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NUMERICAL SIMULATION OF BASE-ISOLATED SYSTEMS SLIDING ON CONCAVE SURFACES

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ABSTRACT

Base isolation is an acknowledged means to reduce the transmission of earthquake accelerations from the ground into the structure. Of particular interest here are systems where the bearing slides on a concave surface which produces a self-centering force restoring the building to its original position. While the idea behind the technology is straightforward, the numerical simulation of the structural response is difficult, since the contact between the bearing and the sliding surface changes continually between sticking and sliding in a nonlinear process. The research presented here aims at developing a computational model for base-isolated building which would allow the engineer to simulate the structural response in the time domain in order to find an optimum design of both building and isolator. It has been shown that a compact 1-DOF system suffices to express the general isolated layer response during the excitation. For more general details such as structural accelerations and drift a set of coupled differential equations must be solved. Finally, through comparison of the behavior of an isolated structure with a fixed-base one, the efficiency of sliding isolation is demonstrated.

Keyword: Base isolation; numerical simulation; 1-DOF system; nonlinear process

1. INTRODUCTION

The idea of protecting buildings from damages of earthquakes through sliding isolated systems has a long history. The first documented innovative design seems to belong to J. A. Calantarients (1909), an English medical doctor, proposing construction of buildings on a layer of fine sand, mica, and talc, that would allow the building to slide in case of strong earthquakes [Naeim, 1999]. There are a large number of systems all working more and less according to this sliding mechanism uncoupling the building from their supporting ground, by using rollers, balls, and cables. Because of the practical problems such as large permanent deformations and differential settlements after earthquake, it was just in the last thirty years that base isolation got a practical functionality. The governing mechanism of all

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the above mentioned systems is the friction and sliding between different layers which are rubbed under structural weight together.

The idea of using an articulated slider sliding on a concave surface, Friction Pendulum Isolation, was first proposed by Zayas et al. (1987). The main advantage of the proposal is the recentering force produced by the geometric form of the surface, resulting in less permanent displacements of the structure after the earthquake. Despite the simplicity of the idea, because of the nonlinear behaviour of the system between sticking and sliding phases, the numerical simulation of the system is challenging. As long as the resultant force acting on the bearing is less than the static friction force, the bearing would not slide and the structure behaves the same as a fixed-base structure. When this static friction force is overcome the bearing would slide, resulting in less transmitted earthquake induced acceleration of the ground to the structure.

In this study friction is considered to be of Coulomb type, therefore the friction coefficient is considered to be constant during the analysis. Defining it as a function of contact pressure or the velocity of sliding [Constantinou et al. and Mocha et. al., 1990 & 1993], however, does not change the generalities of the presented formulation.

2. NUMERICAL SIMULATION WITH A 1-DOF SYSTEM

The simplest structural model uses only one degree of freedom (Figure 1). The structure mass is concentrated in one point, and the structural stiffness and damping are modelled by a linear spring and viscous damper. The friction properties of the base isolator are captured by the friction coefficients μ_0 and μ for sticking and sliding, respectively. This simple model aims primarily as demonstrating the phase change between sticking and sliding.

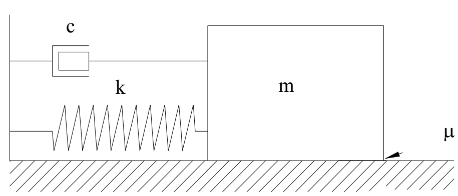


Figure 1. SDOF model

Small time steps allow a linear approximation of the ground excitations in every time increment, and the general differential equation governing such system will be [Tsai, 2005]:

$$m\ddot{u} + c\dot{u} + ku = -\mu mg\,\mathrm{sgn}(\dot{u}) - m\ddot{u}_g^{i-1} - \frac{m\ddot{u}_g^{i} - m\ddot{u}_g^{i-1}}{\Delta t}\tau \tag{1}$$

Where $u, \dot{u}, \ddot{u}$ are the relative displacement, velocity, and acceleration of the mass with respect to the ground, $\ddot{u}_g^{i-1}$ and $\ddot{u}_g^{i}$ are the absolute ground acceleration at the beginning and the end of the current time step, respectively, and Δt is the time increment. The homogeneous and non-homogeneous solutions of the second order differential equation are [Tsai, 2005]:

$$u_c(\tau) = (A\cos\omega_d\tau + B\sin\omega_d\tau)e^{-\xi\omega_n\tau} \tag{2}$$

$$u_p(\tau) = \frac{1}{\omega_n^2}\left[-\mu g\,\mathrm{sgn}(\dot{u}_b) - \ddot{u}_g^{i-1} + \frac{2\xi}{\omega_n\Delta t}(\ddot{u}_g^i - \ddot{u}_g^{i-1})\right] - \frac{(\ddot{u}_g^i - \ddot{u}_g^{i-1})}{\omega_n^2\Delta t}\tau \tag{3}$$

where ω_n and ω_d denote the undamped and damped circular eigenfrequencies and ξ the damping ratio. The two parameters A and B are calculated from the initial conditions at the beginning of the step. Note that equations (2) and (3) hold only if the mass is sliding. The mass will stop sliding when the velocity gets zero and if at the same time the resultant force acting on the mass is not enough to overcome the static friction force F_f; otherwise the mass continues sliding in the opposite direction. The sticking criterion then reads as:

$$|m\ddot{u} + ku| < |F_f| \tag{4}$$

The static friction force is then given by:

$$|F_f| = \mu_0 mg \tag{5}$$

In case of multi-degree-of-freedom models μ is time-dependant, since the vertical force acting on the sliding surface varies with time.

As first example, the response to an initial displacement is studied. All parameters are taken from [Petersen 1996]; here an initial displacement of .125m is imposed and the mass is abruptly released. The structural viscous damping has been set to zero. Since the spring force acting on the mass overcomes the static friction force, the mass begins to slide from a standstill; state (a) in Figure 2. The mass continues sliding until it reaches its maximum velocity, then it slows again until it comes to rest at state (b). The absolute value of displacement at state (b) is smaller than the initial displacement due to friction damping. At this moment the sticking condition is checked and, if satisfied, the mass remains fixed; otherwise it starts to move in the opposite direction.

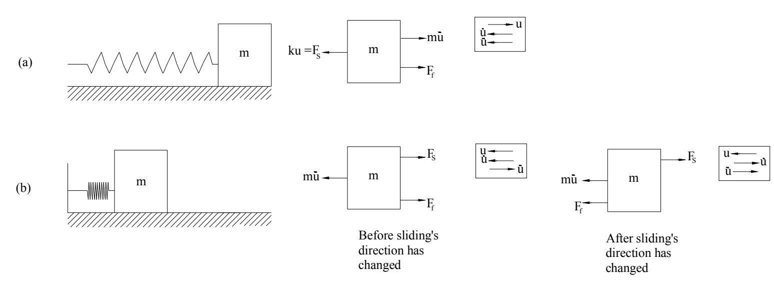


Figure 2. Mass spring system and the forces acting on the set

Time histories of displacement, velocity and acceleration of the mass are presented in Figure 3. Note that in contrast to the classical viscous damping, in case of friction-damped oscillations, the decay of amplitude is linear, and this damping has no effect on the period of oscillation (Figure 4) [Chopra, 2001]. The permanent displacement and the moment of sticking in this case is a function of the spring stiffness, the friction coefficient, and the initial displacement.

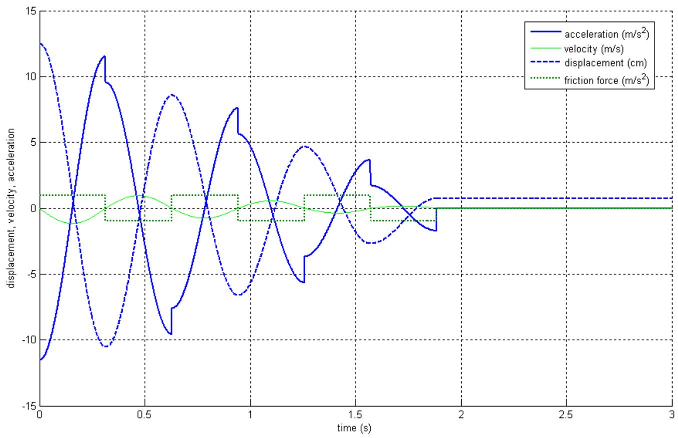


Figure 3. Structural response of the mass and friction force

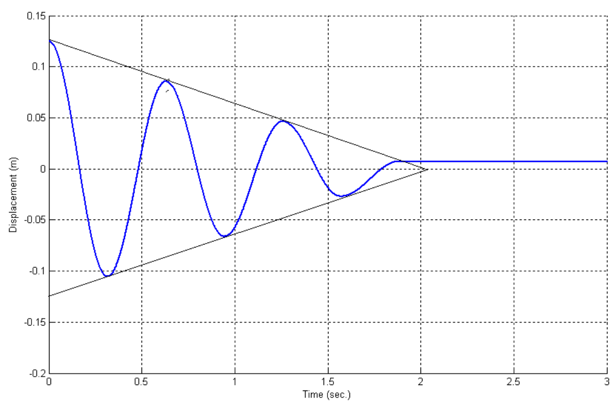


Figure 4. Linear amplitude decay due to the friction-induced damping

As second case a slider is simulated, which experiences succeeding phases of sticking and sliding under an external harmonic excitation. The set consists of a mass and a damper on a concave surface with friction. The differential equation governing the system is the same as before; only the stiffness term in equation (1) is given by a purely geometric stiffness, due to the concavity of the surface:

$$K = \frac{W}{R} \tag{6}$$

where W is the weight of the slider and R the radius of curvature of the sliding surface. The natural period T_0 of such a system is independent of the mass of the slider:

$$T_0 = 2\pi\sqrt{\frac{W}{Kg}} = 2\pi\sqrt{\frac{R}{g}} \tag{7}$$

The excitation acting on the mass is formulated as:

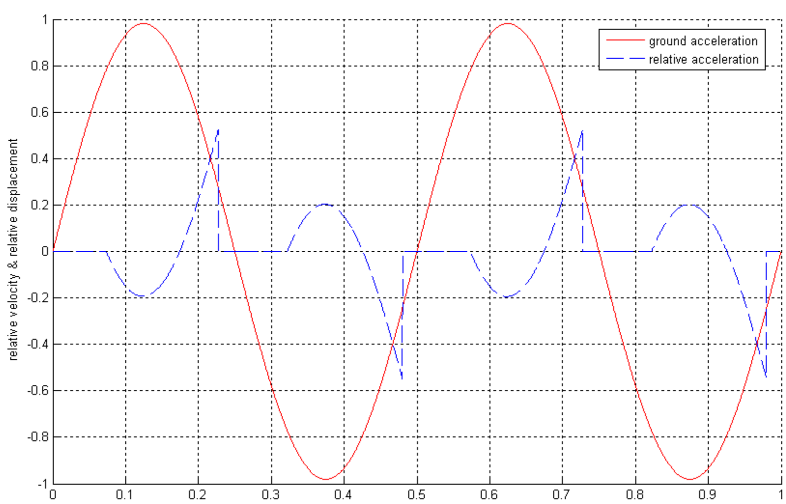
$$F(t) = 0.1g\sin(\frac{2\pi}{T}t) \tag{8}$$

Taking R = 1.0 m, T = 0.5 s, the natural period of the set will be 2.0 s. Time step in this analysis is 0.0005 s, and no adaptive time stepping (section 5) is used. Diagrams of relative displacement, velocity, and acceleration of the set are presented in Figure 5.

The slider remains sticking until the ground excitation is large enough to overcome the static friction force. From this moment the absolute acceleration of the slider is the vectorial sum of the ground acceleration and the relative acceleration of the mass to the ground. Since the relative acceleration of the mass is always in the opposite direction of the ground excitation, the absolute acceleration of the mass, and consequently the inertia force exerted to the mass, are smaller than the ground acceleration and the ground acceleration induced force. The slider continues sliding until the ground acceleration gets so small that is not enough to overcome the friction force between the sliding surfaces. As soon as the relative velocity gets zero the sticking criterion is checked, and if satisfied it sticks, until the ground acceleration gets again large enough to overcome the friction force.

Note that the total force acting on the mass is:

$$\sum F = m\ddot{u}_{absolute} = m(\ddot{u}_g + \ddot{u}) \tag{9}$$



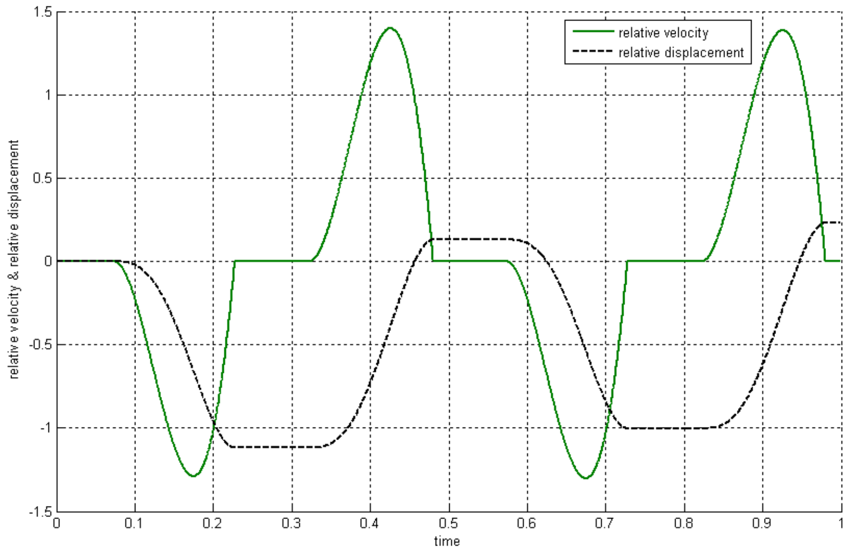


Figure 5. Slider acceleration, velocity, and displacement relative to the base under harmonic excitation

3. NUMERICAL SIMULATION WITH 2-DOF SYSTEM

Based on the same principles stated above, a two-degree-of-freedom system is implemented, which can be interpreted as a model for a typical base-isolated one story building (Figure 6). The response of such a system is governed by the following coupled differential equations [Mostaghel, 1988]:

$$m\ddot{s} + c\dot{s} + ks = -(\ddot{u} + \ddot{u}_g)m \tag{10}$$

$$\ddot{u} + \frac{C}{M+m}\dot{u} + \frac{K}{M+m}u = -\ddot{u}_g - \mu g\, sign(\dot{u}) - \alpha\ddot{s} \quad , \qquad \alpha = \frac{m}{M+m} \tag{11}$$

where:

s : structural displacement relative to the base
$\dot{s}$: structural velocity relative to the base
$\ddot{s}$: structural acceleration relative to the base
m : structural mass
c : structural damping
k : structural stiffness
u : base displacement relative to the ground
$\dot{u}$: base velocity relative to the ground
$\ddot{u}$: base acceleration relative to the ground

M : base mass
C : base damping
K : base stiffness

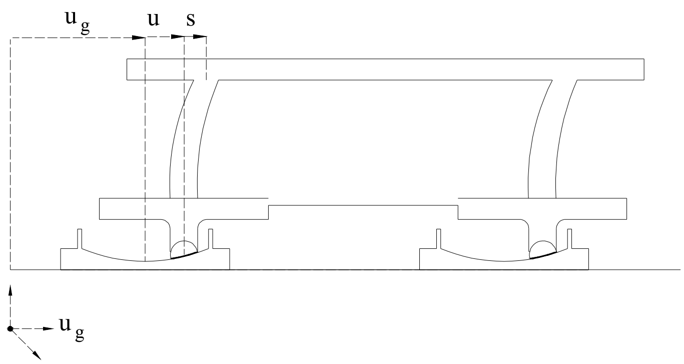


Figure 6. 2-DOF system configuration

As in the previous case, because of the very small time steps used in the analysis, it is supposed that the ground acceleration and relative structural acceleration are changing linearly during every time increment. So the ground acceleration and the relative structural acceleration within a specified time step can be computed as:

$$\ddot{u}_g(t_i+\tau)=\ddot{u}_g(t_i)+\Delta\ddot{u}_g(t_{i+1})\frac{\tau}{\Delta t} = \ddot{u}^i_g + \frac{\ddot{u}^{i+1}_g - \ddot{u}^i_g}{\Delta t}\tau \tag{12}$$

$$\ddot{s}(t_i+\tau)=\ddot{s}(t_i)+\Delta\ddot{s}(t_{i+1})\frac{\tau}{\Delta t} = \ddot{s}(t_i)+\frac{\ddot{s}(t_{i+1})-\ddot{s}(t_i)}{\Delta t}\tau \tag{13}$$

Using equations (12) and (13), the incremental relative structural acceleration can be computed in an implicit manner. Substituting it in equations (10) and (11), base and structural relative accelerations can be both computed [Mostaghel, 1988].

4. STICK-SLIP CRITERION

The system will not move as long as the resultant force acting on the mass is smaller than the static friction force. The sticking criterion is then given by:

$$\left|\ddot{u}_g+\ddot{u}+\alpha\ddot{s}+\omega^2 u\right| < \mu g \tag{14}$$

The forces acting on the sliding mass are inertia forces, including relative base acceleration, relative structural acceleration, and the absolute ground acceleration, and the self-centering force. So the absolute structural acceleration is:

$$\ddot{s}_{absolute} = \ddot{u}_g + \ddot{u} + \ddot{s} \tag{15}$$

and the absolute displacement:

$$s_{absolute} = u_g + u + s \tag{16}$$

In which u_g is the ground displacement due to the excitations. As long as the static friction force is not overcome, the base velocity and acceleration relative to the ground remain zero and the system will behave the same as a fixed base structure according to equation 17 (sticking phase):

$$m\ddot{s} + c\dot{s} + ks = -\ddot{u}_g m \tag{17}$$

5. ADAPTIVE TIME INCREMENTATION

The most prominent factor controlling the precision of the numerical simulation is the prediction of the phase change between sliding and sticking. To get good results, time steps should be exceedingly small, in the order of 10^{-5}-10^{-6} seconds. But on the other hand for practical simulations where the duration of the excitation is about 20-40 seconds, it would be extremely time-consuming and even sometimes not feasible. As an example consider an earthquake with duration of 30 seconds. To simulate it with a time increment of 10^{-5} seconds, the simulation should be done in 3000000 time steps. To overcome this problem, a model with an adaptive time increment has been developed. The strategy consists in using a refined time step only when the system changes from sticking to sliding or vice versa. In the sliding case the time increment will be refined only if the relative velocity of the slider with respect to the ground changes direction. Then the computations will be repeated from the beginning of that time step with a refined time increment until the point of zero velocity is found. In case of sticking, the sticking criterion is checked at the end of the time step, and if it is not satisfied, the time increment will be refined, so that the moment of initiation of sliding is found. In this way the computations can be done with larger time steps and only in case of a phase change, the time steps will be automatically refined. So the amount of numerical computations will be 10-100 times smaller than with the first strategy, without sacrificing the desired accuracy.

6. RESULTS

To evaluate the capabilities of the aforementioned numerical model, a two-degree-of-freedom system, consisting of base mass and structural mass, is simulated. It has been assumed that: M=700 kg, m=350 kg, K=10500 Nm^{-1}, k=87000 Nm^{-1}, R=1 m, μ=.08, ζ_b=.05, and ζ_s=.02.

The N-S component of the El-Centro earthquake (1940) is considered as input ground excitation, in which the maximum amplitude is scaled to one g (9.81 $m.s^{-2}$).

Ground excitations and the structural response are shown in Figure 7. The self-restoring effect of the concave sliding surface can be clearly seen in the history of the base displacement. Although noticeable displacements of up to 24 centimetres occur during the earthquake, the structure comes almost to its original position at the end of excitations.

To demonstrate the efficiency of the base isolation the same analysis has been undertaken with such a large friction coefficient so that no sliding occurs, corresponding to a fixed-base structure. Total structural accelerations in both cases are compared (Figure 8). Obviously the acceleration, and consequently the force acting on the set, in the isolated case (max acceleration 0.6g) are smaller than the same structure anchored to the ground (max acceleration 3g).

The same problem has also been simulated with a one-degree-of-freedom formulation to study the precision of a 1-DOF model in comparison to a 2-DOF one (Figure 9). Although the general trend and the amplitudes are similar in both cases, details are different. To design the structure itself extra degrees of freedom should be used. If however, only an estimation of the behaviour of the isolators is desired, e.g. maximum sliding displacement and velocity, a simulation of the set with a one-degree-of-freedom model seems to be sufficient.

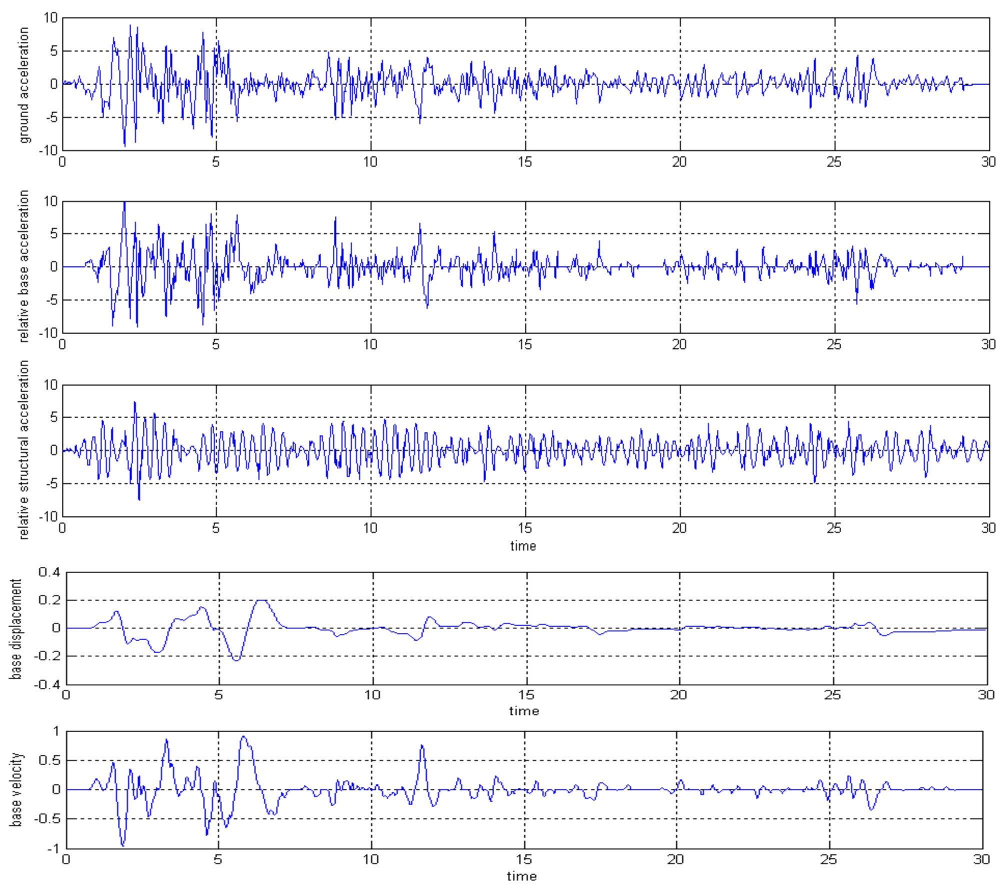


Figure 7. 2-DOF system response to El-Centro earthquake

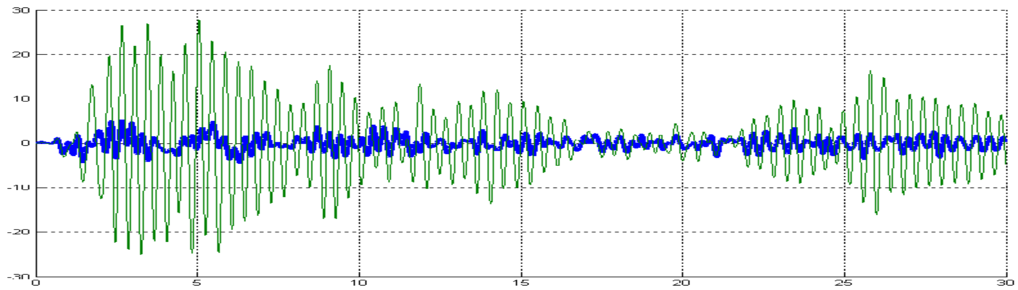


Figure 8. Total structural acceleration for fixed-base and isolated sets

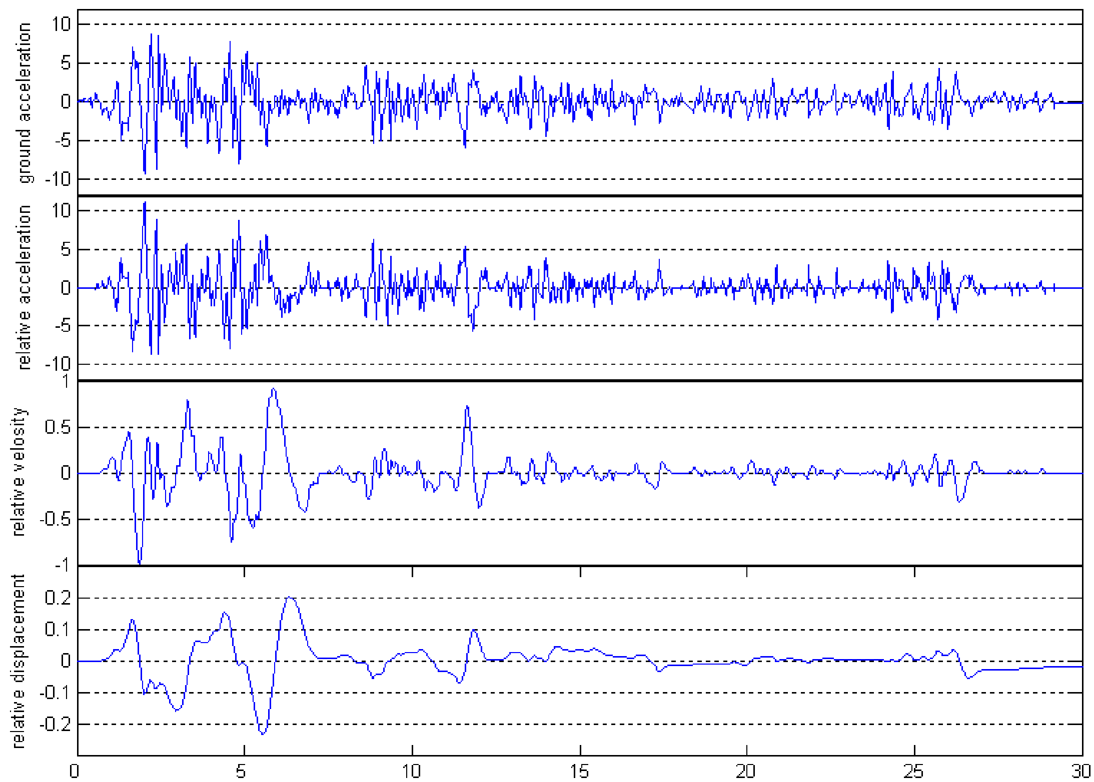


Figure 9. 1-DOF system response to El-Centro earthquake

7. CONCLUSION

A base-isolated system on a concave surface is simulated through the analytical solution of the coupled differential equations of dynamic equilibrium. As long as the resultant force acting on the slider is smaller than the static friction force between the slider and the concave plate, no sliding would happen and the model will behave exactly as a system anchored to the ground. When this static force is overcome, because of increasing ground excitation, the slider will begin to slide relative to the plate which remains fixed to the ground. Because of this relative sliding action the acceleration transmitted to the structure will be considerably decreased. The slider will continue sliding until the sliding velocity changes its direction, in which case the time steps would be refined and the calculation will be repeated from the beginning of the last time step, so that the moment of sticking, i.e. zero sliding velocity, could be exactly located. At this point it is controlled whether the static friction force is again overcome or not. If yes, the set will continue sliding in the opposite direction; otherwise it would remain sticking until the static friction force is again exceeded. It has been shown that approximating the whole set with a 1-DOF system yields the required general information for the primary design steps such as maximum sliding displacement and velocity. For more details regarding the behavior of the structure and time history response it is essential to simulate the set with more degrees of freedom.

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