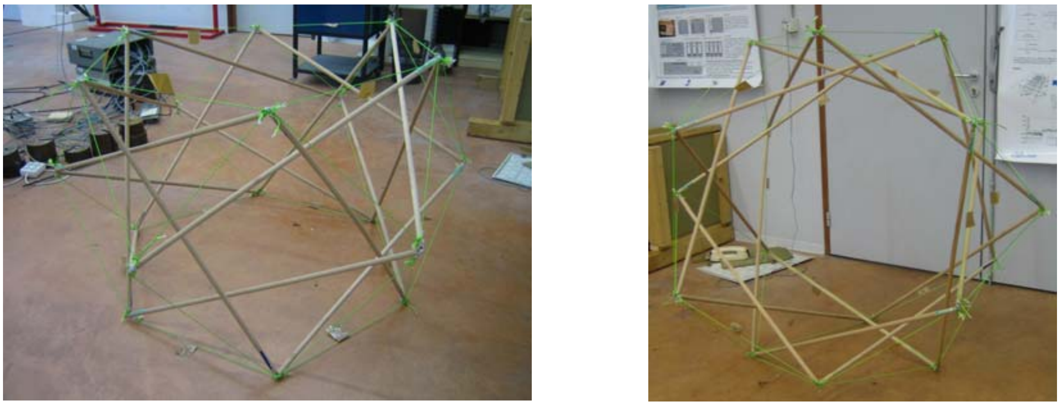
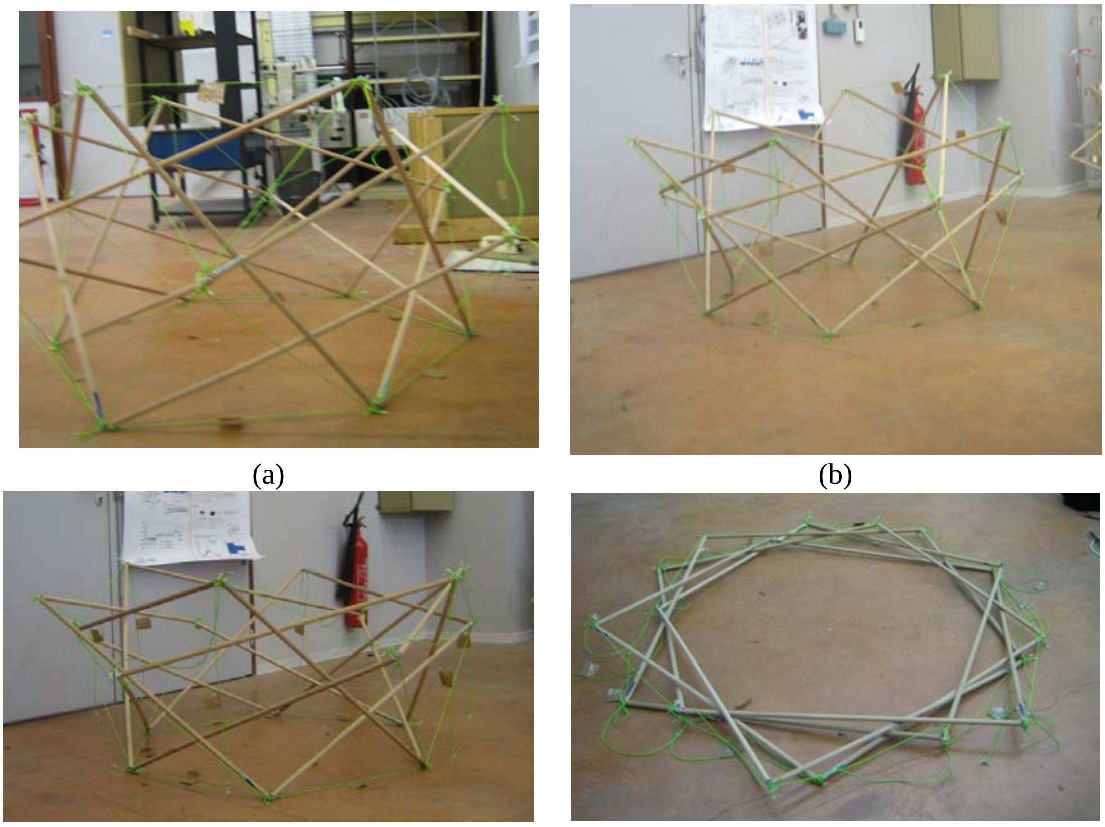


Figure 21. Hexagonal tensegrity ring

5.3.3 Foldable tensegrity ring

These models allowed us to verify a hypothesis on the possibility of folding procedures. Generally the introduction of finite mechanisms which lead to more compact systems can be realized either by struts shortening or cables lengthening. Mixed solutions may also be used.

Our hypothesis concerned the folding policy. We chose to act only on the polygonal circuits lying on the two bases. We begin (Figure 22(a)) by removing the upper polygon of cables. When the top polygon is completely removed (Figure 22(c)), the lower half part of the ring is still rigid at first order. When the lower polygonal circuit of cables is removed (Figure 22(d)), the tensegrity ring is completely flat. It will, of course, be necessary to validate this experiment with a numerical model. But it appears that two possibilities can be investigated: the first one corresponds strictly to the above description. A second one could be to act simultaneously on the two bases: in this case the whole cell would be folded on its median plane, which could be of interest for some applications



(a) (b) (c) (d)

Figure 22. Folding of an hexagonal tensegrity ring

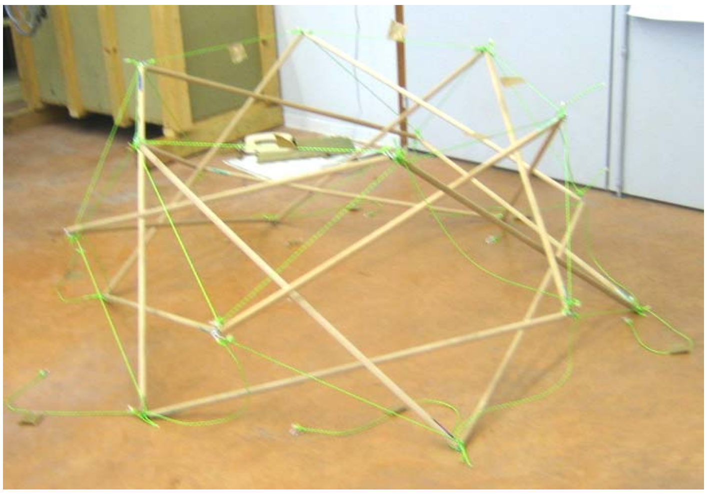


Figure 23. Unfolding a tensegrity ring

The reverse process has been tested: the Figure 23 illustrates this experiment, which begins by the top. When the top polygon of cables is put in place the half top part recovers its rigidity.

5.4 Perspectives of the "hollow rope"

This study could have been done a long time before, if we look to the book of Pugh. Perhaps some people took interest in it, but it seems a comprehensive study could be very promising since many applications can take benefice of the properties of these tensegrity rings. Several ideas are now investigated. “The hollow rope” is one of them, architectural applications seem also to interest people.

The simplest application is to add several tensegrity units by their basis creating so a kind of “hollow rope”. The units can be identical or not in terms of height. If the two bases are not parallel, new curved mean fibber are created. A spatial curve could be designed, provided some overall stability cables are added to the whole tube. Many solutions are available.

The idea of “hollow rope” was soon described with other structural compositions, which did not rely on tensegrity principle. Robert Le Ricolais, and also Maraldi developed their own solutions. Some descriptions of their projects are provided in Ref. [11].

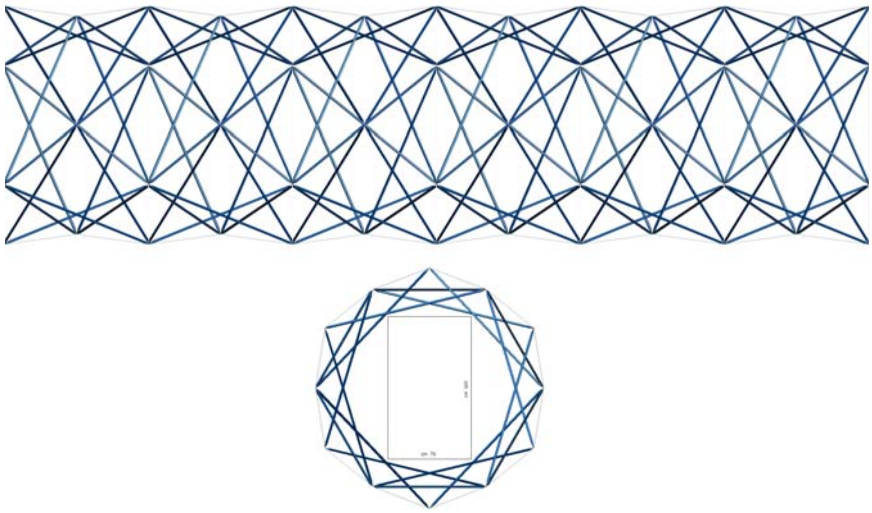


Figure 24. The hollow rope

Several parameters can be adjusted. According to the size of the global system, and to an appropriate size of tubes and cables, a pedestrian bridge could be designed on this structural composition, since the inner free space could receive the walking floor. An optimization of the involved parameters (height, inner radius, outer radius) has to be achieved, with possible addition of longitudinal stiffening cables. A pertinent utilization of irregular cells would allow to designing curve shapes.

At another scale, our studies on cytoskeleton of human cells lead us to model several components like actine filaments and microtubules, which are chains of polymers. The hollow rope would certainly model correctly these microtubules, taking into account fluids interaction.

These first studies on rings provided the roots for more intensive research, which is carried on in our laboratory. The foldabilitty of these rings is tested on more sophisticated models.

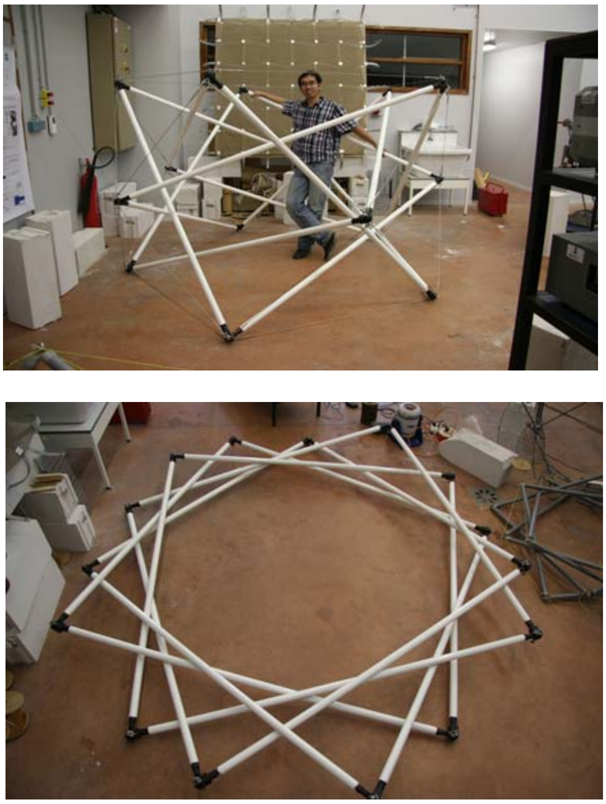


Figure 25. Physical model for a tensegrity ring

6. Conclusion

In this paper the structural morphology of tensegrity systems is presented from the simplest cell, the so-called “simplex”, to more complex ones like pentagonal and hexagonal tensegrity rings. The assembly of tensegrity rings provides interesting structural solutions like the “hollow rope”, but one of their main features is their foldability which could be the key for pertinent applications. Other assemblies like woven double layer tensegrity grids can be derived from simple cells, constituting a way from simplicity to complexity.

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