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DISCOUNTED CASH FLOW TIME-COST TRADE-OFF PROBLEM OPTIMIZATION; ACO APPROACH

K. Aladini, A. Afshar* and E. Kalhor
School of Civil Engineering, Iran University of Science and Technology, Tehran, Iran

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ABSTRACT

Traditional time-cost trade-off (TCTO) analysis in construction management problems disregards time value of money. In fact, the value of money decreases with time and, therefore, discounted cash flows should be considered when solving TCTO optimization problem. As a combinatorial optimization problem one may apply heuristics and/or optimization techniques to solve time–cost trade-off problems. The combinatorial nature of the discrete TCTP, in which the solution space of the problem increases exponentially with the increment in the number of activities and/or the number of potential implementation modes, demands special solution algorithm. A multi-objective ant colony optimization (ACO) based model for project TCTO problem is developed, which minimizes project direct cost taking into account discounted cash flows. The model locates the near optimum Pareto front with a set of non-dominated solutions in which precise discrete activity time-cost function may be used. No simplifying assumptions are needed to implement the discount factor into the modeling structure and solution procedure, as required with mathematical optimization techniques. Details of model formulation are illustrated by an example project. The results show that inclusion of discounted cash flow results in different modes of construction as well as activities' durations and costs and consequently optimal project duration. The proposed approach can help the practitioners in considering net present value in time-cost decisions leading to identification of the best option.

Keywords: Optimization; discounted cash flow; time-cost tradeoff; multiobjective ant colony

1. INTRODUCTION

In any construction project, an activity may be implemented not only with different quantities of the same resource but also with various types of resources. Selection of appropriate resources, including crew sizes, equipment, methods, and technologies to perform a project are challenging decisions to be made by construction planners. These

*E-mail address of the corresponding author: a_afshar@iust.ac.ir (A. Afshar)

decisions ultimately affect the duration and cost of the project.

Major construction projects often involve numerous activities, therefore evaluation of all possible combinations within a short period of time and a reasonable cost may not be feasible. As an example, the total number of alternative combinations of time and cost for a project with only 18 activities and 5 options for each activity has been estimated as 3.81×10^{12} cases. The implementation modes of activities and their time and cost parameters in a construction project have been identified as major factors of the decision making process. Since cost can be expressed as a function of time, it is possible to determine the project time-cost trade-off (TCTO) curve which provides the minimum possible cost of completing project in its feasible time range.

The total cost of a project consists of direct and indirect costs. Direct costs are incurred because of the performance of project activities, while indirect costs include those items that are not directly related to individual project activities. In general, indirect cost increases as the project duration increases and usually assumed as a percentage of project direct cost.

In traditional TCTO analysis, the value of money is assumed to remain constant whatever the project time span is. In other words, costs of activities are summed up to form project direct cost although activities are executed at different times as their start and finish times are scheduled. However, money has a time value and therefore, it is important to consider the time at which costs are incurred. For projects with longer duration consideration of time value of money becomes more important. Among many economical measures the net present value is most commonly used one. Most of existing techniques for solving TCTO problem do not consider time value of money.

Discounted cash flow (DCF) analysis of TCTO problem is of great importance for both owners and contactors. Generally speaking, for a fixed value of total bid, it is preferable for owners to select the bid with the minimum net present value. On the other hand, contactors prefer deciding on the tender with the minimum value to bid while maximizing the net present value as in unbalanced bidding strategy.

In brief, a TCTP is a combinatorial problem which involves finding implementation modes for activities such that the optimal balances between project time and cost are found [1], Eshtehardian et al.). Early attempts to solve TCTP were mathematical and heuristic-based approaches (Feng et al., [2]). Despite their ability to produce optimal and near optimal results for TCTP with linear time-cost relationship, such solutions have revealed a main weakness in solving discrete TCTP. It has been claimed that any exact solution algorithm for discrete time-cost trade-off problem would very likely exhibit an exponential worst case complexity (De et al., [3]). This is mainly due to the combinatorial nature of the discrete TCTP in which the solution space of the problem increases exponentially with the increment in the number of activities and/or the number of potential implementation modes.

The variety of assumed time-cost functions for activities has led the problem to different solution approaches (Yang [4]). These include linear programming, integer programming, a hybrid of linear and integer programming, and dynamic programming. Recently, artificial intelligence (AI) techniques such as genetic algorithms (GAs), ant colony optimization (ACO), and particle swarm optimization (PSO) are introduced to overcome the problems associated with (1) large number of variables and constraints; (2) nonlinearity of time-cost functions; and (3) multi-objective optimization.

Meta-heuristic and evolutionary algorithms have shown relatively higher efficiency in handling these problems. Although they do not necessarily guarantee the global optimal solutions, their ability to search the solutions space intelligently, rather than completely, makes them capable of producing relatively good solutions to large-sized problems. Among them algorithms, the genetic algorithms (GAs) and ant colony algorithm (ACO) have received more attention. In recent works, Feng et al. [2], Li et al., [5] and Hegazy [6] adopted GAs for Time- cost optimization problem. Employing modified adaptive weight approach (MAWA) introduced by Zheng et al. [7], Xiong and Kuang [8] presented an ACO model for solving TCTP. They showed that the proposed ACO results in better solutions compared to GAs. In a similar work, Ng and Zhang [9] adopted ACO to solve TCTP using a MAWA approach in order to integrate optimization of time and cost of the project into a weighted objective function. Their model showed higher efficiency over its GA counterpart. Recently, Afshar et al. [10, 11] proposed a multi-objective version of an ACO algorithm to solve TCTP. Their model showed satisfactory performance in locating the Pareto front with non-dominated solutions. Ammar [12] points out that the heuristic methods may not guarantee optimal solutions, hence proposes a nonlinear mathematical optimization model for project TCTO problem which minimizes project direct cost and takes into account discounted cash flows. Discussing the complications in handling Discount factor in the exponential form in a mathematical optimization model, he uses a simplified form of DCF which introduces an approximation into the model.

This paper extends the traditional TCTO problem to account for time value of money. Realizing the drawback of the commonly used mixed integer nonlinear programming algorithms to solve the general form of discounted cash flow TCTO problem, a general modeling structure and solution methodology is introduced. Previously tested "Non-dominated Archiving Ant Colony Optimization" (NA-ACO) algorithm is employed to solve the TCTO problem with DCF. The so called NA-ACO has the merit of considering time and cost of the project in a multi-objective sense rather than integrating them into a weighted objective function.

The subject should be of interest both for researchers and industry practitioners in academic and/or real field problems. Discounted cash flow (DCF) analysis of TCTO problem is of interest for both owners and contactors. Developing highly efficient and robust algorithms to solve highly complex time–cost-tradeoff problems is still a challenging subject for researchers. Practitioners are also willing to have a reliable trade-off function between the time and cost with discounted cash flow option available for performing the project.

2. PROBLEM STATEMENT AND FORMULATION

Consider a project with n activities, where utility data for project activities are represented by discrete functions. Each discrete point represents a given mode to implement the activity. The normal and crash discrete points represent the two extremes of the activity time-cost function. Obviously, for an activity with only one discrete point, the normal and crash points coincide.

The primary information obtained from traditional scheduling are basically activities' early and late start and finish times. The best mode to carry out the activities, as well as their durations and corresponding costs, are selected optimally by the TCTP model from their utility data to optimize the objective function and satisfy the imposed constraints.

In its general form, the objective function is usually set to minimize the project cost. Assuming that indirect cost increases linearly with project duration, project direct cost only needs to be minimized. The project discounted cash flow direct cost is the summation of all activities' present value costs expressed mathematically by (Ammar [12]):

$$Minimize : discounted \;\; PDC = \sum_{i=1}^{n} \sum_{j=1}^{m_i} (1+r)^{-SF_i} c_{ij} x_{ij} \tag{1}$$

In which m_i= available construction modes for activity i (discrete points in cost function), x_{ij}=zero-one variable belongs to the discrete point number j for activity I , c_{ij}= cost of activity i under mode j(discrete point number j), r=rate of interest, SF_i= scheduled finish time for activity i, *and* $(1+r)^{-SF_i}$ is discount factor expressed in terms of interest rate (r). The zero-one variables are introduced to ensure that only one construction method (discrete point) may be selected per activity. The objective function is subject to the following constraints:

$$\sum_{j=1}^{m_i} x_{ij} = 1, \;\; i = 1,2,...,n \tag{2}$$

$$SF_i - SF_p - \sum_{i=1}^{m_i} d_{ij} x_{ij} \geq 0 \quad p = 1,2,...,NP_i \tag{3}$$

$$SF_k \leq \lambda, \;\; k = 1,2,...,NE \tag{4}$$

The constraint number (2) is included to force a single construction method per activity at a time for an activity i. The number of zero-one variables needed is the sum of discrete points for all activities (number of available construction modes for all activities), while required number of zero-one constraints equals number of project activities, n. Network Logic Constraints are defined by Eq. (3), in which the logical relationship between any two consecutive activities i and its immediate predecessor, p, is expressed mathematically. Constraint number (4) defines the Project completion is time which is controlled by the latest finish time of ending activities. If the number of ending activities is denoted by NE, the project completion constraint(s) is given by Eq. (4) in which λ is the desired project duration. To develop the set of non-dominated solutions one has to vary the project duration and run the proposed mixed integer NLP model accordingly.

Generally speaking, TCTP is a special case of multi-objective optimization problem with two objectives which mainly focuses on selecting options with corresponding time and cost to complete an activity so as to simultaneously minimize the time and total cost in large-scale or complex networks. An efficient approach to a TCTP requires a multi-objective optimization algorithm to allow for greater freedom in exploring possible solutions to reduce the likelihood of being trapped in local optima (Knowles et al., [13]). The proposed approach in this study utilizes a multi-objective optimization approach which not only could provide the satisfactory solution, but also determine the non-dominated set that is beneficial for the further decision-making process. In its multi-objective form a TCTP may be presented as:

$$\textit{Minimize}\ \mathrm{T} = \mathrm{Min}\left[\underset{L_r \in L}{\mathrm{Max}}\left(\sum_{i \in L_r} t_i^{(k)} x_i^{(k)}\right)\right] \tag{5}$$

$$\textit{Minimize PDC} = \sum_{i \in A}(1+r)^{-SF_i}\ dc_i^{(k)} x_i^{(k)} \tag{6}$$

$$\textit{ST:} \qquad \sum_{k=1} x_i^{(k)} = 1 \tag{7}$$

where $t_i^{(k)}$ represents the duration of activity i when employing the *kth* mode of construction; and $x_i^{(k)}$ is the zero-one variable for activity *i* when employing the *kth* mode of construction (=1, if activity i is constructed with option k ; zero otherwise). The activity sequence on the r^{th} r^{th} path is demonstrated by L_r={*ilr, i2r, ..., inr*} where *ijr* represents the sequence number of activity i on the rth path. The set of all paths of a network is presented by L={L_r | r=1,2,..., m}, in which m symbolizes the total number paths in the network; $dc_i^{(k)}$= *direct* cost of activity i under the k^{th} option; ic = indirect cost (daily cost); and A= set of activities in the network. As defined by Afshar et al. [10], equation 5 may be presented in the form of Eqs. (8 and 9):

$$\textit{Minimize T(project Duration)} = \max_{\forall p \in path} [T_1, T_2, ..., T_p, ..., T_P] \tag{8}$$

In which, all paths of a network are presented by p ({p | p = 1,2,..., p}) ; P symbolizes the number of all paths of the network. Each path connects the start activity to the final activity passing a series of directly connected activities. T_p is the total duration of path p and can be shown as:

$$T_P = \sum_{i=1}^{n_p} t_{ij} \tag{9}$$

Where n_p represents the number of activities on path p and t_{ij} denotes the duration of activity i when implemented in mode j .

3. NON-DOMINATED ARCHIVING ANT COLONY OPTIMIZATION ALGORITHM

First proposed by Dorigo [14], Ant Colony Optimization (ACO) applies several generations of artificial ants to search for good solutions of an optimization problem. Good solutions are the product of the ants' cooperative communication. Ants make a solution by adding beneficially defined solution components to partial solution under construction. Consequently, ACO is a suitable procedure for problems of combinatorial type.

In order to optimize a combinatorial problem via ACO, a graph should be defined as to present potential alternatives at each decision space (i.e. activities) in the way that the entire solution space is comprehensively covered. The main body of ACO is comprised of tour construction, and update of pheromone trails.

First proposed by Afshar et al. [10, 11], Non-dominated Archiving Ant Colony Optimization (NA-ACO) is a version of multi-objective ant colony based algorithm. Generally, when applying ACO to an optimization problem, a graph should represent the problem so that the ants can travel between nodes available at each decision point (i.e. Decision variables). In this paper, the decision variables are mode of construction for each activity which must be selected from a given list of feasible modes. This paper intends to minimize the project duration and total discounted cost simultaneously. To do it, combinations of modes of construction and timing for different activities must be evaluated for two nominated objectives.

Associate with each objective, a colony of ant searches for optimum solutions. In the proposed model, one colony attempts to minimize the duration time and the other concerns minimization of total discounted cost of the project. Hereafter, these colonies are called "Tcolony" and "Ccolony", respectively. Adapted from Afshar et al [10, 11], the structure of NA-ACO for time-cost optimization can be briefly addressed in following 5 steps;

Step 1: Each agent of "Tcolony" constructs a path from first activity to the final activity passing one and only one node at each decision stage (i.e. activity). Each agent may select any mode of construction at each decision point. The solutions found by this colony are transferred to the "Ccolony" for evaluation and pheromone updating.

Step 2: In the "Ccolony", the discounted project costs resulted from imported solutions are evaluated, and the best solution is marked. The complete path for the best solution receives pheromone, and pheromone on all edges is updated as defined by Eq.9:

$$\tau_{ij}^{k} \leftarrow \tau_{ij}^{k-1} \times (1-\rho) + \Delta\tau_{ij}^{k} \tag{9}$$

Where ρ∈(0,1] is the evaporation rate, and τ_{ij}^{k} is the pheromone concentration on edge *ij* at K^{th} iteration. To reduce the pressure of past generations, the pheromone on all the edges is partially evaporated. The increment in pheromone conncentration on edges associated with the best solution found at K^{th} iteration is defined as $\Delta\tau_{ij}^{k}$; which is mathematically shown as:

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{Q_k}{f^k(B)} & \text{if path ij is travelled by } k^{th} \text{ iteration's best ant} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Where Q_k is a constant, and $f^k(B)$ is the best fitness value found in k^{th} iteration.

Step 3: In the "Ccolony", agents select new edges using pheromone information. In fact, ants select each edge using a probability function. Standing at i^{th} option of S^{th} activity, the probability of choosing an option for $(s+1)^{\text{th}}$ activity is represented by P_{ij} where:

$$P_{ij}^{n} = \begin{cases} \dfrac{\tau_{ij}^{\alpha} \times \eta_{ij}^{\beta}}{\sum_{j} \tau_{ij}^{\alpha} \times \eta_{ij}^{\beta}} & \textit{if j is an allowable start time} \\ 0 & \textit{otherwise} \end{cases} \tag{11}$$

Where η_{ij} is a heuristic value, α and β are parameters which induce the planners view towards the importance of selection using pheromone or heuristic value. j is considered allowable start only if starting activity (s+1) at j does not violate either network logic or other constraints. The solutions produced in the "Ccolony" are sent to "Tcolony" for evaluation and pheromone updating. This reciprocal exchange is called a cycle-iteration.

Step 4: At the end of any cycle iteration, all the solutions found are sent to an offline archive. In the archive, solutions are evaluated based on both objectives and non-dominated solutions are found. In a minimization problem, a vector $x^{(1)}$ is partially less than another vector $x^{(2)}$, ($x^{(1)} < x^{(2)}$) when no value of $x^{(2)}$ is less than $x^{(1)}$ and at least one value of $x^{(2)}$ is strictly greater than $x^{(1)}$. A solution the vector of which is partially less is a dominated solution and a solution which cannot be dominated throughout an existing solution set is called a non-dominated solution or a Pareto optima. A set of all non-dominated solutions found within a solution set is called Pareto front.

Step 5: Next, the pheromone values on all edges are re-initiated to τ_o and pheromone trails are updated according to the non-dominated solutions in the archive. The total process continues until the stopping criterion is met.

Once again the pheromone matrix is updated using the non-dominated solutions in the offline archive, and edges are explored by the "Tcolony".

4. MODEL APPLICATION

In order to illustrate the concept and performance of the proposed algorithm, an 18-activity network configuration is used as presented in Figure 1. The number of available options for each activity along with the cost and time for each mode of construction are given in Table 1. For this simple example, there are more than 3.6 billion possible combinations of construction modes for delivering the entire project (El-Rayes and Kandil [15]).

The previously described 18 activity problem is solved and without indirect cost. The number of ants, cycle iteration, and total iteration are set to 100, 30, and 200, respectively. Disregarding the indirect cost Figure 2 shows the Pareto front with the sets of non-dominated solutions, both for total direct cost and discounted cash flow cost with different interest rates. As it was expected, DCF analysis has significant impact on project direct cost and may change the optimum mode of construction and their sequencing. It is interesting to note that very many non- dominated solutions are identified when the total project duration changes from 120 to 169 weeks for which the total discounted cost ranges from \$51204 to \$49529.

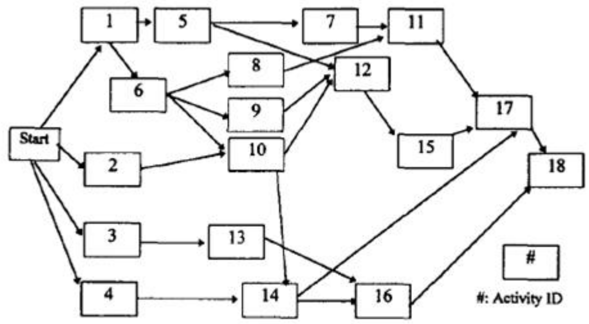

Figure 1. Network of test problem

In other words, one may speed up (or slow down) the project implementation by 49 weeks with a minor increase (or decrease) in the total discounted cost (i.e. \$51204− \$49529= \$1675). On the other hand, for duration of 110 weeks, there is a discountinuity in the Pareto front indicating that one week increase (or decrease) in project duration will significantly decrease (or increase) the total discounted cost. As an example, for interest rate of 9% the change in discounted cost will be in the range of \$8000 (\$68000 to \$60000). In order to have some detailed information about the generated Pareto solutions corresponding activities of generated solutions for interest rate of 10%, 8% and 0% have been selected from the archive and shown in Table 2. When the discounted cash flow is of interest, the solutions are sensitive to the nominated options in a few more activities compared to the case with non –discounted total cost (Table 2). As illustrated, modes of construction for activities number 1 and 17 have changed from 1 and 3 to 5 and 1, from total cost case and discounted cost case, respectively. In other words, when discounted cash flow is considered, the options with less costs and higher durations are more preferred at the early stages of the project implementation, whereas, the options with high costs and lower durations are of more interest in later stages. This is well illustrated for activities number 1 and 17 for which construction options number 5 and 1 are selected, respectively.

When time value of money is disregarded (r=0%), the optimal project duration (corresponding to minimum total cost with weekly indirect cost of \$1000) is 110 weeks. At a value of 10% interest rate, the corresponding optimal project duration is 117 weeks. These results are depicted graphically in Figure 3. It is clearly apparent from Figure 3 that there is a distinct value for optimal project duration if time value of money is ignored.

From the results obtained, it is obvious that ignoring time value of money in the analysis of TCTO problem can produce misleading decisions. The selected duration for activities and consequently the optimum project duration depend on interest rate value chosen and indirect cost rates. For the example project on hand, the optimum project duration for DCF exceeds that of ignoring time value of money. This may not be the case for projects with different characteristics. One should therefore be careful in deciding the values of interest rate and indirect cost values.

Table 1: Details of the case example (adapted from Eshterardian et al. [1])

Activity	Option	Time	Cost	Activity	Ooption	Time	Cost
	1	14	2400		1	15	450
	2	15	2150	10	2	22	400
1	3	16	1900		3	33	320
	4	21	1500		1	12	450
	5	24	1200	11	2	16	350
	1	15	3000		3	20	300
	2	18	2400		1	22	2000
2	3	20	1800	12	2	24	1750
	4	23	1500		3	28	1500
	5	25	1000		4	30	1000
	1	15	4500		1	14	4000
3	2	22	4000	13	2	18	3200
	3	33	3200		3	24	1800
	1	12	45000		1	9	3000
4	2	16	35000	14	2	15	2400
	3	20	30000		3	18	2200
	1	22	20000	15	1	12	4500
5	2	24	17500		2	16	3500
	3	28	15000		1	20	3000
	4	30	10000		2	22	2000
	1	14	40000		3	24	1750
6	2	18	32000	16	4	28	1500
	3	24	18000		5	30	1000
	1	9	30000		1	14	4000
7	2	15	24000	17	2	18	3200
	3	18	22000		3	24	1800
	1	14	220		1	9	3000
	2	15	215	18	2	15	2400
8	3	16	200		3	18	2200
	4	21	208				
	5	24	120				
	1	21	208				
	2	24	120				
9	3	15	300				
	4	18	240				
	5	20	180				

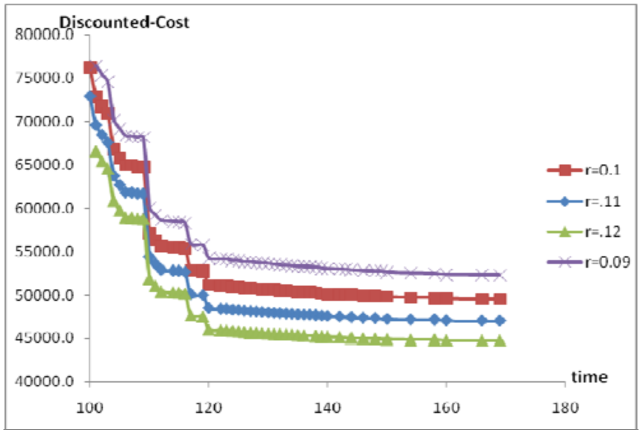

Figure 2. Non-dominated solutions for different interest rates for discounted cash flow case

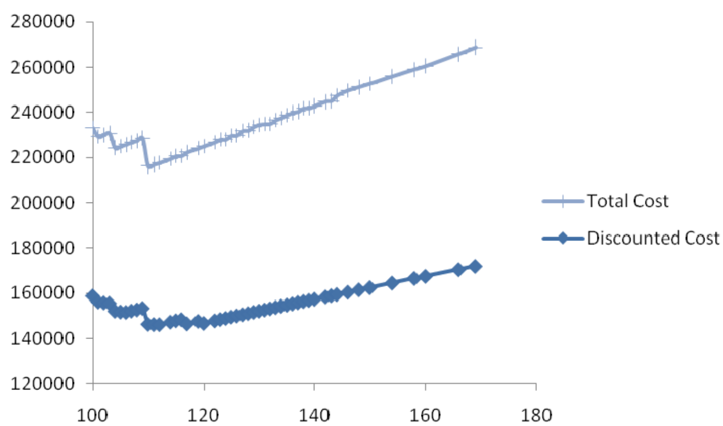

Figure 3. Total cost and discounted cash flow cost with $1000 weekly indirect cost

Table: 2 Modes of constructions for selected decision ponds from non-dominated solutions

				Activity number																	
T	**r %**	**TC**	**DC**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
124	0	127870	-	1	5	3	3	4	3	3	5	1	1	3	1	3	3	2	5	3	1
125	0	127820	-	2	5	3	3	4	3	3	5	1	1	3	1	3	3	2	5	3	1
126	0	127770	-	3	5	3	3	4	3	3	5	1	1	3	1	3	3	2	5	3	1
124	8	104070	57261	5	5	3	3	4	3	3	5	1	1	3	1	3	3	2	5	1	1
126	8	103820	57105	5	5	3	3	4	3	3	5	1	1	3	2	3	3	2	5	1	1
124	10	104070	45817	5	5	3	3	4	3	3	5	1	1	3	1	3	3	2	5	1	1
125	10	104760	45786	5	5	3	3	4	3	3	5	2	1	3	2	3	3	1	5	1	1
126	10	103820	45719	5	5	3	3	4	3	3	5	1	1	3	2	3	3	2	5	1	1

5. CONCLUSIONS

Most of the time-cost trade-off (TCTO) analysis in construction management has disregarded time value of money. In this study a multi-objective ant colony optimization based model for TCTO problem was developed and tested which minimizes the discounted cash flow of direct cost. The general form of discount factor was embedded into the modeling structure without any simplifying assumptions. The model accepts the precise discrete activity time-cost relationship, effectively generates non-dominated set of solutions, and accounts for time value of money. Although the model is formulated to account for DCFs in the deterministic environment, effect of inflation and stochastic nature of the problem can be incorporated in further studies. Application of the model showed that inclusion of discounted cash flow would result in different modes of construction as well as activities' durations and costs and consequently optimal project duration. From the results obtained, it was obvious that ignoring time value of money in the analysis of TCTO problem could produce misleading decisions. The proposed approach can help the practitioners in considering net present value in time-cost decisions leading to identification of the best option. It can be concluded that DCF should be considered in the analysis of TCTO problem, especially for projects span over time periods more than 1 year. TCTO analysis with DCF produces realistic results and consequently sound decisions

REFERENCES

1. Eshtehardian E, Afshar A, and Abbasnia R. Fuzzy-based MOGA approach to stochastic time-cost trade-off problem, *Automation in Construction,* **18**(2009) 692–701.
2. Feng CW, Liu L, and Burns SA. Using Genetic algorithm to solve Construction Time-Cost Trade-Off problems, *Journal of Computing in Civil Engineering*, **11**(1997) 184–9.
3. De P, Dunne EJ, and Wells CE. The discrete time-cost trade-off problem revisited, Europian, *Journal of Operational Research*, **81**(1995) 225–38.
4. Yang, I. Using elitist particle swarm optimization to facilitate Bi-criterion time-cost trade-off analysis, *Journal of Construction Engineering and Management* No. 7, **133**(2007) 498–505.
5. Li H, Cao JN, and Love PE. Using machine learning and GA to solve time-cost trade-off problems, *Journal of Construction Engineering and Management*, **125** (1999) 347–53.
6. Hegazy T. Optimization of time-cost trade-off analysis: using genetic algorithms*, Canadian Journal of Civil Engineering*, **26**(1999) 685–97.
7. Zheng DXM, Ng ST, Kumaraswamy MM. Applying a genetic algorithm-based multi-objective approach for time–cost optimization, *Journal of Construction Engineering and Management*. **130**(2004) 168–76.
8. Xiong Y, and Kuang Y. Applying an ant colony optimization algorithm-based multi-objective approach for time-cost trade-off, *Journal of Construction Engineering and Management,* **134**(2008) 153–6.
9. Neg ST, and Zhang Y. Optimizing construction time and cost using ant colony optimization approach, *Journal of Construction Engineering and Management*, No. 4,

134(2008) 721–8.

10. Afshar A, Ziaraty AK, Kaveh A, and Sharifi F. Non-dominated archiving multi-colony ant algorithm in time-cost trade-off optimization, *Journal of Construction Engineering and Management*. **135**(2009) 668–74.
11. Afshar A, Kaveh A, and Shoghli OR. Multi-objective optimization of time-cost-quality using multi-colony ant algorithm, *Asian Journal of Civil Engineering (Building and Housing)*, No. 2, **8**(2007) 149–60.
12. Ammar, Mohammad A. Optimization of project time-cost trade-off problem with discounted cash flows, *Journal of Construction Engineering and Management*, No. 1, **137**(2010) 65–71.
13. Knowles JD, Watson, RA, and Corne DW. Reducing local optima in single-objective problems by multi objection, in Zitzler, E., Deb, K. and Thiele, L. (eds), Lecture Notes in Computer Science, Vol. 1993 (2001), Springer, Berlin, pp. 269–83.
14. M. Dorigo, Ant System: Optimization by a colony of cooperating agents, *IEEE Transaction of System Management, Cybernetics*, No. 1, **26**(1996) 29–41.
15. El-Rayes K, Kandil A. Time-cost-quality trade-off analysis for highway construction, *Journal of Construction Engineering and Management*, No. 4, **131**(2005) 477–86.