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PLASTIC BEHAVIOUR AND STABILITY CONSTRAINTS IN THE RELIABILITY BASED SHAKEDOWN ANALYSIS AND OPTIMAL DESIGN OF SKELETAL STRUCTURES

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ABSTRACT

This paper includes description of reliability based design optimization of elasto-plastic skeletal structures under multi-parameter static loading. Presented approach is based on the assumption that the complementary strain energy of the residual forces is considered as an overall measure of the plastic performance for plastic shakedown analysis and optimal design of the structure, and this measure is an uncertain quantity responsible for resistance of the structure by assuming Gaussian distribution. The problems yield to nonlinear mathematical programming which are solved by the use of bi-level sequential quadratic algorithm. Simple portal frames are optimized to illustrate the proposed approaches.

Keywords: Reliability-based design optimization; complementary strain energy; shakedown analysis; gaussian distribution

1. INTRODUCTION

Structures of mechanical engineering for instance power plants, reactors, pressure vessels, etc. or civil engineering for instance trusses, frames, grids, bridge decks etc. are exposed to variable loading, particularly cyclic or repeated loading. In these situations the step-by-step elastic-plastic calculation is somewhat very costly in computing times. The most efficient way to handle the problem is to apply the shakedown theory. This theory is based on experimental facts obtained from realistic structures or laboratory specimen. It offers a direct method as like as limit analysis to perform the analysis of the problem [4].

There are several engineering problem where the designer should face to the problem of limited load carrying capacity of the connected elements of the structures [1, 2, 3]. Such problem can be found during the rehabilitation of the old buildings with composite plates (floors) or in case of steel frame structures. Due to the different behaviour of the beam

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elements of the skeletal structures for compression and tension the stability problems need a certain care. The national standards give appropriate tool to solve the problem. In structural analysis the static and kinematic theorems provide appropriate tools to solve these complex problems [4, 5, 6].

In classical plasticity, optimal plastic analysis and design are among the most important basic problems [7]. At the application of the plastic analysis and design methods the control of the plastic behavior of the structures is an important requirement. Since the shakedown analysis provides no information about the magnitude of the plastic deformations and residual displacements accumulated before the adaptation of the structure, therefore for their determination several bounding theorems and approximate methods have been proposed. Among others Kaliszky and Lógó [1, 4] suggested that the complementary strain energy of the residual forces could be considered an overall measure of the plastic performance of structures and the plastic deformations should be controlled by introducing a limit for magnitude of this energy. In engineering the problem parameters (geometrical, material, strength, manufacturing) are given deterministically or considered with uncertainties. The obtained analysis and/or design task is more complex and can lead to reliability analysis and design [5,8,9].

This paper is a revised and extended version of the CST2010 Conference presentation of Lógó et al. [3]. The aim of this work is to take into consideration the influence of the limited load carrying capacity of the connections on the plastic limit state of elasto-plastic steel (or composite) trusses and frames under multi-parameter static loading and probabilistically given conditions during the design. In addition to the plastic shakedown analysis and optimal design to control the plastic behaviour of the structure, bound on the complementary strain energy of the residual forces is also applied. This bound has significant effect for the load parameter [2]. The different behaviours of the beam elements in compression and tension are taken into consideration by the reduced load carrying capacity in compression. The calculation is based on the requirements of the Eurocode [11].

The second objective of this paper is to analyze the influence of the limited load carrying capacity of the connections on the optimal frame designs. The main structural elements of steel framed multi-storey structures are columns, beams and their connections. Especially, inaccuracies in manufacturing and later deterioration of the connections change their limited load carrying capacity, which in consequence influences the behavior of the structure, see [2], and has to be taken into account in the analysis and design. It is assumed that the stiffness of the connections is semi-rigid [2, 10, 20], i.e. somewhere between rigid and pinned connection limits.

If the design uncertainties (manufacturing, strength, geometrical) are expressed by the calculation of the complementary strain energy of the residual forces, the reliability based extended shakedown analysis and optimal design problems can be formed. Numerical procedures are elaborated which are based on a direct integration technique and the uncertainties are assumed to follow Gaussian distribution. The formulations of the problems yield to nonlinear mathematical programming which are solved by the use of sequential quadratic algorithm. The nested optimization procedure is governed by the reliability index calculation. The applications are illustrated by some numerical examples.

2. ELEMENTS OF THE MECHANICAL MODELING AND THE ANALYSIS

2.1 Notations and loadings

In the paper the following notations are used:

$\mathbf{P}_d$: dead load;

$\mathbf{P}_1$, $\mathbf{P}_2$: static working loads;

$\mathbf{M}_h^e$, $\mathbf{M}_d^e$: fictitious elastic moments calculated from the live and dead loads assuming that the structure is purely elastic;

$\mathbf{Q}^r$, $\mathbf{M}^r$: residual internal forces and moments;

$\mathbf{M}_d^p$, $\mathbf{M}_h^p$: plastic moments;

$\overline{\mathbf{M}}^p$: limit moments of the bounded beam to column joints;

W_{p0} : allowable complementary strain energy of the residual forces;

$\boldsymbol{\sigma}_y$, E : yield stress and Young's modulus;

A_i, I_i, S_{0i} and ℓ_i : areas, moment of inertias of the cross-sections and length of the finite elements (*i=1,2,…,n*), respectively;

$\overline{\mathrm{S}}_{\mathrm{j}}$: stiffness of the *j-th* semi-rigid connection, (*j=1,2,…,k*) is the number of semi-rigid connections, they are subsets of (*i=1,2,…,n*);

$\mathbf{F}, \mathbf{K}, \mathbf{G}, \mathbf{G}^*$: flexibility, stiffness, geometrical and equilibrium matrices;

β : reliability index;

Φ^{-1} : inverse cumulative distribution function (so called probit function) of the Gaussian distribution;

$f\left(\mathrm{W}_{p0}\right)$: the Gaussian probability density function of the complementary strain energy of the residual forces;

V_0 : represents the total limit volume of the structure.

The problem class which is considered is based on the assumption that the dead and working (pay) loads are deterministic. The structure is subjected to a dead load $\mathbf{P}_d$ and two independent, static working loads $\mathbf{P}_1$ and $\mathbf{P}_2$ with multipliers $m_1 \geq 0$, $m_2 \geq 0$. In the analysis five loading cases (*h=1,2,…,5*) shown in Table 1 are taken into consideration. For each loading case a shakedown multiplier m_{sh} can be calculated. Making use of these multipliers a limit curve can be constructed in the plane m_1, m_2 (Figure 1). The structure does not shake down, under the action of the loads $m_1\mathbf{P}_1$, $m_2\mathbf{P}_2$, if the points corresponding to the multipliers m_1, m_2 lies inside or on the limit curve.

Table 1: Load combinations

h	Multipliers	Loads	Load multipliers
1	$m_2 = 0$	$\mathbf{Q}_1 = \mathbf{P}_1$	m_{s1}
2	$m_1 = 0$	$\mathbf{Q_2} = \mathbf{P_2}$	m_{s2}
3	$m_1 = 0.5m_2$	$\mathbf{Q}_3 = [0.5\mathbf{P}_1,\ (0.5\mathbf{P}_1 + \mathbf{P}_2),\ \mathbf{P}_2]$	m_{s3}
4	$m_1 = m_2$	$\mathbf{Q}_4 = [\mathbf{P}_1,\ (\mathbf{P}_1 + \mathbf{P}_2),\ \mathbf{P}_2]$	m_{s4}
5	$m_1 = 2m_2$	$\mathbf{Q}_5 = [2.0\mathbf{P}_1,\ (2.0\mathbf{P}_1 + \mathbf{P}_2),\ \mathbf{P}_2]$	m_{s5}

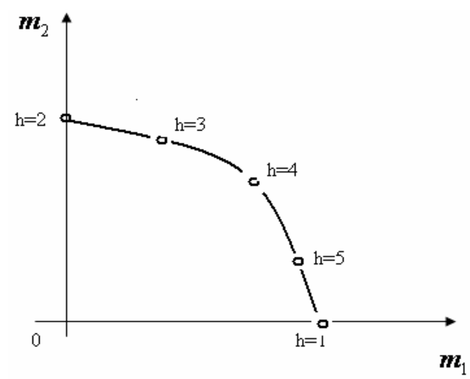


Figure 1. Limit curve and safe domain

2.2 Modeling of the beam-to-column connections

The main structural elements of steel framed structures are the columns, the beams and their connections. Conventionally the beam-to-column connections are considered to be either pinned or rigid. In case of pinned connections, the frames have to be stabilized by appropriate bracing systems. Such frames are named brace frames by Eurocode [11]. The term rigid in this context implies that the connection is capable of resisting moments with a high stiffness. When the connections are rigid, the overall stability may be provided by the frame itself without the inclusion of specific bracing systems. Although the idealization of connection stiffness as pinned or rigid has been applied exclusively in the past. It is generally recognized that the real behaviour of the connections is never as ideal as assumed in the analysis.

Analysis and design of steel frames with semi-rigid connections have been extensively examined Monforton and Wu [12]; Frye and Morris [13]; Lui and Chen [14]; Cunningham [15]; King [16]; King and Chen [17]; Dhillon and Malley [18]; Sekulovic and Salatic [19]; Kaveh and Moez [20]; Lógó et al. [2], Wang [21]; Ihaddoudene et al. [22]. Optimum design of steel frames with semi-rigid connections has also been investigated using mathematical programming techniques Xu and Grierson [23]; Almusallam [24]; Alsalloum and Almusallam [25]; Simoes [26]; Kameshki and Saka [27] and [28]; Hayalioglu and Degertekin [29], [30] and [31]; Csebfalvi [32]; Degertekin and Hayalioglu [33].

2.2.1 Modeling of the Semi-Rigid Connections

In case of rigid connections between the nodes and elements are defined as default. However in case of semi-rigid connections can be taken into consideration by reducing elementary stiffness matrix $\hat{\mathbf{K}}$:

$$\hat{\mathbf{K}} = \mathbf{K} - \frac{1}{k_{pp}} \mathbf{k}_p \mathbf{k}_p^T \tag{1}$$

where k_{pp} is a main diagonal element of $\hat{\mathbf{K}}$, $\mathbf{k}_p$ and $\mathbf{k}_p^T$ are the column and the row in $\mathbf{K}$ that involve k_{pp}.

The typical general behaviour of the semi-rigid connection can be illustrated by a moment-rotation relationship shown in Figure 2. In this paper this relationship will be approximated by three different elasto-plastic models given in Figure 3. Here $\overline{\mathbf{M}}^{\mathbf{p}}$ is the plastic limit moment and $\overline{\mathrm{S}}$ is the stiffness of the semi-rigid connection. Their magnitudes can be assumed from the results of experiments. These models are incorporated in the elementary stiffness matrix of the beam elements. In this research the assumptions are mentioned here will be considered for modeling of the semi-rigid connections.

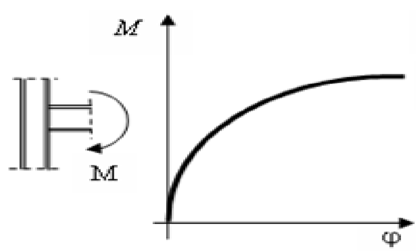
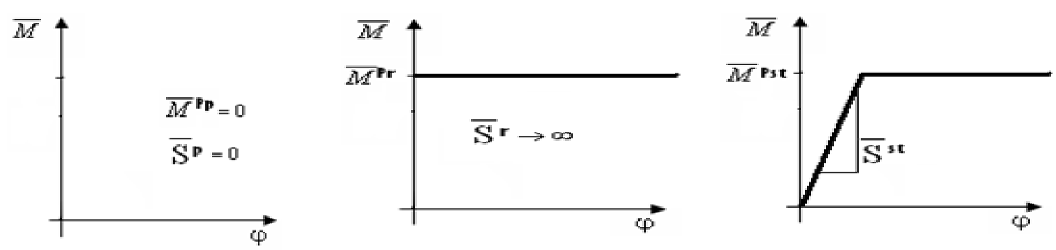


Figure 2. Real behaviour of the semi-rigid connection



a. Pinned connection b. Rigid connection c. Semi-rigid connection

Figure 3a-c. Models of the semi-rigid connections

2.3 Reliability-based control of the plastic deformations

At the application of the plastic analysis and design methods the control of the plastic behaviour of the structures is an important requirement. Following the suggestions of Kaliszky and Lógó [1, 4] the complementary strain energy of the residual forces could be considered as an overall measure of the plastic performance of structures and the plastic deformations should be controlled by introducing a bound for magnitude of this energy:

$$\frac{1}{2}\sum_{i=1}^{n} \mathrm{Q}_i^r \mathbf{F}_i \mathrm{Q}_i^r \le \mathrm{W}_{\mathrm{p0}} \tag{2}$$

Here W_{p0} is an assumed bound for the complementary strain energy of the residual forces. This constraint can be expressed in terms of the residual moments $\mathrm{M}_{i,a}^r$ and $\mathrm{M}_{i,b}^r$ acting at the ends (*a* and *b*) of the finite elements as follows:

$$\frac{1}{6\mathrm{E}}\sum_{i=1}^{n} \frac{\ell_i}{\mathrm{I}_i}\left[(\mathrm{M}_{i,a}^r)^2 + (\mathrm{M}_{i,a}^r)(\mathrm{M}_{i,b}^r) + (\mathrm{M}_{i,b}^r)^2\right] \le \mathrm{W}_{\mathrm{p0}}. \tag{3}$$

By the use of Eq. (3) a limit state function can be constructed:

$$g\left(\mathrm{W}_{\mathrm{p0}}, \mathbf{M}^r\right) = \mathrm{W}_{\mathrm{p0}} - \frac{1}{6\mathrm{E}}\sum_{i=1}^{n} \frac{\ell_i}{\mathrm{I}_i}\left[(\mathrm{M}_{i,a}^r)^2 + (\mathrm{M}_{i,a}^r)(\mathrm{M}_{i,b}^r) + (\mathrm{M}_{i,b}^r)^2\right]. \tag{4}$$

The plastic deformations are controlled while the bound for the magnitude of the complementary strain energy of the residual forces exceeds the calculated value of the complementary strain energy of the residual forces. On similar way a limit state function can be determined in case of axially loaded structures. Neglecting the details the formulation are as follows:

$$g\left(\mathrm{W}_{\mathrm{p0}}, \mathbf{N}^r\right) = \mathrm{W}_{\mathrm{p0}} - \frac{1}{2\mathrm{E}}\sum_{i=1}^{n} \frac{\ell_i}{\mathrm{E}_i}(\mathrm{N}_i^r)^2 \rangle 0. \tag{5}$$

Introducing the basic concepts of the reliability analysis and using the force method the failure of the structure can be defined as follows:

$$g\left(\mathbf{X}_{\mathrm{R}}, \mathbf{X}_S\right) = \mathbf{X}_{\mathrm{R}} - \mathbf{X}_S \le 0; \tag{6}$$

where $\mathbf{X}_{\mathrm{R}}$ indicates either the bound for the statically admissible forces $\mathbf{X}_{\mathrm{S}}$ or a bound for the derived quantities from $\mathbf{X}_{\mathrm{S}}$. The probability of failure is given by

$$\mathrm{P}_f = F_g\left(0\right); \tag{7.a}$$

and can be calculated as

$$\mathrm{P}_f = \int\limits_{g\left(\mathbf{X}_\mathrm{R},\mathbf{X}_S\right)\le 0} f\left(\mathbf{X}\right)dx. \tag{7.b}$$

Let assumed that due to the uncertainties the bound for the magnitude of the complementary strain energy of the residual forces is given randomly and for sake of simplicity it follows the Gaussian distribution with given mean value $\overline{\mathrm{W}}_{\mathrm{p}0}$ and standard deviation σ_w. Due to the number of the probabilistic variables (here only single) the probability of the failure event can be expressed in a closed integral form:

$$\mathrm{P}_{f,calc} = \int\limits_{g\left(\mathrm{W}_{\mathrm{p}0},\mathbf{M}^r\right)\le 0} f\left(\overline{\mathrm{W}}_{\mathrm{p}0},\sigma_w\right)dx \text{ or for trusses } \mathrm{P}_{f,calc} = \int\limits_{g\left(\mathrm{W}_{\mathrm{p}0},\mathbf{N}^r\right)\le 0} f\left(\overline{\mathrm{W}}_{\mathrm{p}0},\sigma_w\right)dx. \tag{7.c}$$

By the use of the strict reliability index a reliability condition can be formed:

$$\beta_{target} - \beta_{calc} \le 0\,; \tag{7.d}$$

where β_{target} and β_{calc} are calculated as follows:

$$\beta_{target} = -\Phi^{-1}\left(\mathrm{P}_{f,target}\right); \tag{7.e}$$

$$\beta_{calc} = -\Phi^{-1}\left(\mathrm{P}_{f,calc}\right). \tag{7.f}$$

Due to the simplicity of the present case the integral formulation is not needed, since the probability of failure can be described easily with the distribution function of the normal distribution of the stochastic bound W_{p0}.

3. EXTENDED SHAKEDOWN DESIGN OF SKELETAL STRUCTURES

3.1 Basic design formulations of axially loaded structures

According the recommendations of Eurocode [11] the design resistance of the compressed members should be reduced. The formulation is as follows:

$$N_{b,Rd} = \chi A \sigma_y \tag{8.a}$$

Here χ is the reduction coefficient defined by the relevant buckling mode. For axial compression in members the value of χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ should be determined from the relevant buckling curve according to:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \le 1 \tag{8.b}$$

where $\Phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$. The imperfection factor α corresponds to the appropriate buckling curve.

The mechanical model can be given by the following conditions: determine the maximum load multiplier m_{sh} and cross-sectional dimensions under the conditions that (*i*) the structure with given layout is strong enough to carry the loads ($\mathbf{P}_d + m_{sh}\mathbf{Q}_h$), (*ii*) satisfies the constraints on the self equilibrated residual forces and limited strength capacities, (*iii*) satisfies the constraints on plastic deformations and residual displacements, (*iv*) safe enough and the required amount of material does not exceed a given limit. The design solution method based on the static theorem of shakedown analysis is formulated as below:

$$\text{Maximize} \quad m_{sh} \tag{9.a}$$

Subject to

$$\mathbf{G}^*\mathbf{N}_h^r = \mathbf{0}; \tag{9.b}$$

$$\mathbf{N}_d^e = \mathbf{F}^{-1}\mathbf{G}\mathbf{K}^{-1}\mathbf{P}_d; \tag{9.c}$$

$$\mathbf{N}_h^e = \mathbf{F}^{-1}\mathbf{G}\mathbf{K}^{-1}m_{sh}\mathbf{Q}_h; \tag{9.d}$$

$$-\chi A\sigma_y \le (N_{di}^e + N_{hi}^r + \max N_{hi}^e) \le A\sigma_y, (i = 1, 2, ..., n); \tag{9.e}$$

$$-\chi A\sigma_y \le (N_{di}^e + N_{hi}^r + \min N_{hi}^e) \le A\sigma_y, \ (i = 1, 2, ..., n); \tag{9.f}$$

$$\beta_{target} - \beta_{calc} \le 0; \tag{9.g}$$

$$\sum_i A_i \ell_i - V_0 \le 0. \tag{9.h}$$

Here Eq. (9.b) is equilibrium equations for the residual forces and Eqs. (9.c-d) express the calculations of the elastic fictitious internal forces from the dead load and from the live (pay) loads, respectively. Eqs. (9.e-f) are the yield conditions. Eq. (9.g) is the reliability condition which control the plastic behaviour of the truss by the use of the residual strain energy. The material redistribution is controlled by Eq. (9.h). The goal is to find the maximum of the statically admissible load multiplier m_{sh}.

As it was stated before this is a constrained nonlinear mathematical programming problem and it can be solved by any appropriate solution method (e.g. SPQ method) which is governed by an iterative procedure for Eq. (9.g). Starting with a reliable structure (feasible solution) and selecting one of the loading combination $\mathrm{Q}_h;(h=1,2,...,5)$ a shakedown multiplier m_{sh} can be determined where the target reliability index is reached iteratively. Then the limit curve and the safe loading domain of the shakedown can be constructed and the unknown cross-sectional dimensions can be obtained.

In details the highly nonlinear problem is solved by applying a sequential quadratic mathematical programming algorithm (SQP) with the following iterative procedure:

Step 1. Assuming the initial cross-sectional dimension x_i with almost failure free solution [$\mathrm{P}_{f0} = F(\bar{W}_{p0} - 3\sigma_w)$] determine m_{sh} using Eqs. (9.a-h).

Step 2. Using m_{sh} determine the optimal cross-sectional dimension x_i as solution of the mathematical programming problem (9.a-h).

Step 3. Using x_i obtained in step 2 determine the new reliability index $\beta_{calc,i}$ in problem (9) increase the probability of failure P_{f0} by appropriate $\Delta\mathrm{P}_f$.

Step 4. Using m_{sh} and x_i obtained in step 3 repeat steps 2-3 as long as the difference of the results $\beta_{target} - \beta_{calc,i}$ of two consecutive steps 4 are acceptable small.

3.1.1 Alternative design formulation for trusses

Interchanging the objective function -Eq.(9.a)- and the last constraint -Eq. (9.h)- an alternative design formulation can be formulated. This formulation yields to the classical minimum volume problem:

$$\text{Minimize } V = \sum_i A_i \ell_i \tag{10.a}$$

Subject to

$$\mathbf{G}^* \mathbf{N}_h^r = \mathbf{0}\,; \tag{10.b}$$

$$\mathbf{N}_d^e = \mathbf{F}^{-1}\mathbf{G}\mathbf{K}^{-1}\mathbf{P}_d\,; \tag{10.c}$$

$$\mathbf{N}_h^e = \mathbf{F}^{-1}\mathbf{G}\mathbf{K}^{-1} m_{sh}\mathbf{Q}_h\,; \tag{10.d}$$

$$-\chi A\sigma_y \le (N_{di}^e + N_{hi}^r + \max N_{hi}^e) \le A\sigma_y\,, (i=1,2,...,n)\,; \tag{10.e}$$

$$-\chi A\sigma_y \le (N_{di}^e + N_{hi}^r + \min N_{hi}^e) \le A\sigma_y\,,\ (i=1,2,...,n)\,; \tag{10.f}$$

$$\beta_{target} - \beta_{calc} \le 0; \tag{10.g}$$

$$m_{sh} - m_0 \le 0. \tag{10.h}$$

Here all the equations have the same meanings as it was before in Eqs. (9.b-g) while Eq. (10.h) gives an upper bound for the external loads.

Trough the optimality condition it can be proved that this nonlinear mathematical programming problem Eqs. (10.a-h) leads to the same optimal solution as problem Eqs. (9.a-h).

3.2 Basic design formulations of frames

A simplified method is introduced here for the design of frames with limited rotational capacity at the beam-column connections. For sake of simplicity only the bending moments are considered. According the recommendation of Eurocode [11] the design buckling resistance moment of a laterally unrestrained beam should be taken as:

$$M_{b,Rd} = \chi_{LT} S_0 \sigma_y . \tag{11.a}$$

Here χ_{LT} is the reduction factor for the lateral-torsional buckling. The calculation of the value of χ_{LT} for the appropriate non-dimensional slenderness $\bar{\lambda}_{LT}$ should be determined as follows:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \le 1 \tag{11.b}$$

where $\Phi = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}{}^2\right]$. The imperfection factor α_{LT} corresponds to the appropriate buckling curve.

Similarly to the truss problem also a mechanical model can be created where one has to determine the maximum load multiplier m_{sh} and cross-sectional dimensions under the conditions that (*i*) the structure with given layout is strong enough to carry the loads ($\mathbf{P}_d + m_{sh}\mathbf{Q}_h$), (*ii*) satisfies the constraints on the limited beam-to-column strength capacity, (*iii*) satisfies the constraints on plastic deformations and residual displacements, (*iv*) safe enough and the required amount of material does not exceed a given limit. The solution formulation based on the static theorem of shakedown analysis is formulated as below:

$$\text{Maximize} \quad m_{sh} \tag{12.a}$$

Subject to

$$\mathbf{G}^*\mathbf{M}_h^r = \mathbf{0}\,; \tag{12.b}$$

$$\mathbf{M}_d^e = \mathbf{F}^{-1}\mathbf{G}\mathbf{K}^{-1}\mathbf{P}_d\,; \tag{12.c}$$

$$\mathbf{M}_h^e = \mathbf{F}^{-1}\mathbf{G}\mathbf{K}^{-1} m_{sh}\mathbf{Q}_h\,; \tag{12.d}$$

$$-\chi_{LT}\mathrm{S}_{0i}\sigma_y \le (M^e_{di} + M^r_{hi} + \max M^e_{hi}) \le 2\mathrm{S}_{0i}\sigma_y \,, (i = 1,2,...,n)\,; \tag{12.e}$$

$$-\chi_{LT}\mathrm{S}_{0i}\sigma_y \le (M^e_{di} + M^r_{hi} + \min M^e_{hi}) \le 2\mathrm{S}_{0i}\sigma_y \,,\ (i = 1,2,...,n)\,; \tag{12.f}$$

$$-\overline{M}^p_j \le (M^e_{dj} + M^r_{hj} + \max M^e_{hj}) \le \overline{M}^p_j \,,\ (j = 1,2,...,k)\,; \tag{12.g}$$

$$-\overline{M}^p_j \le (M^e_{dj} + M^r_{hj} + \min M^e_{hj}) \le \overline{M}^p_j \,,\ (j = 1,2,...,k)\,; \tag{12.h}$$

$$\beta_{target} - \beta_{calc} \le 0; \tag{12.i}$$

$$\sum_i A_i \ell_i - V_0 \le 0. \tag{12.j}$$

Here Eq. (12.b) expresses the self equilibrated internal force (bending) conditions. Eqs. (12.c-d) are the calculations of the elastic fictitious internal forces (moments) from the dead load and from the live (pay) loads, respectively. Eqs. (12.e-f) are the yield conditions for shakedown. Eqs. (12.g-h) are used as yield conditions of the semi-rigid connections. Eq. (12.i) is the reliability condition. The material redistribution is controlled by Eq. (12.j). The goal is to find the maximum of the statically admissible load multiplier m_{sh}.

Similarly as before this is a constrained nonlinear mathematical programming problem and it can be solved by any appropriate solution method (e.g., SPQ method) which is governed by an iterative procedure for Eq. (12.i). Selecting one of the semi-rigid connection models for each loading combination $\mathrm{Q}_h; (h = 1,2,...,5)$ gradually a shakedown multiplier m_{sh} can be determined, then the limit curve of shakedown can be constructed and the unknown cross-sectional dimensions can be obtained.

3.2.1 Alternative design formulation for frames

The “classical” minimum volume design model can be created similarly to Section 3.1.1. Interchanging the objective function - Eq. (12.a) - and the last constraint - Eq. (12.j) - an alternative design formulation can be formulated as bellow:

$$\text{Minimize } V = \sum_i A_i \ell_i \tag{13.a}$$

Subject to

$$\mathbf{G}^*\mathbf{M}^r_h = \mathbf{0}\,; \tag{13.b}$$

$$\mathbf{M}^e_d = \mathbf{F}^{-1}\mathbf{G}\mathbf{K}^{-1}\mathbf{P}_d\,; \tag{13.c}$$

$$\mathbf{M}^e_h = \mathbf{F}^{-1}\mathbf{G}\mathbf{K}^{-1} m_{sh}\mathbf{Q}_h\,; \tag{13.d}$$

$$-\chi_{LT}\mathrm{S}_{0i}\sigma_y \le (M^e_{di} + M^r_{hi} + \max M^e_{hi}) \le 2\mathrm{S}_{0i}\sigma_y \,, (i = 1,2,...,n)\,; \tag{13.e}$$

$$-\chi_{LT} S_{0i}\sigma_y \le (M^e_{di} + M^r_{hi} + \min M^e_{hi}) \le 2S_{0i}\sigma_y \,,\ (i = 1,2,...,n)\,; \tag{13.f}$$

$$-\overline{M}^p_j \le (M^e_{dj} + M^r_{hj} + \max M^e_{hj}) \le \overline{M}^p_j \,,\ (j = 1,2,...,k)\,; \tag{13.g}$$

$$-\overline{M}^p_j \le (M^e_{dj} + M^r_{hj} + \min M^e_{hj}) \le \overline{M}^p_j \,,\ (j = 1,2,...,k)\,; \tag{13.h}$$

$$\beta_{target} - \beta_{calc} \le 0; \tag{13.i}$$

$$m_{sh} - m_0 \le 0. \tag{13.j}$$

Here all the equations have the same meanings as it was before in Eqs. (12.b-i) while Eq. (13.j) gives an upper bound for the external loads.

Trough the optimality condition it can be proved that this nonlinear mathematical programming problem Eqs. (13.a-j) leads to same optimal solution as problem Eqs. (12.a-j).

4. NUMERICAL EXAMPLES

To demonstrate the theories and solution strategy introduced above, a nonlinear mathematical programming procedure is elaborated where one has to determine the safe loading domain and cross-sectional dimensions of a simple frame with deterministic loading data and with probabilistic bound for the magnitude of the complementary strain energy of the residual forces.

4.1 Example 1.

The application of the design method is illustrated by a simple test example shown in Figure 4. At the joints 2 and 4 the portal frame has semi-rigid connection. The working loads are $P_1 = 10\ kN$, $P_2 = 15\ kN$ and $P_d = 0$.

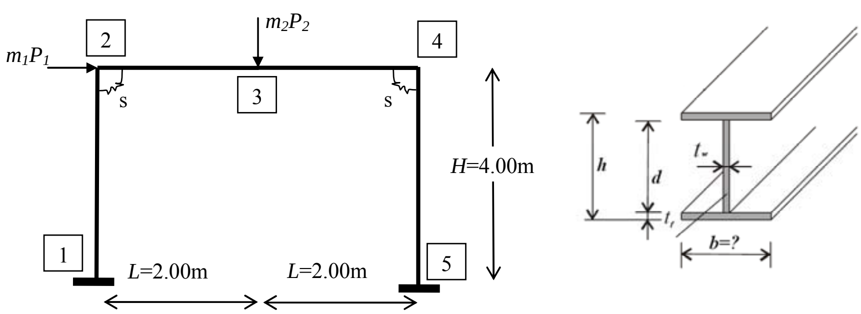


Figure 4. Portal frame as test problem

The yield stress and the Young's modulus are σ_y=21kN/cm^2 and E=2.07.10^6kN/cm^2. The non-variable cross-sectional dimensions are (see Figure 4): d=22cm, t_w=2cm, t_f=2cm. The applied beam-to-column rigidity values are varied gradually from fix to hinge connection ($\overline{S}_r$ =*1.0e+10,...,0.0kNm/rad*).

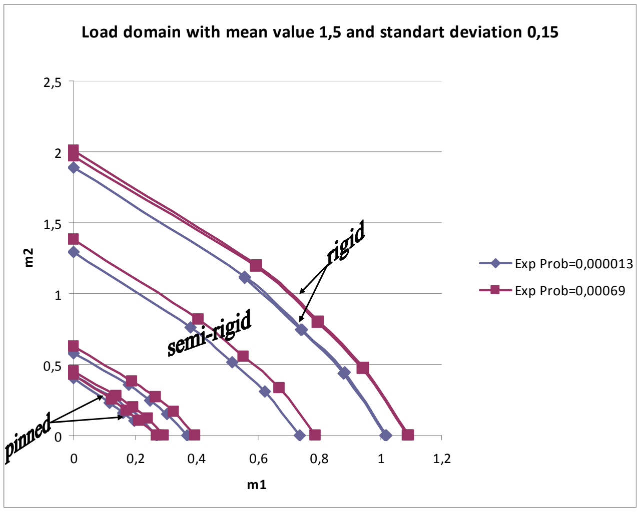


Figure 5a. Safe loading domain for the design

The results are presented in Figures 5.a-b. and 6. In Figure 5.a one can see the safe loading domains with different expected probability and beam-to-column connection rigidity, respectively. In Figure 5b the safe loading domains are presented in function of the different structural volume and rigidity, respectively. As it is seen the stiffnesses of the semi-rigid connection influence significantly the plastic behaviour of the frame. The results are in very good agreement with the expectations that the safe loading domain is convex and the increase of the safety level decrease the safe loading domain in case of the same volume limit.

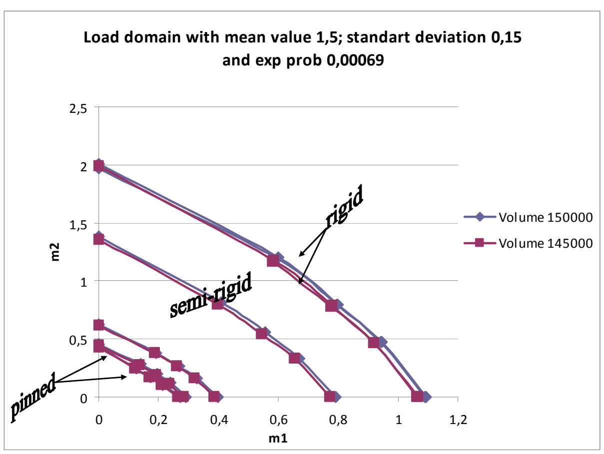


Figure 5b. Safe loading domain for the design

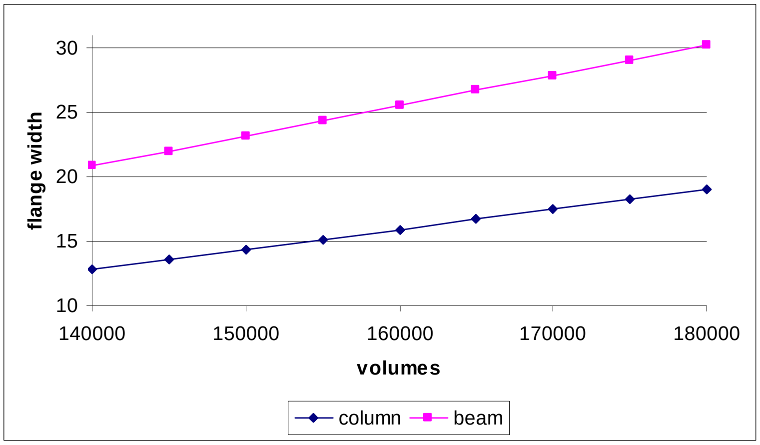


Figure 6. Variation of the flange width

In Figure 6. optimal cross-sectional widths are presented in function of the total volume. The beam-to- column rigidity is connected to some semi-rigid case ($\overline{S}_r$ =1.0e+6 *kNm/rad*) and the design is elaborated by the target reliability index $\beta = 3.2$.

4.2 Example 2

A two-storey frame is investigated as an analysis example shown in Figure 7. At the beam to column connections (joints 1, 3, 4 and 6) the portal frame has semi-rigid connection. The working loads are $P_1 = 10\ kN$, $P_2 = 30\ kN$ and $P_d = 0$.

The yield stress and the Young's modulus are $\sigma_y = 21 kN/cm^2$ and $E = 2.07 \cdot 10^6\ kN/cm^2$. The assumed connection rigidity is $\overline{S}_r$ =*0.1kNm/rad*. The cross-sectional data of the beam are: $A_{Beam} = 28.5 cm^2$, $I_{Beam} = 1943\ cm^4$, $S_{Beam} = 130.0\ cm^3$, while for the columns are : $A_{column} = 39.0 cm^2$, $I_{column} = 3891\ cm^4$, $S_{column} = 210.0\ cm^3$.

The results of the solution are presented in Figures 8.-10. where different target reliability indexes, mean values and standard deviations are considered. In Figure 8. the safe limit load domain is presented in case of different mean values of the complementary strain energy of the residual forces ($\overline{W}_{p0} = 90;\ 80;\ 70;\ 60 kNcm;$) with standard deviation $\sigma_w = 6 kNcm$ and target reliability index $\beta_{target} = 3.2$. One can see that increasing the mean values result in bigger safe loading domain. In Figure 9. the evaluation of the load multipliers are presented in function of the mean values of the complementary strain energy of the residual forces ($\overline{W}_{p0} = 90;\ 80;\ 70;\ 60 kNcm;$), different standard deviations ($\sigma_w = 2;\ 6;\ 10;\ 14 kNcm$) and target reliability index $\beta_{target} = 3.2$.

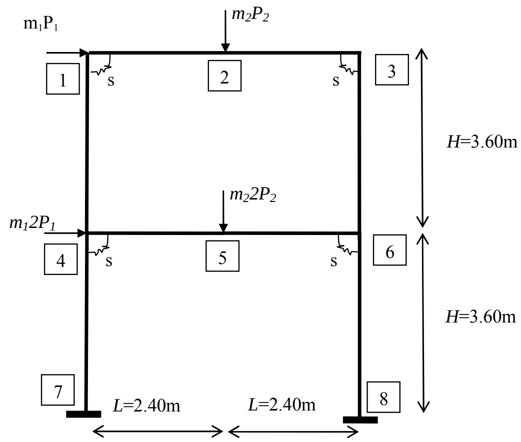


Figure 7. Two-storey frame

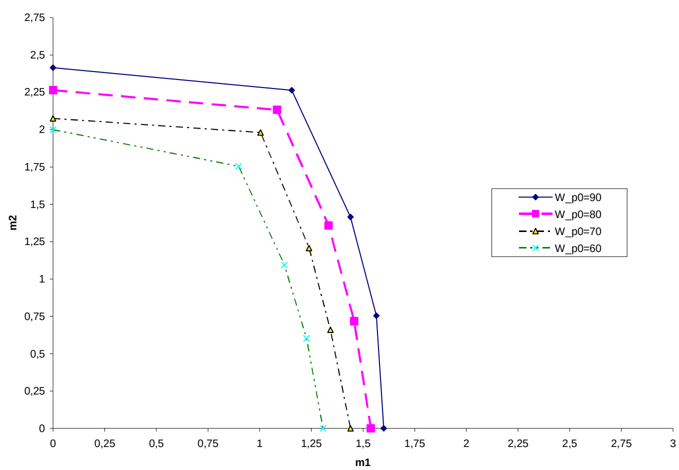


Figure 8. Safe loading domain

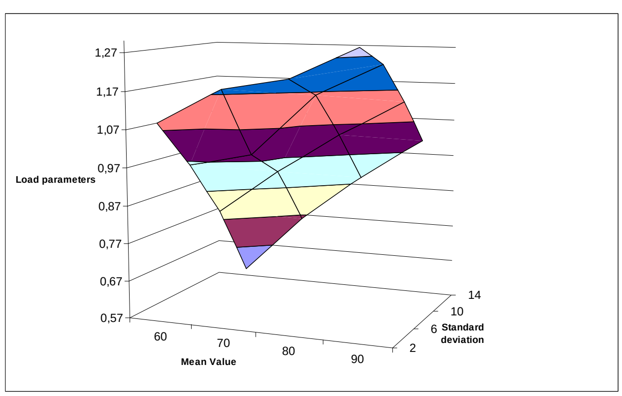


Figure 9. Safe loading surface 1

In Figure 10. the variation of the load multipliers are presented in function of different target reliability indexes (β_{target}=3.2; 4.2;)), fix mean values of the complementary strain energy of the residual forces ($\bar{W}_{p0} = 60$) and different standard deviations ($\sigma_w = 2;\ 6;\ 10;\ 14;$).

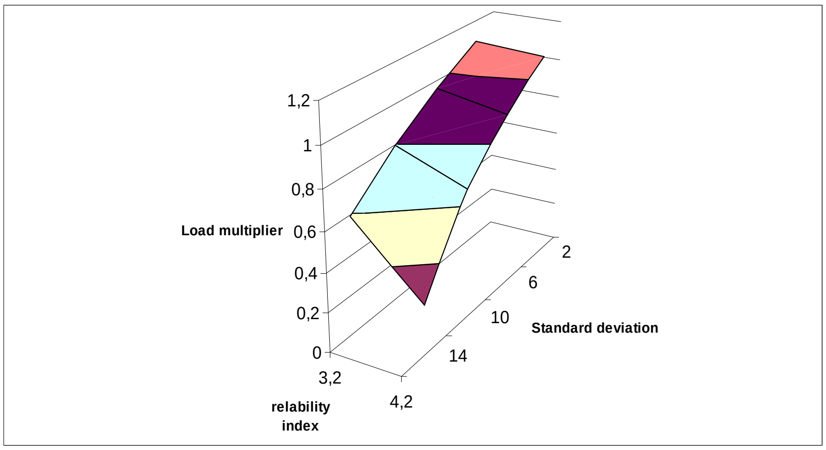


Figure 10. Safe loading surface 2

The shapes of the surfaces are in good agreement with the expectation that the increase of the reliability level and the amount of the complementary strain energy of the residual forces results in bigger load multipliers.

5. CONCLUSIONS

In this paper mechanical models are introduced for the extended shakedown design. The semi-rigid behaviour is described by appropriate models and to control the plastic behaviour of the structure probabilistically given bound on the complementary strain energy of the residual forces is applied. Limit curves and optimal cross-sections are presented for the shakedown multipliers. The numerical analysis shows that the stiffness of the semi-rigid connections, the mean value and the standard deviation of the bound of the complementary strain energy of the residual forces can influence significantly the magnitude of the shakedown multipliers and in some cases the results are very sensitive on the stiffness of the semi-rigid connections. The presented investigation drowns the attention to the importance of the problem but further investigations are necessary to make more general statements.

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