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RELIABILITY-BASED OPTIMIZATION OF STEEL FRAME STRUCTURES USING MODIFIED GENETIC ALGORITHM

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ABSTRACT

Reliability-based optimization of two and three dimensional frame structures is the subject of this study. For this purpose, a computer program was developed and tested over a number of examples for validation. Since similar studies have been made previously for trusses and reliably documented in the literature, optimization of such structures based on reliability analysis could therefore be confidently relied on, and thus, designing of such structures could be considered with less value for safety factors.

This probabilistic optimization technique can well substitute that of the deterministic one where a considerable factor of safety and therefore, a heavy structure as always is a must. For this purpose, one may take into account the probabilistic behavior for load, yield stress, young modulus, etc, using parameters such as standard deviation and variance, through which safety remarks can be embedded into the design procedure by some mathematical relations, resulting to a probabilistic optimization technique. In this technique, one must first define the failure criterion, followed by the computation of safety zone (Z), reliability index (β) and lastly, the failure probability (P_f).

In this paper, the applied load and the yield stress are considered probabilistic, while the violation of interior forces from the member ultimate strength is the failure criterion. For each of the interior axial, shear, bending and torsion reactions, the failure probability is calculated and the maximum value is constrained through optimization process.

During the optimization process using Genetic Algorithm (GA), the failure probabilities are some boundary constraints and minimizing the weight of structure is the objective of the problem. The profiles of I-shaped cross-sections are selected from a data file.

Finally, the probabilistic technique and deterministic one are investigated and compared applied to some structural problems.

Keywords: Reliability analysis; safety factor; probabilistic optimization; standard deviation; variance; failure criterion; safety zone; reliability index; failure probability; genetic algorithm

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1. INTRODUCTION

Mankind has always been after safety and certainty. To understand it as an external parameter to guard a safe environment, it is obviously fair to state that tendency to safety is an instinctive desire within a person, to reduce danger or risks.

A structure under the study of a designer is not always under a set of predefined or approximate loadings, and since these approximations and uncertainties are not meant to ignore, to assure reliability, generally safety factors are considered. Major the safety factor ensures a more relaxed mind. However, by considering a safety factor, yet the variations of load (or structural resistance) distribution factor or changes of load probability density will introduce different failure probabilities, which indicate that safety factors do not account for the uncertainties existing within the load or even material properties. In some cases however, a smaller safety factor than unity may even handle the uncertainties due to failure probability, as well fulfilling economical purposes.

The uncertainties in structural parameters such as material properties, external loads, geometry, etc., have caused serious attentions to reliability in structural design and analysis. Thus, reliability theory, as a branch of theory of probability, provides a firm framework which can introduce a proper factor of safety when required [15]. Thus, any system made of a satisfied reliability index, may be referred to as safe. Now, higher safety factors do not necessarily lead to reliable and safe designs. They may even tend to either over-design cost wise or under-design reliability-wise.

Obviously, some of the most recent developments in reliability-based civil engineering analysis and design have been covered and authored by some of the most active scholars in their respective areas, representing some of the most recent research, upcoming interests and challenges to the profession. To mention only some of these valued researchers, one may refer to [1-11, 14, 23, 24]. However, civil engineers have not yet been so fast in adapting to new probabilistic methods developed and effectively use stochastic design procedures for economic and reliable designs of systems.

In this paper, the structural member or system failure probability of 2D and 3D frame structures are computed using reliability theory. They will then be embedded into a discrete-based stochastic design model developed here where GA-based optimization technique is taken place.

Thus, the outcome of this work is a computer program which optimizes 2D and 3D frame structures using GA technique with a major attention paid to reliability of those structures included as constraints in the optimization model.

2. SECOND MOMENT METHODS IN DETERMINING RELIABILITY INDEX

Evaluating safety of a structure depends on incoming loads, resistance of materials, executive issues and other probabilistic variables. In addition to them, they need time and design parameters which are definite and numerical variables. Loads generally have a probabilistic process in time and physical space Z(t, x, y, z). In such a space, resistance of structure appears in form of limit surfaces (Figure 1) and since the resistance is a probabilistic variable, resultant surfaces are also probabilistic and in this case, failure means passing from limit surface of safety region in *Z* space [16].

A structure which is affected by a certain position of loads can be considered as a point of sample space *Z*. The reliability of a structure P_S is the resting probability of that sample point in safety region of above space, while the failure probability P_f is the placing of that point in failure region.

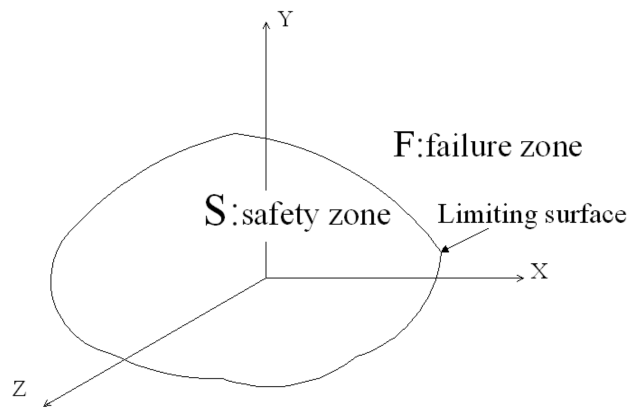

Figure 1. Structural resistance as the limiting surface within the loading space

Basic idea of second moment theorem is based on this point that all uncertainties related to structure reliability should be expressed in terms of mean (first moment) or covariance (second moment) of internal parameters. These parameters are called basic variables that are specified here with notation Z_i. Basic variables include loading and resistance parameters, as well as geometric variables and uncertainties related to a selected model. One may use second moment method of reliability when numbers of basic variables are limited. In addition, we should be able to say if in exchange for a group of these values, structure places in failure region or safety region. This may result in dividing space *Z* to two regions; namely safety and failure regions, noted as *S* and *F*, respectively. These two regions are separated by failure surface, termed also as limiting surface. For more clarity of the reliability indices used here, two distinct methods are paid attention to.

2.1 Cornell reliability index (CRI)

If failure surface is shown by L_Z , one can define failure function as [16]:

$$
\begin{aligned}
g(z_i) > 0 \ , \quad & z_i \ \varepsilon \ S \\
g(z_i) = 0 \ , \quad & z_i \ \varepsilon \ L_Z \\
g(z_i) < 0 \ , \quad & z_i \ \varepsilon \ F
\end{aligned}
\tag{1}
$$

Where, function g that is better to be chosen derivable for computational reasons, is usually obtained by structural analysis. If probabilistic variables Z_i corresponding to parameters z_i are replaced in failure function, the resultant probabilistic variable is called safety margin and is specified with *M*.

$$M = g(Z_i) \tag{2}$$

According to definition, this safety margin reflects arbitrariness of choosing function g .

Cornell in 1969 [16], defines the reliability index or safety index β_c as follows:

$$\beta_c = \frac{E[M]}{D[M]} \tag{3}$$

CRI in form of distance measured from region $E[M]$ to failure surface provides an appropriate evaluation of reliability. This distance is measured in form of a multiple of uncertainty parameter $D[M]$. $E[M]$ and $D[M]$ are in fact mean and standard deviations of safety margin, respectively. In Cornell first formulation, failure function is expressed in form of resistance difference r from load effect s :

$$g(r,s) = r - s \tag{4}$$

Similar safety margin of above relation is:

$$M = R - S \tag{5}$$

If *R* and *S* are independent, reliability index (1) will be as follows:

$$\beta_c = \frac{E[R]-E[S]}{\sqrt{Var[R]+Var[S]}} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{6}$$

The above relation is used for a distinct member with normal distribution.

If the reliability index is a hyper-plane, we can define linear failure function as follows:

$$g(z_i) = a_0 + \sum_{i=1}^{n} a_i z_i = a_0 + \mathbf{a}^{\mathbf{T}}\mathbf{z} \tag{7}$$

Where, $\mathbf{a}^{\mathbf{T}}$ is a row vector with entries a_i and $\mathbf{Z}$ is a column vector with entries Z_i.

Similar safety margin of failure function in eq. (7) is:

$$M = a_0 + \sum_{i=1}^{n} a_i Z_i = a_0 + \mathbf{a}^{\mathbf{T}}\mathbf{Z} \tag{8}$$

Thus, the reliability index (3) will be:

$$\beta_c = \frac{a_0 + \mathbf{a}^{\mathbf{T}} E[\mathbf{Z}]}{\sqrt{\mathbf{a}^{\mathbf{T}} \mathbf{C}_z \mathbf{a}}} \tag{9}$$

Where $E[Z]$ and C_z are the mean and covariance matrices for vector $\mathbf{Z}$, respectively.

In equation (9) one may notice that β remains unchanged under any linear transformation of basic variables.

2.2 The first order second moment reliability index

The failure function (4) is not unique for a failure surface, and there are various alternatives for that function. Basic variables R and S are often limited to positive values for physical reasons. So, a simple alternative for relation (4) is as follows [16]:

$$g(r,s) = \log(r/s) \tag{10}$$

Therefore, reliability index (3) will be defined as:

$$\beta_{RE} = \frac{E[\log(R/S)]}{D[\log(R/S)]} \tag{11}$$

The safety margin *log* (*R/S*) is a nonlinear function of *R* and *S*, and therefore, mean and standard deviation are not obtained from second moment representation (*S, R*) only.

One method of settling such issues is linearization of safety margin. This work requires choosing linearization method. The simplest method is applying linear expressions of Taylor expansion around one point. If linearization operation is done around a point related to mean values (μ_R, μ_S), we will then have the following result:

$$M_{FO} = \log \mu_R - \log \mu_S + \frac{R - \mu_R}{\mu_R} - \frac{S - \mu_S}{\mu_S} \tag{12}$$

Using above linear safety margin and regarding relation (9), the following reliability index is obtained:

$$\beta_{FO} = \frac{\log \mu_R - \log \mu_S}{\sqrt{V_R^2 + V_S^2}} \tag{13}$$

Where, V_R and V_S are dispersion coefficients of R and S, respectively.

β_{FO} is known as the First order-second moment reliability index. The above relation is to compute the reliability index of a separate member. However, it is also used for hyper-planes similar to CRI.

In evaluating structure reliability that has been discussed here, we usually use CRI. This is because failure functions in structures are generally in form of linear combination of structure member resistance and external loads effect [17]. In this order safety margins of structural systems are similar to equation (8) where modal resistance R and modal external load effect S are present as probabilistic variables. This is true when geometric parameters of a structure such as length, cross section, moment of inertia and also elastic modulus have definite values. It is to be noted that in the present study, only external loads and elements resistance of structure have been considered as probabilistic parameters.

2.3 Steps to determine reliability formulations [1]

1. Failure definition: First step in using reliability theory for structures is to specify boundaries between safety and failure criteria. For instance, member, nodal and/or system displacements excessive of the allowable ones may indicate the failure.
2. The second step is to choose a model which relates the essential variables to failure criterion or the system safety index.
3. Determining the uncertainties in essential variables.
4. Obtaining failure probability distribution functions of the variables.

3. USING RELIABILITY THEORY IN 2D AND 3D STRUCTURAL FRAMES

The first step is to determine the failure criterion for frames. Then, by computing the difference between internal forces and ultimate resistance in critical sections, a safety margin (Z) is found. Structural resistance space is then appeared as limit surfaces. Since the load and the resistance are probabilistic variables, the resulted surfaces are also probabilistic. Now, if in this domain, the safety, failure and the separating surface of those two are shown by S, F and L_Z respectively, then:

$$\begin{aligned} Z > 0 \quad & \Rightarrow S \\ Z = 0 \quad & \Rightarrow L_Z \\ Z < 0 \quad & \Rightarrow F \end{aligned}$$

Having determined Z, then the reliability and safety indices will be obtained. To compute the reliability index denoted by β, one must determine the parameters of standard deviation, variance and mean safety margin $E[Z]$, so that [4]:

$$\beta = E[Z]/\sigma[Z] \tag{14}$$

3.1 Reliability theory in 2D frame structures

Since bending moments may be considered as the highest effective load in 2D frames, the formation of plastic hinges at the location of joints and critically affected bending moment sections will therefore be defined as failure criteria. Thus, the difference between internal forces caused by environmental loads and bending resistance at the joints or critical sections is regarded as safety margin. In 2D frames, internal forces are the bending moments at the joints which, using matrix methods at the left and right of each member are determined as [17]:

$$S_i^L = \sum_{i=1}^{l} Lij^L{}_{(I_i)} L_j \quad \& \quad S_i^R = \sum_{i=1}^{l} Lij^R{}_{(I_i)} L_j \qquad (i = 1,2,3,...,n) \tag{15}$$

Where, $Lij_{(I_i)}$ is the member reaction forces matrix in critical sections affected by a unit load, L_j is the vector of external loads and $(I=I_1, I_2, ..., I_n)^T$ contains member moment of inertia. Using the following relation, one may express the members bending resistances according to a complete plastic moment as [7]:

$$R_i = AZP_i \, C_{yi} \tag{16}$$

Where A_i, C_{yi} and AZP_i are the cross sectional area, allowable stress and modulus of plastic section for the i^{th} member, respectively. According to Figure 2, AZP_i for an I-section may be obtained using the following formula:

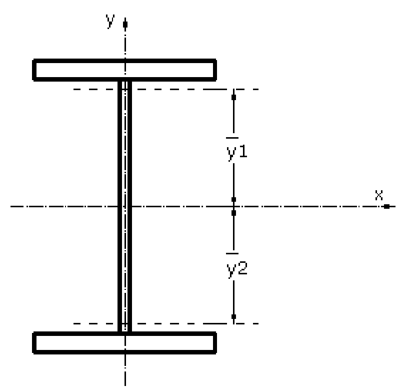

Figure 2. Axles of I-section for determining Modulus of plastic section about horizontal axis

Modulus of plastic section about horizontal axis=

$$\left| \begin{array}{ll} AZP_i = (A_i / 2) \times (\bar{y}_1 - \bar{y}_2)_i & (a) \\ \bar{y}_1 = \bar{y}_2 & (b) \end{array} \right\rangle \; (a)\;,\;(b) \quad \Rightarrow AZP_i = A_i \times (\bar{y}_1)$$

Now, the safety margin at the left and right ends of members may be defined as [17]:

$$Z_i^L = R_i - S_i^L \qquad \& \qquad Z_i^R = R_i - S_i^R \tag{17}$$

In a 2D frame structure consisting of n members under a number of l loads, if allowable stresses C_{yi} and applied loads L_j are probabilistic variables, then the reliability margin will be a probabilistic variable and failure probability at the two ends of each member will be [11]:

$$P_{f_i} = \phi\,(-\beta_i) \quad \Rightarrow \quad P_{f_i} = \phi\,(-E[Z_i]/\sigma_{Z_i}) \tag{18}$$

Where ϕ may be referred to as the standard normal probability distribution function. Also $E[Z_i]$ and σ_{Z_i} are the mean and standard deviations of safety margin of i^{th} element which are computed using the following relations [15]:

$$E[Z_i] = \overline{C}_{yi}(AZP_i) - \sum_{i=1}^{l} Lij_{(I_i)}\ \overline{L}_j \tag{19}$$

$$\sigma_{Z_i}{}^2 = \sigma_{C_{yi}}{}^2 (AZP_i)^2 + \sum_{i=1}^{l} Lij_{(I_i)}{}^2\ \sigma_{Lj}{}^2 \tag{20}$$

In equations (19) and (20) C_{yi} and L_j are the mean yield stress and load, respectively. $\sigma_{C\,yi}{}^2$, and $\sigma_{L\,j}{}^2$ are the Variance Coefficients of Yield Stress (VCYS) and External Load (VCEL), respectively. Also l indicates total number applied loads to the structure.

3.2 Reliability theory in 3D frame structures

In such a structure each of the bending moments about horizontal or vertical axes, also axial forces or shear forces could cause total or member failures. Therefore, in 3D frame structures failure probabilities caused by all types of internal forces in all directions are computed and the maximum value is considered as the dominant (appointed) value for failure probability. In Eq. (16), instead of using the coefficient AZP_i for axial and shear resistive forces, cross sectional area A_i of the i^{th} member is taking place, and C_{yi} as the allowable axial and shear stress is read from data files. Besides, according to Figure 3, computation of AZP_i with regard to the I-shaped cross-sectional as discrete variables is done as follows [7]:

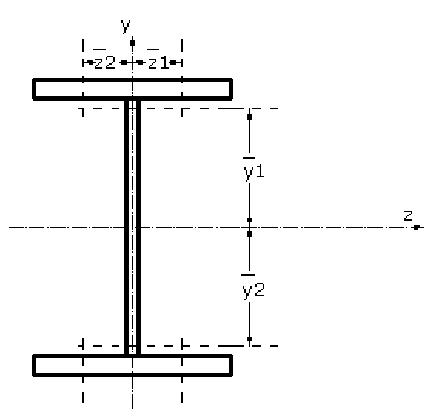

Figure 3. I-section for determining Modulus of plastic section about horizontal and vertical axles

Modulus of plastic section about horizontal axis=

$$\left|\begin{array}{ll} AZP_i = (A_i/2)\times(\bar{y}_1+\bar{y}_2)_i & (a) \\ \bar{y}_1 = \bar{y}_2 & (b) \end{array}\right\rangle \ (a)\ ,\ (b) \quad \Rightarrow \ AZP_i = A_i \times (\bar{y}_1)_i$$

Modulus of plastic section about vertical axis=

$$\left|\begin{array}{ll} AZP_i = (A_i/2)\times(\bar{z}_1+\bar{z}_2)_i & (a) \\ \bar{z}_1 = \bar{z}_2 & (b) \end{array}\right\rangle \ (a)\ ,\ (b) \ \Rightarrow \ AZP_i = A_i \times (\bar{z}_1)_i$$

Therefore, values for the mean and variance of the safety margin for the i^{th} member are computed as followed [10, 22]:

1. *Axial force in x direction:*

$$E[Z_i] = \overline{C}_{yi}(A_i) - \sum_{j=1}^{l} Lij_{(I_i)}\,\overline{L}_j \quad , \quad \sigma_{Zi}^{\ 2} = \sigma_{Cyi}^{\ 2}(A_i)^2 + \sum_{j=1}^{l} Lij_{(I_i)}^{\ 2}\sigma_{Lj}^{\ 2}$$

2. *Shear force in z direction:*

$$E[Z_i] = 0.55\overline{C}_{yi}(2\times b_{f_i}\times t_{f_i}) - \sum_{j=1}^{l} Lij_{(I_i)}\,\overline{L}_j \quad , \quad \sigma_{Zi}^{\ 2} = (0.55)^2\sigma_{Cyi}^{\ 2}(2\times b_{f_i}\times t_{f_i})^2 + \sum_{j=1}^{l} Lij_{(I_i)}^{\ 2}\sigma_{Lj}^{\ 2}$$

3. *Shear force in y direction:*

$$E[Z_i] = 0.55\overline{C}_{yi}(d_i\times t_{w_i}) - \sum_{j=1}^{l} Lij_{(I_i)}\,\overline{L}_j \quad , \quad \sigma_{Zi}^{\ 2} = (0.55)^2\sigma_{Cyi}^{\ 2}(d_i\times t_{w_i})^2 + \sum_{j=1}^{l} Lij_{(I_i)}^{\ 2}\sigma_{Lj}^{\ 2}$$

4. *Bending about horizontal z axis:*

$$E[Z_i] = \overline{C}_{yi}(A_i\times \bar{y}_i) - \sum_{j=1}^{l} Lij_{(I_i)}\,\overline{L}_j \quad , \quad \sigma_{Zi}^{\ 2} = \sigma_{Cyi}^{\ 2}(A_i\times \bar{y}_i)^2 + \sum_{j=1}^{l} Lij_{(I_i)}^{\ 2}\sigma_{Lj}^{\ 2}$$

5. *Bending about vertical y axis:*

$$E[Z_i] = \overline{C}_{yi}(A_i\times \bar{z}_i) - \sum_{j=1}^{l} Lij_{(I_i)}\,\overline{L}_j \quad , \quad \sigma_{Zi}^{\ 2} = \sigma_{Cyi}^{\ 2}(A_i\times \bar{z}_i)^2 + \sum_{j=1}^{l} Lij_{(I_i)}^{\ 2}\sigma_{Lj}^{\ 2}$$

Now, using Eq. (14), reliability index (β) for each member under each of the internal forces are determined, and by using Eq. (18), failure probability (P_{f_i}) for all cases are computed. Maximum failure probability of the above five cases will be a dominant value for each member.

Since the structure is designed in plastic limit, cautions must be taken to avoid torsion, as it will cause collapse of the system, and therefore cannot be reliable. Of course, if failure probability is defined as violation of allowable strength, then the mean and variance values of torsion moment should first be computed.

6. *Torsion:*

$$E[Z_i] = 0.4\overline{C}_{yi}\times(j_i/t_{wi}) - \sum_{j=1}^{l} Lij_{(I_i)}\overline{L}_j \ , \quad \sigma_{Zi}^{\ 2} = (0.4)^2\sigma_{Cyi}^{\ 2}(j_j/t_{wi})^2 + \sum_{j=1}^{l} Lij_{(I_i)}^{\ 2}\sigma_{L_j}^{\ 2}$$

Then also, in all five above cases, the allowable stress may substitute the yield stress and its coefficients, if constrained by failure probability in elastic limit.

4. RELIABILITY THEORY BASED ON MEMBERS AND SYSTEM FAILURE PROBABILITIES

4.1 Reliability theory based on member failure probability

To optimize a structure by emphasizing on the safety of all members, one should determine failure probability of each member, while constraining it to a minimum value expressed as the allowable. Thus, failure probability of each member is considered as a constraint that should not exceed allowable failure probability related to each member. Thus [4]:

$$P_{f_i} \le P_{fa_i} \qquad (i = 1,2,\ldots,n_e) \tag{21}$$

Where P_{f_i} and P_{fa_i} are the failure and the allowable failure probabilities one for i^{th} member, respectively.

One of the features of such a method is that member's failure probability under probabilistic external loads will be reduced to a minimum one. However, from the economical point of view, there are also members whose failure will not affect the whole system a great deal, and so higher allowable failure probabilities for such members may be permitted, and this obviously reduces the total weight of the structure as the objective function.

4.2 Reliability theory based on system failure probability

In order to design a large structure with a minimum failure probability for the whole system, members' failure probabilities will be determined and then added to be constrained. Thus, this sum value should not exceed the system allowable failure probability. This statement may also be expressed in terms of the following formula [1]:

$$P_f = \sum_{i=1}^{n_e} P_{fi} \tag{22}$$

$$P_f \le P_{fa} \tag{23}$$

Where (P_f) and (P_{fa}) indicate total failure and total allowable failure probabilities, respectively.

Based on [1], the relation (22) is an upper conservative bounding relation, aiming for the reliability of indeterminate structures. Now, since this method has indicated logical responses to especially very large indeterminate space structures, it might also be fair to consider the same approach for computing failure probabilities for frame structures. Therefore, to avoid a very complex, tedious and long process of true reliability analysis of frame structures, the upper bound is considered active. Besides, seeking an acceptable solution, some different approaches have been investigated, details of which are listed below, assuring for the validated results obtained.

First: in this study, by converting the ends of each member to a hinge and also by introducing a hinge in the middle of beams, 3 hinges for each beam and 2 hinges for each column in the structure would be formed. Now, by computing failure probabilities for each case individually, and then summing them all up, a significant failure probability for the whole system would be obtained. This would obviously generate a big failure probability that could definitely lead to an increase in the weight of the structure. This approach may be considered very conservative. To avoid such a problem, and as well as guarding for the critical failure point while seeking the betterment of structure weight, only the maximum failure probability for each member enters the computation. Thus:

$$\left| \begin{array}{l} P_{S1} = \sum_{i=1}^{neb} (MAX(\ P_f(3i-2),\ P_f(3i-1),\ P_f(3i))) \\ P_{S2} = \sum_{i=1}^{nec} (MAX(\ P_f(3i-2),\ P_f(3i-1)\ ) \end{array} \right\rangle \qquad P_S = P_{S1} + P_{S2} \tag{24}$$

Where, P_{S1}, P_{S2} and P_S are the beams, the columns and the total system failure probabilities summation, respectively and *neb* and *nec* are total number of the beams and the columns. $P_f\,(3i\text{-}2)$, $P_f\,(3i\text{-}1)$ are also two ends failure probability of members and $P_f\,(3i)$ is the failure probability of the middle of beams. As one notices, each member enters only one representative in computation.

Second: In this state, we introduce a higher failure probability for the system by introducing and adding the most critical failure probability from each node to the member failure probability. Thus:

$$P_S = P_{S1} + P_{S2} + \sum_{i=1}^{nn} P_{fi\,critical} \tag{25}$$

Third: This time, we add the available whole failure probability without any limitation and as you can see in the examples, the weight of the structure increases significantly.

$$\left| \begin{array}{l} P_{S1} = \sum_{i=1}^{neb} (\ P_f(3i-2) +\ P_f(3i-1) + P_f(3i)\ ) \\ P_{S2} = \sum_{i=1}^{nec} (\ P_f(3i-2) +\ P_f(3i-1)\ ) \end{array} \right\rangle \qquad P_S = P_{S1} + P_{S2} \tag{26}$$

Results from above three states in numerical examples will be compared and evaluated with results of elements failure probability method.

5. GENETIC ALGORITHM IN OPTIMIZATION BASED ON RELIABILITY THEORY

Having determined (P_f) for the members and the whole system, by defining constraints and objective function system optimization will be performed using Genetic Algorithm (GA) method. This method is inspired by nature which can replace mathematical methods in particular as a discrete optimization technique, with the feature of hardly trapped in a local optimum. In this work, the objective function is minimization of weight and the constraints are the (P_{fa}) for either structural members or the system.

5.1 Objective Function

In this work, the objective function is to minimize the weight of the structure so that:

$$W = \sum_{i=1}^{ne} w_i \times l_i \qquad (27)$$

Where w_i is the weight per unit length, and l_i and n_e are length of member i and total number of members, respectively.

In the present work design variables considered for frame structures are of discrete type. A number of I-shaped section profiles are arranged in a catalogue list. In this list there are cross-sectional areas as essential variables and also related properties such as moments of inertia, etc., are therefore considered as dependent variables. Due to discrete nature of essential variables for problems under study here, GA was employed as a type of optimizer [18-20]. Thus, the aim is to select the best set of profiles as design variables from the available set in the catalogue list, to minimize the weight of the structure as well as satisfying all the constraints. This is done through an iterative process, details of which may be addressed in [13].

5.2 Constraints

For the problems studied here, the constraints are formed as follows:

$$P_{f_i} \le P_{fa_i} \qquad (i = 1, 2, ..., n_e) \qquad (28.a)$$

$$P_f \le P_{fa} \qquad (28.b)$$

Where n_e is the number of structural members. P_{fa_i} and P_{fa} are the allowable failure probabilities for i^{th} member and for the system, respectively.

On the other hand, to control the slenderness of columns, the following constraint will also be applied:

$$(\frac{KL}{r})_{\max_i} < 200 \qquad ; \; i = 1, 2, ..., n_e \qquad (29)$$

Where $(\frac{KL}{r})_{\max_i}$ is the maximum slenderness coefficient of i^{th} member.

5.3 Penalized objective function

Since GAs are designed for unconstrained maximization problems, the constrained problems should therefore be converted into unconstrained ones [12, 18]. For this purpose first the coefficient of constraint violation (C) will be computed from:

$$C_1 = \sum_{i=1}^{ne} \max(\frac{p_{f_i}}{p_{f_{ai}}} - 1, 0) \tag{30.a}$$

$$C_2 = \max(\frac{p_f}{p_{f_a}} - 1, 0) \tag{30.b}$$

Where, C_1 and C_2 are used for the failure probability of members and system, respectively. Then, using penalization method [12], the modified fitness function will be as follows:

$$\left| \begin{array}{l} C = C_1 \\ \quad or \\ C = C_2 \end{array} \right\rangle \qquad Gol_i = W \times (1 + R_p \times C) \tag{31}$$

Where, Gol_i is the modified objective function of the i^{th} design, and R_p is the penalty function not exceeding:

$$R_P = r_1 \times \left[1 - 0.2 \times (ngen - 1)\right] \leq 4r_1 \tag{32}$$

Where *ngen* is generations counter and r_1 is a constant penalty function coefficient selected arbitrary. Here, its value was taken as:

$$r_1 = 25 \times (1 + 0.2 \times (ngen - 1)) \leq 100 \tag{33}$$

Using GA to minimize an objective function such as the weight of the frame structure as for all the cases under study here, fitness function for the i^{th} design will then be computed using the equation below [6]:

$$Fit_i = Gol_{\max} + Gol_{\min} - Gol_i \tag{34}$$

Where for any generation, Fit_i is the i^{th} fitness design, Gol_{max} and Gol_{min} are the largest and smallest penalized objective functions, respectively.

Programs developed in this work, are performed to eventually optimize a 2D/3D frame structure according to reliability-based design, results through which may be presented using some examples.

A. Unit-load analysis program for 2D and 3D frames
To compute the safety margin, it is necessary that first 2D-frames or 3D-frames are analyzed based on a unit-load approach and then intrinsic reaction forces and moments are determined.

B. Reliability programs for 2D and 3D frames
Elements and/or system failure probabilities will then be computed using RELIABILITY-2D and RELIABILITY_3D programs developed for the purpose of this study (see section3).

C. Reliability-based minimization of weight using GA
Elements and/or system failure probabilities computed will then be used as constraints for weight minimization of frames.

6. BENCHMARK EXAMPLES

Here, the aim is to carry out the procedure of optimum design on a number of problems. Since there was no means of comparison in the literature, the attempt was made to verify the program developed and solutions obtained by studying different types of problems using different aspects and viewpoints, details of which are embedded within the examples sections.

6.1 Example (1) – A five storey frame with rigid joints
This example is a five-storey five-span plane frame structure with a given set of loadings as shown in Figure 4. Each span length is set as 5.6m and each storey as 2.8m height. The uniform distributed load on each beam span is approximated to a mean value of 39.45 N/cm. Table 1 lists categorized set of design variables assumed and some related data for the problem are shown in Table 2.

Table 1: Categorizing frame members for example 1

No. Type	**1**	**2**	**3**	**4**	**5**
No. El	1, 6, 11, 16, 21	2, 7, 12, 17, 22	3, 8, 13, 18, 23	4, 9, 14, 19, 24	5, 10, 15, 20, 25
No. Type	6	7	8	9	10
No.El	26, 51	27, 5	28, 53	29, 54	30, 55
No. Type	11	12	13	14	15
No.El	31, 46	32, 47	33, 48	34, 49	35, 50
No. Type	16	17	18	19	20
No.El	36, 41	37, 42	38, 43	39, 44	40, 45

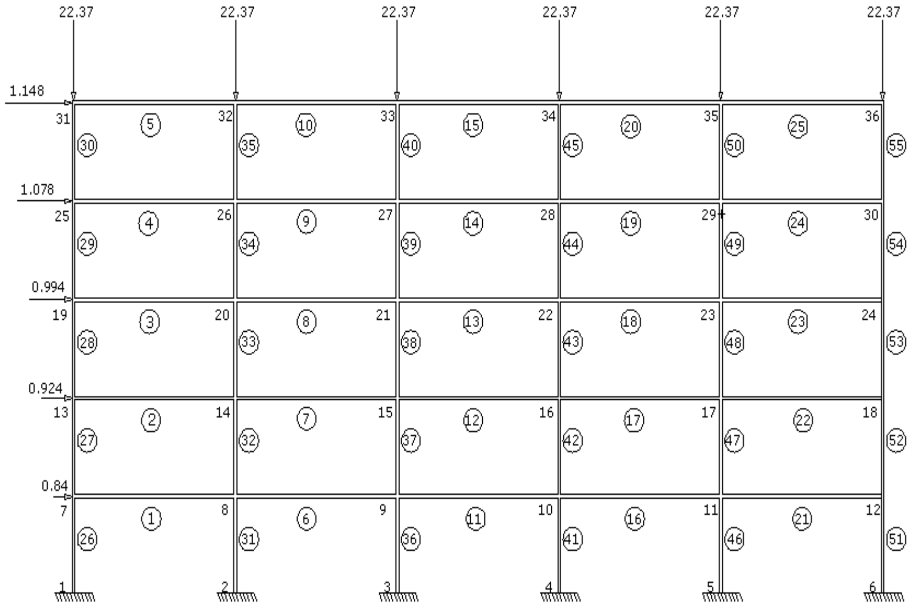

Figure 4. Structural geometry, members numbering and loading, Example 1

Table 2: Optimization input data for example 1

Control parameters to GA	Mutation probability	0.004
	Cross probability	1
	Population size	50
Another parameters	Materials density	0.0785 (N/cm^3)
	Elasticity module	21e6 (N/cm^3)

Three different cases of optimum designs were studied. For cases two and three, designing under external forces and probabilistic yield stress, the load distribution factor is taken as 0.1 and members yield stress distribution factor as 0.05. The reason for carrying out these comparisons is to verify whether the solutions will match while under the same limited conditions applied to both probabilistic and deterministic models. Thus, it may well assure correctness of the theory and program developed.

1) In this case, assumption is made to almost take the allowable failure probability to an epsilon value close to zero to prevent a division by zero. Similarly, the load variance and members yield stress factors are also taken as nearly zero. This is in fact a case similar to deterministic one.

Now, by defining the violation of the allowable resistance as failure criterion, the structure is attempted for optimization while elastically constrained. Thus, a comparison of the optimum design weights of the latter with deterministic approach is made while the same constraints were used. The results obtained were illustrated in Figure 5 where, probabilistic optimum weight is recorded as 11050kg, while for the deterministic case the optimum weight is equal to 10418kg. This shows an acceptable 6% drift of the results. In elastic probabilistic method, maximum of lateral drift is an epsilon value close to zero but maximum displacement in middle of beams spans is 8.41 cm. It is clear that one should define a constraint of nodes failure probability to prevent nodes failure in beams and columns.

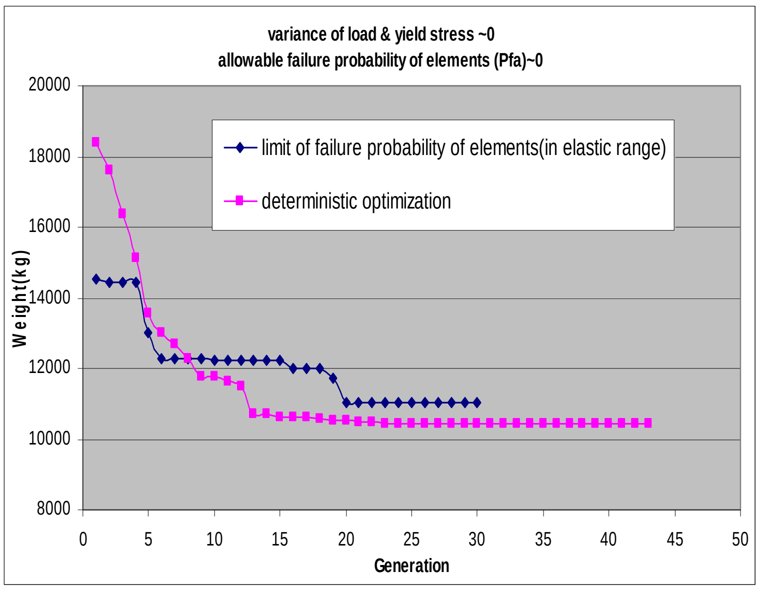

Figure 5. Convergence history of optimum weight for both models under elastic limit, Example 1

Same assumptions as above, however this time members plastic failure criterion is constrained for probabilistic optimization procedure, and the optimum weigh at each generation were compared with those of deterministic model while constrained in the non-elastic region. Figure 6 shows the convergence histories of the above cases where the probability-based design leads to an optimum 8804kg weight while the deterministic one records an optimum weight of 8718kg, with a 0.98% drift. As anticipated, non-elastic optimum weight is less than that in elastic limit. In the aforementioned probabilistic method, maximum of lateral drift is close to zero and maximum displacement in middle of beams spans is 1.26 cm. it shows that in this case, drift and displacement have been controlled well, but however, it is necessary that one defines a constraint for nodal failure probability.

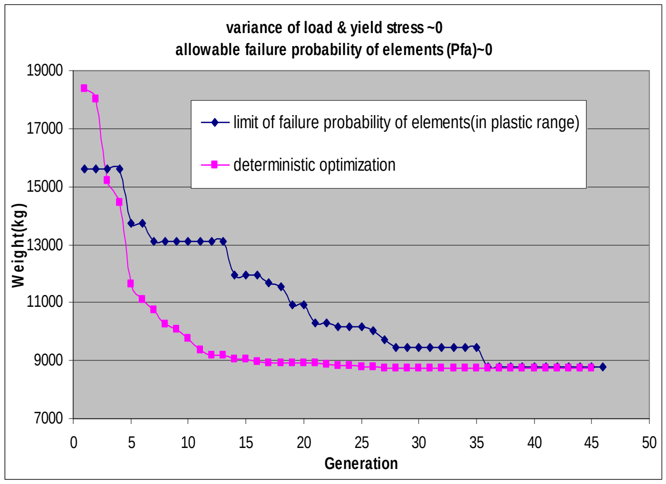

Figure 6. convergence history of optimum weight for both models under non-elastic limit, Example 1

2) Using the same conditions as in case B above, the structure was again attempted for optimization. This time permissible values for failure probabilities of members and system, being respectively equal to 1.8×10^{-6} and 1×10^{-4} were allowed. The optimum weights were determined as 6864kg and 6699kg, respectively. Figure 7 illustrates the corresponding convergence histories for the two cases above. For the first case, maximum displacement in middle of beams is 5.35cm and maximum lateral drift is close to zero. Also maximum slender coefficient of columns is 92. For second case, maximum displacement in middle of beams is 1.97cm and maximum lateral drift is close to zero. Also maximum slender coefficient of columns is 132. Thus, the results indicate that constraining nodal failure probability may cause a certain control to the structural failure.

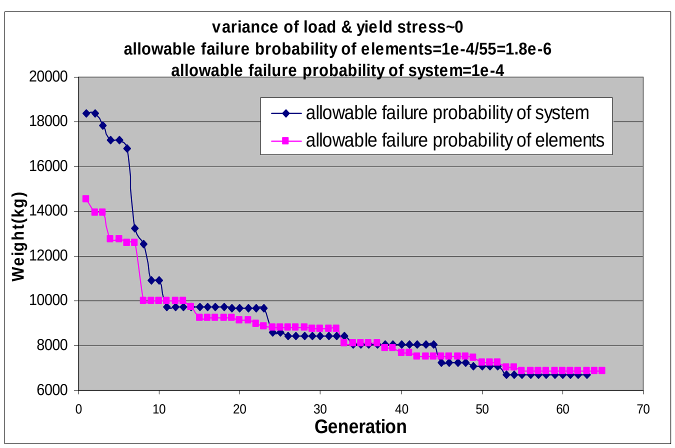

Figure 7. probabilistic weight design in non-elastic limit, with zero VCYS and VCEL and a non-zero allowable failure probability, Example 1

To verify the validity of the developed program, the following examples are presented.

6.2 Example (2) – A one-storey one-span 3D frame

Figure 8 illustrates the problem. The average external loads are shown in this Figure and members mean yield stress is assumed as 24 *kN*/cm^2. Grouping of the members as design variables are listed in Table 3 and some related data for the problem are shown in Table 4.

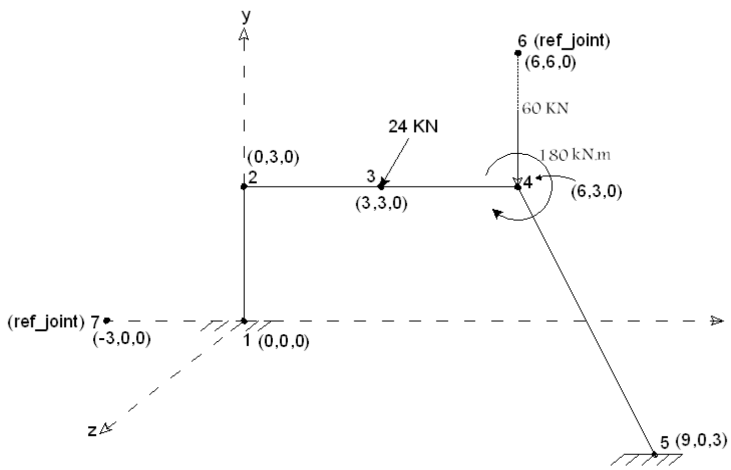

Figure 8. Structural geometry, members numbering and loading for example 2

Table 3: Categorizing frame members for example 2

No. Type	**1**	**2**	**3**
No. El	1	2, 3	4

Table 4: Optimization input data for example 2

Control parameters to GA	Mutation probability	0.004
	Cross probability	1
	Population size	50
Another parameters	Materials density	0.0785 (N/cm^3)
	Elasticity module	21e6 (N/cm^3)

Figure 9-a illustrates that with a nearly zero failure probability and a constant VCYS value equal to 0.05 for a member, with an increase in VCEL as listed in Table V, the

optimum weight will increase as well as the safety factor. In that table one also realizes that a VCEL lower than 0.1 whether does not affect the optimum weight or the affect is so little.

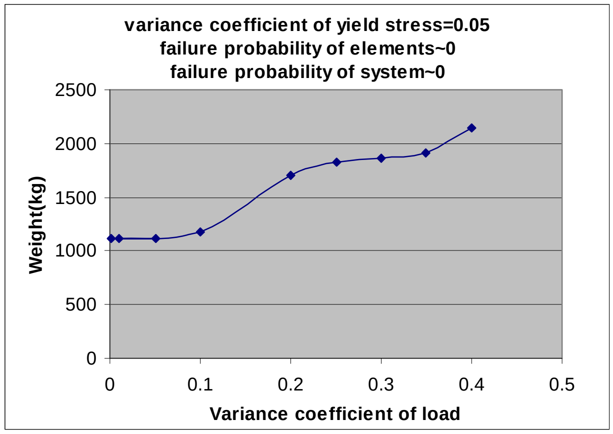

Figure 9-a. Variation of optimum design with a constant VCYS and varied VCEL under allowable failure probabilities of nearly zero, example 2

By increasing the allowable failure probability of members to 1.67×10^{-4}, while maintaining the same value for VCYS, a distinct reduction on the weight will be observed, as illustrated in Figure 9-b. Also in Figure 10, an accumulative set of results indicating variations of optimum weight with different measurements of VCEL and VCYS is shown. Information gathered in this Figure easily indicates that by increasing each of these coefficients, the optimum weight may also increase. They may also be regarded symbolically as a source of reference on predicting the optimum weight of the structure at certain values for VCYS and VCEL using just interpolation.

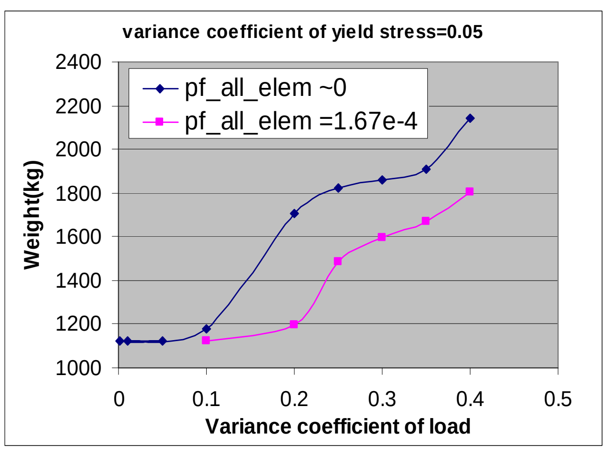

Figure 9-b. Comparison of optimum weight with P of nearly zero and 1.67e-3 for example 2

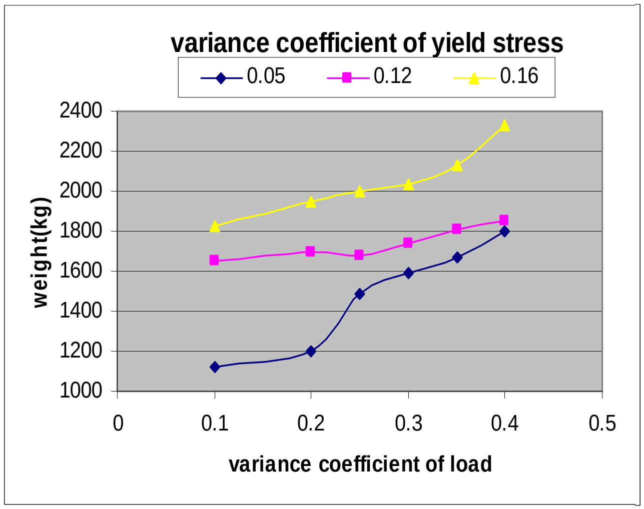

Figure 10. Optimum weight variation with VCYS and VCEL using $P_{f\text{-all-elem}}$= 1.67e-4 and $P_{f\text{-all-system}}$ =1e-3, example 2

Now, by assuming a constant permissible member failure probability equal to 1.67×10^{-4}, and a permissible system failure probability equal to 5×10^{-4}, being in fact the sum of all members permissible failure probabilities, Figure 11 may be presented. It performs variation of weight of the structure as the objective function against number of generations while constraining separately members and system failure probabilities against the allowable values. It is shown that under a system failure probability constraint, the optimum weight is of a further quantity, equal to 1144kg in comparison to the 1140 kg optimum weight of the structure when failure probability of members is constrained, resulting in nearly equal weights.

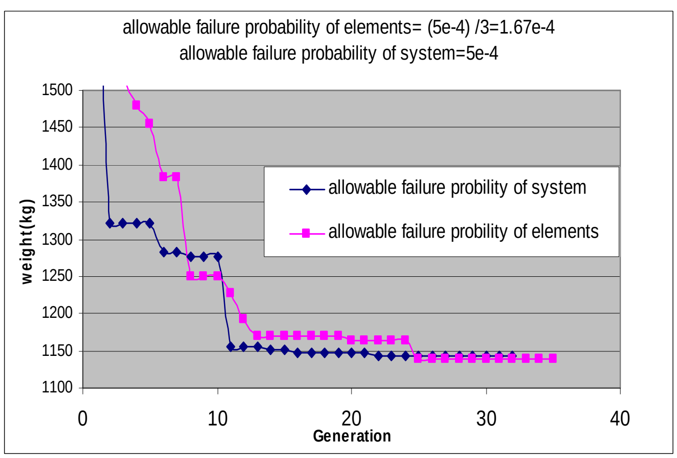

Figure 11. Convergence history of optimum weight against generation under different failure probabilities for example 2

Table 5: Safety coefficient due to increase of vcel according to Figure 6-a

VCYS = 0.05									
VCEL	0.001	0.05	0.01	0.1	0.2	0.25	0.3	0.35	0.4
Optimum weight	1120	1120	1120	1178	1709	1823	1859	1911	2144
Safety coefficient	1	1	1	1.05	1.526	1.627	1.66	1.706	1.914

6.3 Example (3) – A six-storey 3D frame structure with rigid joints

As a third and final benchmark example, a 6-storey 3-span space frame with rigid beam-to-column joints will be considered. The structural geometry of the problem is shown in Figure 12, while member numbering and sizes, including plan & side views are illustrated in Figure 13 and 14. Figure 15 indicate the exerted lateral loading for this example, while there is also a uniform distributed loading equal to 200N/cm applied on the beams. Also GA control parameters and material properties as well as categorized set of design variables for the frame are listed in Tables 6 and 7, respectively.

VCEL and VCYS for the members are arbitrarily taken as 0.1 and 0.05, respectively. Also, member's allowable failure probability is constrained to 4.167×10^{-6} while for the system, this value is chosen as 10^{-3}.

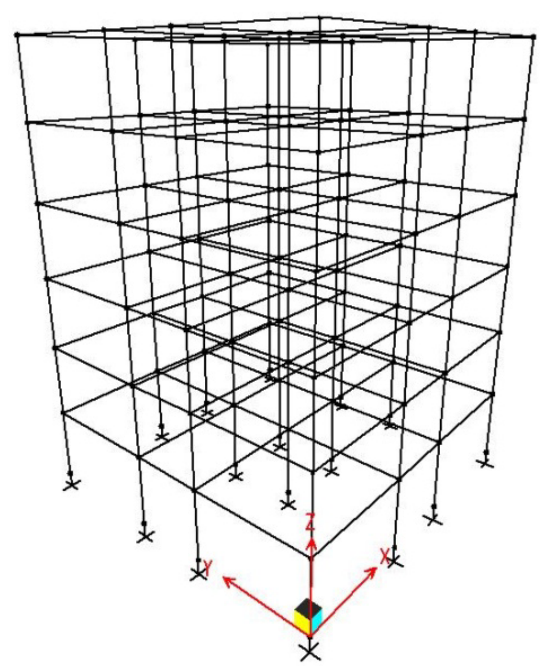

Figure 12. Structural geometry for example 3

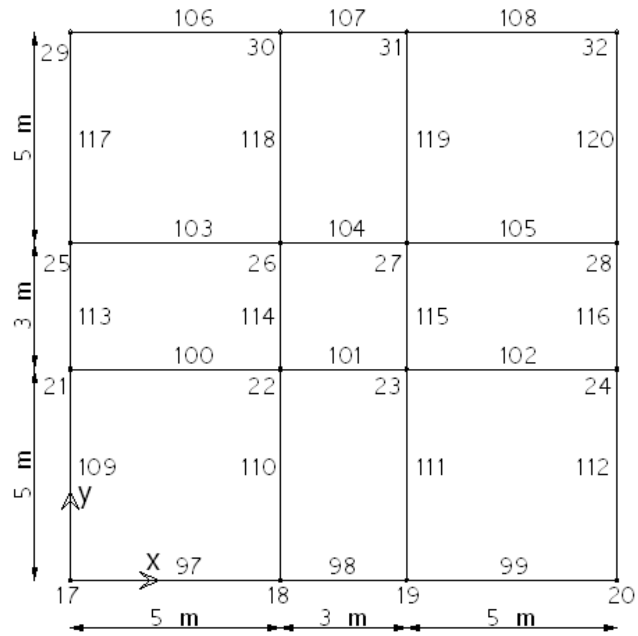

Figure 13. Member numbering and sizes- plan view, example 3

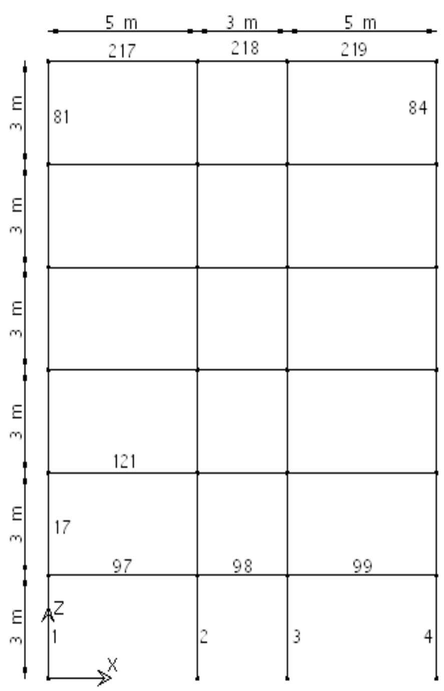

Figure 14. Member numbering–side view, example 3

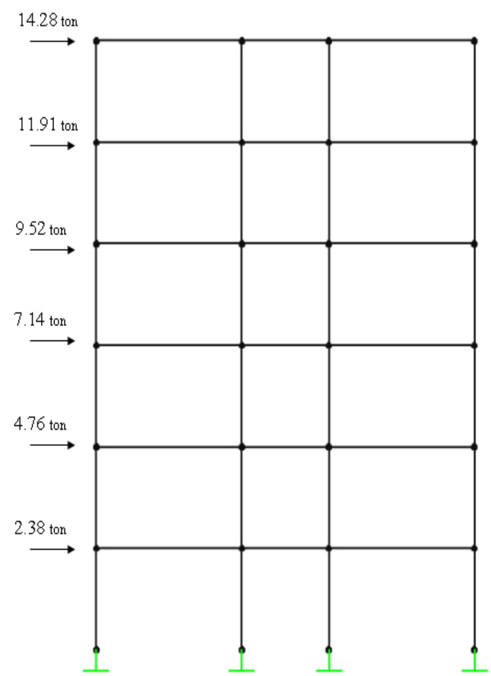

Figure 15. lateral loading, Example 3

Table 6: optimization input data for example 3

Control parameters to GA	Mutation probability	0.004
	Cross probability	1
	Population size	40
Other parameters	Materials density	0.0785
	Elasticity module	21e6 (N/cm^3)

Table 7: Categorized set of design variables for example 3

No.type	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
No.el	1, 2, 3, 4, 5, 8, 9, 12, 13, 14, 15, 16	6, 7, 10, 11	17, 18, 19, 20, 21, 24, 25, 28, 29, 30, 31, 32	22, 23, 26, 27	33, 34, 35, 36, 37, 40, 41, 44, 45, 46, 47, 48	38, 39, 42, 43	49, 50, 51, 52, 53, 56, 57, 60, 61, 62, 63, 64	54, 55, 58, 59
No.type	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
No.el	65, 66, 67, 68, 69, 72, 73, 76, 77, 78, 79, 80	70, 71, 74, 75	81, 82, 83, 84, 85, 88, 89, 92, 93, 94, 95, 96	86, 87, 90, 91	97, 99, 100, 102,103,105, 06, 108, 109, 110, 111, 112, 117, 118, 119, 120	98, 101, 104, 107, 113, 114, 115, 116	121, 123, 124, 126, 127, 129, 130, 132, 133, 134, 135, 136, 141, 142, 143, 144	122,125, 128,131, 137,138, 139, 140
No.type	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
No.el	145, 147, 148,150,151,153,154,156, 157, 158,159, 160, 165, 166, 167, 168	146, 149, 152, 155, 161, 162, 163, 164	169, 171, 172, 174, 175, 177, 178, 180, 181, 182, 183, 184, 189, 190, 191, 192	170, 173, 176, 179, 185, 186, 187, 188	193, 195, 196, 198, 199, 201, 202, 205, 206, 207, 208, 213, 214, 215, 216	194, 197, 200, 203, 209, 210, 211, 212	217, 219, 220, 222, 223, 225, 226, 228, 229, 230, 231, 232, 237, 238, 239, 240	218,221, 224,227, 233,234, 235, 236

Having run the problem while aiming for the minimum weight of the structure, the following constraints were considered separately:

Failure probabilities at each end of the columns and beam sections and also at the midpoints of beams do not exceed the permissible values ($Pf_i \leq Pf_{ai}$). Note that for this example Pf_{ai} is a constant value, while Pf_i is different for every point in the columns or beam sections. The value listed in row 5, column 2 of that table just shows that the maximum pf_{ai} has not exceeded the permissible failure probabilities for all the members pf_{ai}.

Sum of most critical failure probabilities for total elements not to exceed the permissible failure probability for the system ($\sum_{i=1}^{ne} Pf_{i\,critical} \le Pf_a$) (See first state in section IV-B).

Sum of most critical failure probabilities for total elements and nodes, not to violate the corresponding system allowable values ($\sum_{i=1}^{ne} Pf_{i\,critical} + \sum_{j=1}^{nn} Pf_{j\,critical} \le Pf_a$) (See second state in section IV-B).

Total sum of critical sections failure probabilities to be limited not to exceed the system allowable quantity ($(\sum_{i=1}^{neb} Pf_{(3i-2,3i-1,3i)} + \sum_{j=1}^{nec} Pf_{(3j-2,3j-1)}) \le Pf_a$) (See third state in section IV-B).

In above relations, *ne*, *nn*, *neb* and *nec* are the total number of elements, nodes, beams and columns, respectively. Also, 3i-2 and 3i-1 define ends of the members and 3i is the midpoint of the beams.

Procedure of optimization on Example 3 performed a convergence history as indicated in Figure 16. According to Table 8, regarding the first set of constraints, an optimum weight of 75261kg was recorded for the Pf_{ai}. Interesting to note that for the second constraint, a percentage difference of -0.27% was recorded for the optimum weight, when compared with that of the first set of constraints. This might be concluded that by assuming an arbitrary value for the Pf_a, a value for Pf_i May be computed by dividing Pf_i by *ne* as shown below Table 8 [1].

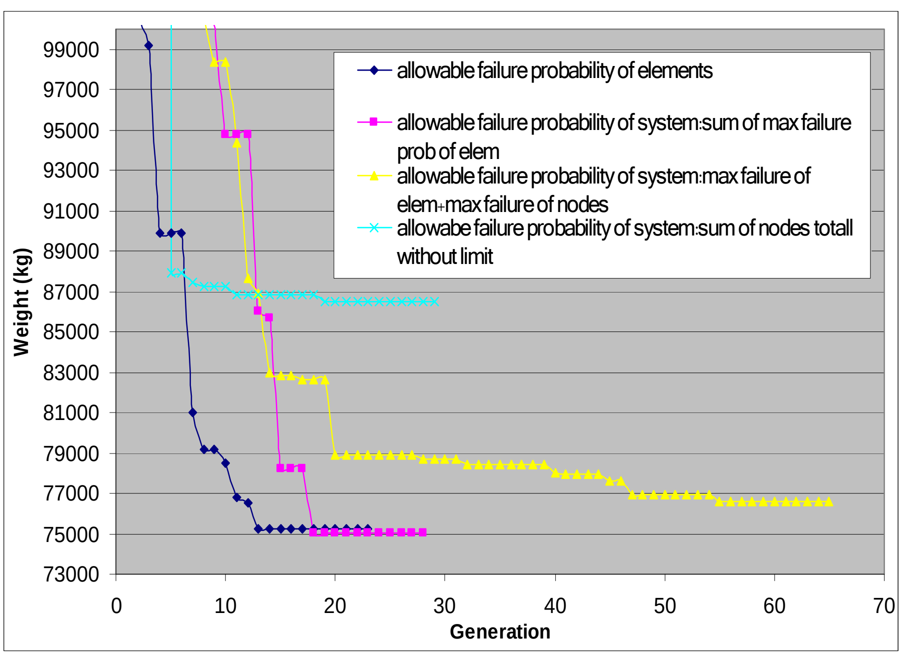

Figure 16. convergence history of optimum weight against generation under different failure probabilities, example 3

Table 8: Variation of displacements and optimum weight under different constraints of failure probability

Kind of reliability / Results	pf_i	$\sum_{i=1}^{ne} Pf_{i\_critical}$	$\sum_{i=1}^{ne} Pf_{i\_critical} + \sum_{j=1}^{nn} Pf_{j\_critical}$	$(\sum_{i=1}^{neb} Pf_{(3i-2,3i-1,3i)} + \sum_{j=1}^{nec} Pf_{(3j-2,3j-1)})$
Max Transverse Disp. (cm) └	1.1	1.49	2.2	1.05
Max Lateral Disp . (cm) ╘	6.3	7.3	4.9	6.96
Max. slender. Coef. ├	61	82	82	53.6
Failure probability ┤	6.5 e-05*	3.2e-4	8.5e-5	1.28e-4
Optimum weight (kg)	75261	75055	76619	86506
Variation of optimum weight the system failure probability methods ratio the members failure probability method	--	-0.27 %	1.8 %	15 %

└ : Allowable transverse disp.[20] = length of member / 240 = 500 cm / 240 = **2.1 cm**

╘ : Allowable lateral disp.[19] = (Height_build * Coef. / No_floor) = 18*0.025/6= 0.075 m **=7.5 cm**
T= 0.08 * (height_build^(3/4)) IF T < 0.7 then Coef.= 0.025
Else Coef. =0.02 ∴ T=0.08*(18^3/4))=0.69 ⇒ Coef.=0.025

├ : Allowable slender. coef. [20] **= 200**

┤ : Allowable failure probability: $Pf_{ai}=Pf_a / ne$, Pf_a **=1e-3,** Pf_{ai} **= 4.167e-6 ,**

*: the maximum pf_i just to show that it has not exceeded the permissible failure probabilities for all the members pf_i

7. CONCLUSIONS

In this work, the attempts were made to introduce ways in which some approximated techniques may well substitute the safety factors used through conventional design approach.

Also, by performing different examples, quality performance of a written program to handle reliability-based optimization of frame structures was observed, results of which may be listed as follows:

1. In case where allowable failure probability as well as VCYS and VCEL are taken as nearly zero, the output through reliability-based optimization and deterministic one will result very close solutions, indicating to some extent the correctness of the approach

taken and the program developed.

2. Exceeding the VCYS and VCEL values, will lead to a higher optimum weight, comparing to point (1) rose above.
3. Interpolation technique may be a reliable source of reference in predicting the optimum weight of the structure for different combinations of VCYS and VCEL at a certain allowable failure probability. Thus as shown in Figure 10, it may essentially be fair to generalize that, by increasing the allowable failure probability of members while maintaining the same value for VCYS, a distinct reduction on the weight will be found. Also an accumulative set of results indicating variations of optimum weight with different measurements of VCEL and VCYS shows that by increasing each of these coefficients, the optimum weight may also increase.
4. Using modern design techniques and reliability-based optimization may be a suitable substitute in deciding for the safety factors as a result of which a more reliable performance of the structure may be achieved.
5. Results obtained may indicate that for prevention of nodal failure, one should define a nodal failure probability constraint assuring that displacement in members and drift in floors do not exceed from allowable values.

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