

OPTIMUM DESIGN OF PLATE STRUCTURES USING BINARY PARTICLE SWARM OPTIMIZATION

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ABSTRACT

Particle swarm optimization is an efficient population based algorithm used to solve real valued and nonlinear continuous optimization problems. To resolve binary optimization problems with PSO, binary particle swarm optimization (BPSO) has been developed. In this paper, optimum design of plates using BPSO is presented. The objective function aims at finding the optimum weight of plates with the nodal displacements selected as constraints. Numerical examples show that BPSO can be a suitable algorithm to solve optimization problems in binary search space.

Keywords: Optimum design; plate structures; particle swarm optimization

1. INTRODUCTION

In today world, optimization as a combination of mathematics and economic issues is used extensively in various branches of science such as engineering, natural sciences, chemistry and etc. Human activities always are performed in a way to save energy and to reach the maximum output or profit by using the limited available resources. Optimization simplifies achieving the best result, while certain constraints are fulfilled.

Among different optimization problems, structural optimization forms one of the most important fields of the optimization. Structural optimization includes size, geometry and topology optimizations [1]. Size optimization involves determining the physical dimensions of structural elements (e.g. thickness, width, or other properties of elements). Geometry optimization determines the optimum location of the joints in the structure in addition to the size of members. Topology optimization determines the best arrangement of a given amount of material in the range of design, by repeated removal and rebuilding them so that maximum performance of structure is achieved [2].

Topology optimization method allows for the introduction or removal of boundaries in designs through additions or deletions of material within the design domain. This results in

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improved performance, and may lead to entirely new design options during product development [3]. In this case, the design variables represent the shape of the body and by a number of constraints, such as stresses or displacements, are limited [4].

This paper focuses on the topology optimization of plates. There are many researches on the optimum design of plates. Rao and Hinton [5] studied the structural optimization of shells and folded plates using two-node Mindlin–Reissner finite strips. Belblidia et al. [6] expressed the topology optimization method for Mindlin–Reissner plates. They determined models with single or three layered material using re-size method based on Bendsoe and Kikuchi [7]. The aim is to create a single or three layered plates for a given volume by distributing material on the plate. Vigdergauzin [8] considered perforated plate in which identical traction-free holes form square or hexagonal periodic arrays. Analysis of floating circular plates has been performed by Andrianov and Hermans [9] while Damaren [10] has studied the rectangular case. Damaren [11] investigated problem of a thin floating plate to maximize radiation damping. Jiang et al. [12] suggested an uncertain optimization method to optimize orientations of a composite laminated plate with uncertain nonlinear material properties to obtain the maximum stiffness in the thickness direction.

Contrary to above mentioned studies, using meta-heuristics for optimizing the plate structures is limited. Wang and Tai [13] and Balamurugan et al. [14] utilized genetic algorithm in structural topology optimization. Ant colony optimization and differential evolution are applied by Kaveh et al. [15] and Wu and Tseng [3] for topology optimization, respectively. Particle swarm optimization (PSO) as another powerful meta-heuristic is considered in this paper for determining the optimum design of plate structures. The remaining of the paper is as follows:

Section 2 presents the particle swarm optimization and its binary version utilized in this paper. Statement of the plate optimization is presented in section 3. Section 4 investigates two benchmark design examples using PSO. In section 5, the conclusion of the paper is presented.

2. PARTICLE SWARM OPTIMIZATION METHOD

2.1 Basic concept of particle swarm optimization

Over the past decade, using intelligent algorithms in the design of advanced, intelligence and nature information systems has been used. Among the most popular methods inspired by nature, while optimizing data and information in the field is performed, one can mention the genetic algorithms, ant colony, charged system search [16] and particle swarm optimization [17].

Particle swarm optimization method is a technique based on population that is developed for nonlinear continuous optimization problems. Population of PSO is called swarm and each member in the population of PSO is called particle. PSO is inspired by social behavior of birds flocking or fish schooling. The scientists found that the individuals of a swarm try to fly with an optimal distance between each other, and therefore they can save their energy. Thus speed plays an important role in setting up optimal distance. The particles feed each member to find his determined pace using two factors: its own previous experience and the experience of other members. This is similar to human behavior in a decision that people

considered their previous experience and the best experience of other people around them. Based on above concepts Kennedy and Eberhart [17] expands PSO methods for continuous nonlinear optimization functions.

Particle Swarm Optimization is a randomly optimization method that each swarm consists of N_P moving particles in n-dimensional space, each particle representing a solution for the problem.

\mathbf{X}_i , position of i th particle is disposed by velocity that depend to i th particle's distance from his previous best position and best location of its neighboring particles [18]. In PSO, the solution of choice is particle location in search space. Each particle is made by two factors: speed and position. Then, particles search the solution space by updating the speed and location. Two better positions in PSO are available: the best position that the particle can to reach (Pbest) and the best position that other particles can have (Gbest) [19].

2.2 Governing regulation for PSO

Each particle, inherent solution, starts with randomly position and velocity in the swarm. Position and velocity for the particle k in an n-dimensional search space are represented by vectors $x_k = (x_{k1}, x_{k2}, \dots, x_{kn})$ and $v_k = (v_{k1}, v_{k2}, \dots, v_{kn})$ that $x_{kd} (d = 1, 2, \dots, n)$ represented position and $v_{kd} (d = 1, 2, \dots, n)$ represented velocity of particle in n-dimensional search space.

Each particle knows the position and objective function value for its position. Thus every particle remembers the position in which his best performance $P_k^i = P_{k1}^i, P_{k2}^i, \dots, P_{kn}^i$ is reached. In addition, each particle can distinguish the best particle of the swarm (shown by subtitle g). This best particle can also be obtained by a subset of particles (local neighborhood) or all particles (overall neighborhood).

Behavior of particles in each iteration is a consensus between three options:

Follow the present pattern, back to the best particle position, returning to the best value of all particles. It is agreed that this can be as follows:

$$v_{id}^{n+1} = wv_{id}^n + c_1r_1(p_{id}^n - x_{id}^n) + c_2r_2(p_{gd}^n - x_{id}^n) \quad (1)$$

$$x_{id}^{n+1} = x_{id}^n + X(v_{id}^{n+1}) \quad (2)$$

where x_{id}^n is th particle position, in n th iteration and d th dimension. v_{id}^n is velocity of i th particle in n th iteration and d th dimension. P_{best} is the best position that the particle can reach and G_{best} is the best location in the swarm. p_{gd}^n as G_{best} is best position in the swarm. c_1 and c_2 are two coefficients and r_1 and r_2 are two random number in the range of $[0,1]$. w is the inertia weight. PSO has been expanded as a simple but effective method for problems with one or more objective functions. In comparison to other social based methods, the benefits of PSO method include a simple structure, easy accessibility for practical applications, the ease of applying, being quick to get a solution and that it is strong. However, in the field for discrete and difficult problem like the one presented in this paper, its discrete version is necessary and among discrete PSO, binary PSO is the useful ones.

This study utilizes binary PSO for optimum design of plate structures.

2.3 Binary particle swarm optimization method

PSO methods only solve the problems that the real number of elements is consistently applied because particles continuous flying in discrete space is impossible. Modified PSO method for solving binary solution value problems was expanded by the Kennedy and Eberhart in 1997, [17]. In this way, a particle moves in a limited research space to 0 or 1 on each dimension. In BPSO, updating a particle represents changes of a bit which should be in either state 1 or 0 and the velocity represents the probability of bit taking the value 0 or 1. A sigmoid function is used to convert all the real velocities to the range [0, 1]. Figure 1 presents a plot of a sigmoid function. In BPSO, the updating velocity, Eq. (1), remains unchanged while the position updating rule is replaced by the following equation:

$$x_i^{k+1} = \begin{cases} 0 & \text{if } rand() \geq S(v_i^{k+1}) \\ 1 & \text{if } rand() < S(v_i^{k+1}) \end{cases} \quad (3)$$

where $S(\cdot)$ is a sigmoid function to convert particle's velocity to the probability as follows:

$$S(v_i^{k+1}) = \frac{1}{1 + e^{-v_i^{k+1}}} \quad (4)$$

and $rand()$ is a random number uniformly distributed between [0, 1], Ref. [20].

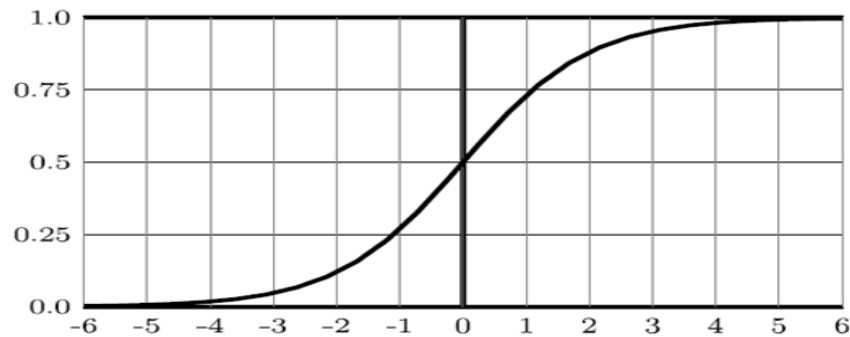


Figure 1. The sigmoid function

3. STATEMENT OF THE PLATE OPTIMIZATION

The purpose of optimization problem is finding the minimum weight of a plate while satisfy the considered design variables. To analyze the plate and find its stiffness matrix, isoparametric four- node elements are used, where for each node, two degrees of freedom

are considered in two directions, horizontal and vertical. Then, considering the constraints and the anchor force entered, all the displacements in both directions of the four nodes of element is calculated.

To create holes in the shape optimization, heterogeneous materials are used in the plate. Thus for empty elements, elasticity modulus is considered as a very small value close to zero, here it is taken as $1e-8$). For filled elements, the elasticity modulus is set to one. Also, the elements which have constraint or are under loading must be kept as full.

The objective function can be expressed as follows:

$$fit = m \times \frac{Um}{UM} \times (1 + \max(\text{penalty}))^{\frac{1}{2}} \quad (5)$$

where fit is objective function, m is number of fill elements in the plate, Um is the maximum displacement of the structure, UM is the maximum displacement in the case that all elements of plate are filled. $penalty$ is the penalty function is used when the displacement of filled elements exceed from allowed displacement and calculated as follows

$$penalty = \begin{cases} \frac{|u - U \max|}{U \max} & \text{if } |u| > U \max \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

where u is the displacement of the filled elements, $U \max$ is the maximum allowed displacement in the plate.

It is possible that the generated solution vector by a particle be non - ontinuous and as a result it will not be useable. Therefore before analyzing the plate, some modification on the solution is necessary to make a continuous structure. One approach is as the following [3]:

1) First element that should be preserved is selected as a starting element. Elements that are constraint or under loading as earlys elements are considered, because those elements to maintain the physically dependency of model must be present. Code "2" is attributed to early elements to control the continuity of the model.

2) The structure is marked with the binary codes and only elements with code "1" for the control of the model's continuousness are considered. If element has code "1" and has shared edge with beginning element, code "1" code is going to change to code "2". This trend should be continued until no element with code "1" remains.

3) If all elements with code "1" change to the code "2", structures will be continuous, but if element with code "1" remains, the structure is discontinuous.

After this trend, if the structure was discontinuous, modification of discontinues elements (distained by code "1") will be performed. In this way, elimination of the discontinuous elements or filling the empty elements around discontinuous elements to improve continuity of structure will be done randomly, as shown in Figure 2.

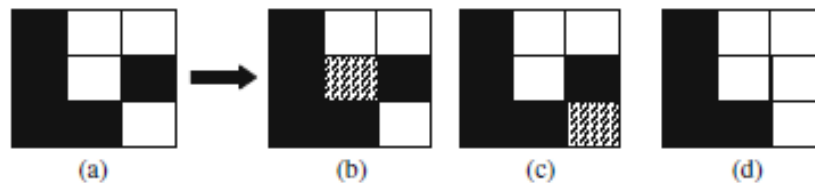


Figure 2. Model modification, [3]

4. NUMERICAL EXAMPLES

4.1 Case 1

This case involves finding a suitable topology structure for a minimum weight that satisfies the maximum displacement constraints. The geometric dimensions of the cantilever structure are a 1 unit width and a 2 unit height. The design domain is divided into 10×20 quadrilateral elements for finite element analysis. It is subjected to a load F with value of 1 unit applied at the center on the right edge. The nodes with all degrees of freedom fixed are located at the top and bottom of the left edge, as shown in Figure 3. It is assumed that $E \cdot t = 1$ for filled elements, with E referring to the Young's modulus and t referring to the thickness of the plate and $E_0 = 1e-8$ for void elements. The maximum displacement is 20 unit. The parameter setting is listed in Table 1 and the optimum result derived by Wang and Tai [21] is shown in Figure 4a and the result obtained by Wu and Tseng [3] using a differential evolution algorithm is shown in Figure 4b. The best result obtained by the BPSO is as shown in Figure 5. As shown in the pictures results of Wu and Tseng [3] and BPSO are same and both of them have same weight that this is the minimum weight of this plate maintaining continuity and the BPSO has better result in comparison to those of Wang and Tai [21].

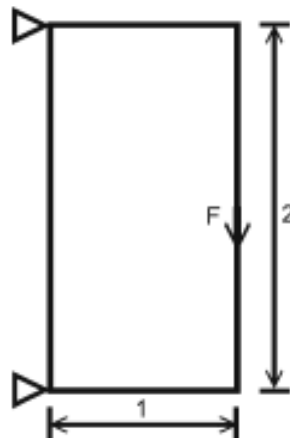


Figure 3. Cantilever structure of case 1

Table 1. Parameter settings of the BPSO for topology optimization of the structure

Parameter	Value
Population size	100
F	1
Maximum number of iteration	100

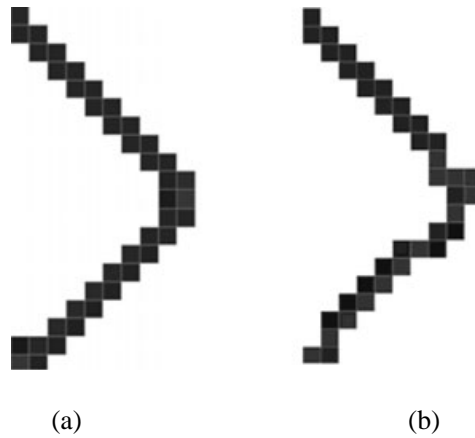


Figure 4. Optimum results of (a) Wang and Tai [21], weight = 0.2, (b) Wu and Tseng [3], weight = 0.19

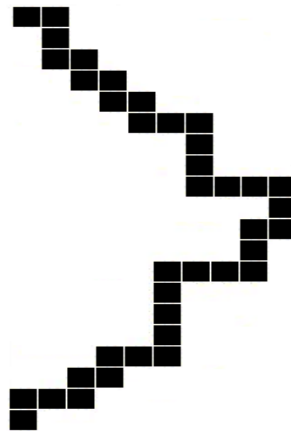


Figure 5. Optimum results of the BPSO, weight = 0.19

4.2 Case 2

The width and height of this example are 2 and 1 units, respectively. The design domain is

divided into 20×10 quadrilateral elements and the maximum displacement is 220. The downward loading F is applied at the center on the right edge, while the remaining conditions and parameter settings are identical to those of case 1. The model of the cantilever structure is shown in Figure 6. This example is solved by genetic algorithms as shown in Figure 7a and b according to Wang and Tai [21] and Balamurugan et al. [22], respectively. The optimized result derived by the differential evolution algorithm [3] is shown in Figure 7c. The result obtained by BPSO is shown in Figure 8. Results of the BPSO in comparison to other applied methods for this problem shows that BPSO is better than other methods to find minimum weight of plate.

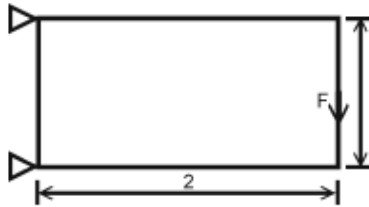


Figure 6. Cantilever structure of case 2

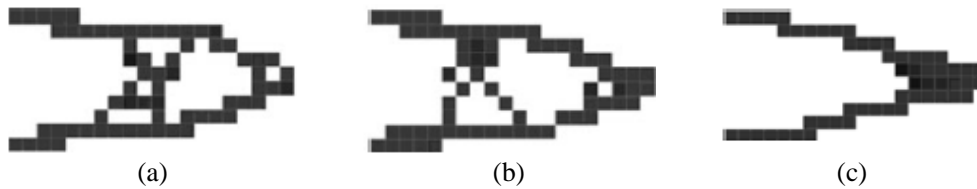


Figure 7. Optimum results of the second example (a) Wang and Tai [21], weight = 0.325, (b) Balamurugan et al. [22], weight = 0.34, (c) Wu and Tseng [3], weight = 0.275

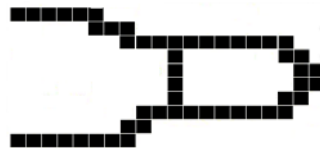


Figure 8. Optimum results of the BPSO after modification, weight = 0.26

5. CONCLUSION

In this study, an investigation into structural shape optimization using a modified binary particle swarm optimization is performed. PSO methods can only solve the problems where the real number of elements is consistently applied because particles continuous flying in discrete space is impossible. The binary PSO is suitable for finding solutions for numerical optimization problems. Thus, the binary PSO is developed for using in 0 or 1 search space. In BPSO, the probability of 0 or 1 being of particles is determined with velocity value that is

used in sigmoid function. Here, problems with one nonlinear objective function that minimized weight of plates with displacement constraint are solved. The results of the numerical examples show that the BPSO can find the results close to the ones obtained by other meta-heuristic algorithms such as GA and DE. As a result, The BPSO is found a suitable algorithm for solving plate optimizing problems.

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