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A NEW METHOD FOR EMPLOYMENT OF DETERMINISTIC ATTENUATION RELATIONSHIP IN PROBABILISTIC SEISMIC HAZARD ANALYSIS

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ABSTRACT

This paper offers a method to use deterministic attenuation relationships in probabilistic seismic hazard analysis (PSHA). For a given magnitude and epicentral distance, deterministic attenuation relationships evaluate strong ground motion definitely, so analyzer must assume a standard deviation for the attenuation relationship in the PSHA. This study proposes a PSHA method which can apply the attenuation relationship without standard deviation. Required uncertainty for PSHA is obtained through the uncertainty in magnitude. Seismic source is modeled as a convex combination of three Gaussian distributions. The method is tested for Milad tower site and the results are in accordance with previous studies.

Keywords: Probabilistic hazard analysis; deterministic seismic attenuation; earthquake epicenter; Gaussian distribution; Milad tower

1. INTRODUCTION

Earthquake like other natural phenomena is uncertain in timing, location and magnitude so it has always been a challenging matter to analyze. Wide damages of destructive earthquakes stimulate the prediction of them and the inherent uncertainties require new probabilistic approaches. Therefore in the recent decades, uncertainty has become an important issue in the probabilistic seismic hazard analysis.

Uncertainty is categorized as either an epistemic or aleatory type. To consider epistemic uncertainty a weighted average of different inputs is calculated, as what the "logic tree analysis" does in the seismic hazard analysis (SHA). Aleatory uncertainty involves the randomness of variables. Location, time and magnitude of the future earthquakes are variables which could be regarded as stochastic [1].

Aleatory uncertainty is usually considered by processing the recorded data, so the validity of assessed stochastic models depends on the correctness and completeness of the applied data. Therefore in the related seismic hazard analyses, it is very important to access a correct and complete catalog of earthquakes or set of records.

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From aleatory point of view, seismic source is not restricted to where the faults are located. That is the source covers all over the region with an evaluated random distribution. The necessity of using such model is that not all earthquakes are tectonic, as some are due to dilatancy in crustal rocks, explosions or volcanism [2]. That is fault maps do not justify occurrence of every earthquake. In a previous study of an Italian area, convex combination of Gaussian distributions is suggested as stochastic seismic source [3,4] and is applied to a PSHA [5] by using a directional stochastic attenuation model for the scale of intensity [6].

Uncertainty is also available in the attenuation of earthquakes. For a definite magnitude and epicentral distance, the experienced strong ground motion for the site is not certain. Many attenuation relationships suggest normal distribution for considering the uncertainty, in other words a standard deviation is offered together with the attenuation relationship while the mean value is calculated via the relationship. The standard deviation of strong ground motion parameter is used in the PSHA to show the uncertainty of estimated values. Uncertainty of attenuation relationships is shown schematically in Figure 1 [7]. Some attenuation relationships have no standard deviation, that is to say they are deterministic and estimate the strong ground motion definitely, for a given magnitude and epicentral distance. Contrary to probabilistic attenuation relationships, no normal distribution is considered for these relationships.

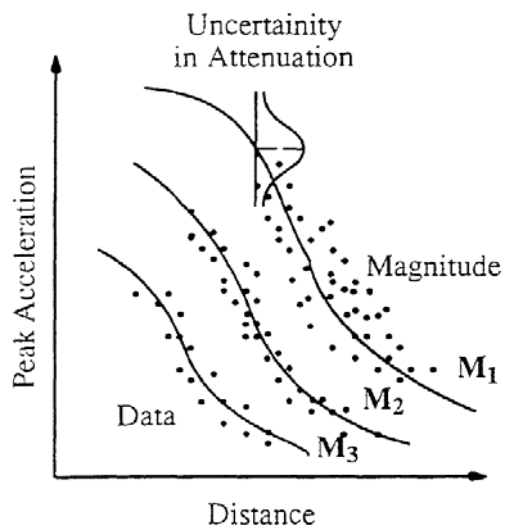

Figure 1. Uncertainty of attenuation relationship [7]

Deterministic attenuation relationships can be applied in any SHA. For deterministic seismic hazard analysis (DSHA) no probability distribution is required for the attenuation relationship so that a deterministic attenuation relationship is directly applicable. On the contrary, PSHA needs a probabilistic form of attenuation relationship, so that in the case of deterministic attenuation relationships an assumption should be made for the uncertainty. In other words, analyzer selects an appropriate standard deviation for the relationship if it does

not have any value by itself.

In order to use deterministic attenuation relationship in the PSHA without any assumption, a method is proposed in this paper for the direct application of it. The uncertainty in the attenuation relationship is not considered by an assumed standard deviation, but it is taken into account through the uncertainty in magnitude. This means that the magnitude density function is responsible for the uncertainty in estimation of the strong ground motion parameter.

It is noteworthy that the uncertainty of seismic attenuation could be modelled in many other ways. For example, a directional model of attenuation could be suggested for a region, in which the direction of site relative to epicenter is considered as a random process [6,8]. The method in this paper is restricted to common non-directional attenuation relationships, but it can be generalized to directional relationships.

In the present paper, seismic source is modelled by a statistical stochastic distribution. A convex combination of three Gaussian (normal) distributions is selected to fit the epicentral coordinates of past earthquakes. This model is selected instead of fault maps to show a better mathematical illustration of the suggested method.

2. METHODOLGY

Here, a mathematical definition of hazard density function is presented. Hazard density function on a site, located on the given Cartesian coordinates of (x_s,y_s), is calculated through the double integral as

$$f(h) = \int_{X_e}\int_{Y_e} f_{H,X_e,Y_e}(h, X_e, Y_e)\,dY_e\,dX_e \tag{1}$$

where h is a given value for the hazard scale H (i.e. strong ground motion parameter), f_H is the hazard density function and (X_e,Y_e) is the Cartesian epicentral coordinates.

As Eq. (1) shows, hazard density function is expressed in terms of a joint distribution of hazard and epicentral coordinates. This joint distribution may be defined according to the definition of conditional distribution as

$$f_{H,X_e,Y_e}(h, x_e, y_e) = f_{H|x_e,y_e}(h) \times f_{X_e,Y_e}(x_e, y_e) \tag{2}$$

where $f_{H|x_e,y_e}$ is the conditional distribution of hazard scale for a given epicentral coordinates and f_{X_e,Y_e} is the epicentral distribution of region.

An attenuation relationship is required to calculate the $f_{H|x_e,y_e}$. Suppose that the general form of attenuation relationship is

$$H = A(M, R) \tag{3}$$

where M is the magnitude of earthquake and R represents the epicentral distance. In other

words:

$$R = \sqrt{(X_s - X_e)^2 + (Y_s - Y_e)^2} \tag{4}$$

For a definite site (x_s,y_s) and the conditional distribution ($f_{H|x_e,y_e}$), epicentral distance has a constant value (r). Hence the attenuation relationship becomes H=A(M,r), which has only two variables (H and M). The attenuation relationship must be inversed in order to calculate $f_{H|x_e,y_e}$:

$$M = A^{-1}(H,r) \tag{5}$$

$$f_{H|x_e,y_e}(h) = f_{H|r,\varphi}(h) = f_M\left(A^{-1}(h,r)\right) \times \frac{dA^{-1}}{dH}(h,r) \tag{6}$$

where f_M is the magnitude density function and (r,φ) are polar coordinates of the site relative to the epicenter. As the attenuation relationship is not directional, the relative angle (φ) has no effect on the calculations. Guttenberg-Richter relationship is selected to model the magnitude density function [9]:

$$f_M(m) = \beta.\exp(-\beta(m - m_0)) \tag{7}$$

where m_0 is the minimum magnitude of earthquakes. Corresponding cumulative distribution is also shown in Figure 2.

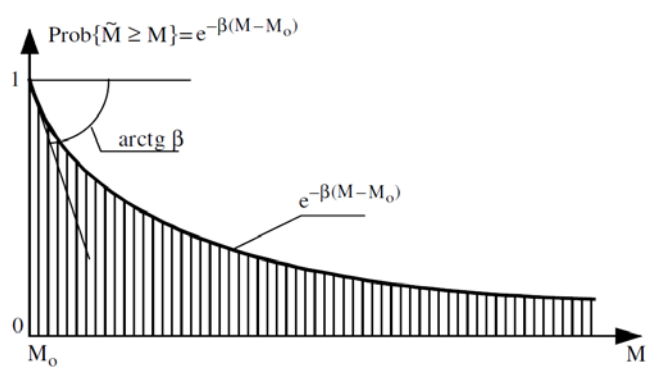

Figure 2. Cumulative density function of the magnitude

For irreversible attenuation relationships, a substitute equation is suggested below:

$$f_{H|x_e,y_e}(h) = f_{H|r,\varphi}(h) = \frac{f_M(m)}{\frac{dA}{dH}(m,r)} \tag{8}$$

where given h and r, m is calculated numerically through the attenuation relationship.

After calculation of the conditional distribution, epicentral distribution (f_{X_e,Y_e}) must be evaluated. In this paper a convex combination of three bivariate Gaussian distributions is selected to model the epicentral distribution. Figure 3 shows a bivariate Gaussian distribution; note that (μ_X, μ_Y) are the same as $(\bar{x}, \bar{y})$. Following equations describes the mentioned model:

$$f_{X_e,Y_e}(x_e, y_e) = \sum_i k_i g_i(x_e, y_e, \bar{x}_i, \bar{y}_i, \sigma_{xi}, \sigma_{yi}, \rho_i) \tag{9}$$

with:

$$\sum_i k_i = 1 \quad ; \quad 0 < k_i \le 1 \tag{10}$$

$$g_i(x_e, y_e, \ldots, \rho_i) = \frac{\exp\left\{\frac{-1}{2(1-\rho_i^2)}\left[\left(\frac{x_e-\bar{x}_i}{\sigma_{xi}}\right)^2 + \left(\frac{y_e-\bar{y}_i}{\sigma_{yi}}\right)^2 - 2\rho\left(\frac{x_e-\bar{x}_i}{\sigma_{xi}}\right)\left(\frac{y_e-\bar{y}_i}{\sigma_{yi}}\right)\right]\right\}}{2\pi\sigma_{xi}\sigma_{yi}\sqrt{1-\rho_i^2}} \tag{11}$$

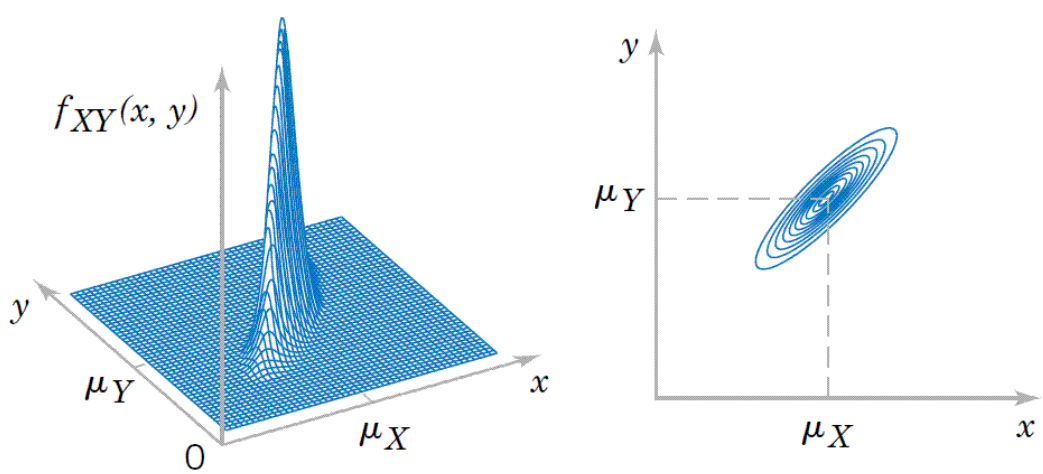

Figure 3. Bivariate Gaussian distribution [10]

Substituting Eq. 6 and Eq. 9 into Eq. 2 and then Eq. 1, one obtains:

$$f_H(h) = \int_{X_e}\int_{Y_e}\left(f_M\left(A^{-1}(h,R)\right) \times \frac{dA^{-1}}{dH}(h,R)\right).\left(f_{X_e,Y_e}(X_e, Y_e)\right)dY_e dX_e \tag{12}$$

3. CASE STUDY AND DATA

A case study is selected to show the steps in methodology. Hazard density function is calculated at the site of Milad tower in metropolitan Tehran for different quantitative scales. A previous study for this site has been made by Boland Payeh Co. [11] by using a different source and attenuation model, the result of which is mentioned in the next section.

Seismicity parameter of the magnitude density function is adopted from Ref. [12]. The parameter has been evaluated by using a catalog in the appendix of that paper and Kijko [2000] software ($\beta = 1.08$ for $M_0 = 4$).

All magnitudes in the mentioned catalog were converted to surface magnitude before modelling the source model (epicentral distribution). For converting M_L to M_s, values in Table 1 are employed and conversion of m_b to M_s is based on the relationship by Iranian committee of large damns (IRCOLD) [13]:

$$M_s = 1.2m_b - 1.29 \tag{13}$$

Table 1: Equivalences among magnitude scales and intensity; plate boundary earthquakes [7]

m_b	M_L	M_s	M_w	M_0 (dyne-cm)	Epicentral intensity (I_0)
4.0	4.3	3.0	4.1	10^{21}	IV
4.5	4.8	3.6	4.5	10^{22}	V
5.0	5.3	4.6	5.2	10^{23}	VI
5.5	5.8	5.6	5.8	10^{24}	VII
6.0	6.3	6.6	6.6	10^{25}	VIII
6.5	6.8	7.3	7.3	10^{26}	IX-X
7.0	7.3	8.2	8.2	10^{27}	XI-XII

Evaluation of epicenteral distribution is based on the catalog in Ref. [12] and the model adopted from Refs. [3,4] (a convex combination of bivariate Gaussian distributions). The model is fitted to all events larger than M_0.

The attenuation relationship offered in Ref. [14] is preferred as it is a recent and comprehensive one for Iran:

$$\ln H = C_1 + C_2 M_s + C_3 \ln R \tag{14}$$

where H is peak ground acceleration (PGA), peak ground velocity (PGV) or effective peak acceleration (EPA), R is the distance between site and epicenter in km, C1, C2 and C3 are coefficients of the attenuation model.

Table 2: Coefficients of the attenuation relationship (Alborz and Centeral Iran) [14]

Site condition	H (scale)	C_1	C_2	C_3
Rock	PGAH	4.15	0.623	−0.96
	PGAV	3.46	0.635	−0.996
	PGVH	−0.71	0.894	−0.875
	PGVV	−2.18	0.99	−0.83
	EPAH	3.45	0.66	−0.88
	EPAV	2.25	0.67	−0.84
Soil	PGAH	3.65	0.678	−0.95
	PGAV	3.03	0.732	−1.03
	PGVH	−1	1.03	−0.93
	PGVV	−2.5	1.08	−0.85
	EPAH	3.69	0.68	−1.01
	EPAV	3.42	0.69	−0.99

Note that EPA is a scale for normalizing the standard response spectrum. The unit of PGA and EPA is cm/s^2 (gal) and PGV is in cm/s. H and V (in the scale) represent horizontal and vertical components. Standard deviations are not included in the table since the uncertainty of attenuation is not considered in this method.

4. RESULTS AND DISCUSSION

Evaluated epicentral distribution is shown in Figure 4. It should be noted that the probability values were calculated in the Cartesian coordinates with the scale of km. A convex combination of Gaussian distributions was used to fit the epicentral coordinates of past earthquakes larger than 4 Richter (considering the surface magnitude). Recorded epicentral coordinate are scattered in Figure 4.

Table 3: Estimated parameters of convex combination of Gaussian distributions

Component (i)	k_i	$\bar{x}_i$	$\bar{y}_i$	σ_{xi}	σ_{yi}	ρ_i
1	0.27	53.08°E	36.21°N	1977.5	1133.0	-5.23e-5
2	0.62	51.24°E	36.09°N	9991.9	7336.7	-7.98e-5
3	0.11	50.71°E	34.23°N	1875.1	741.5	-4.30e-4

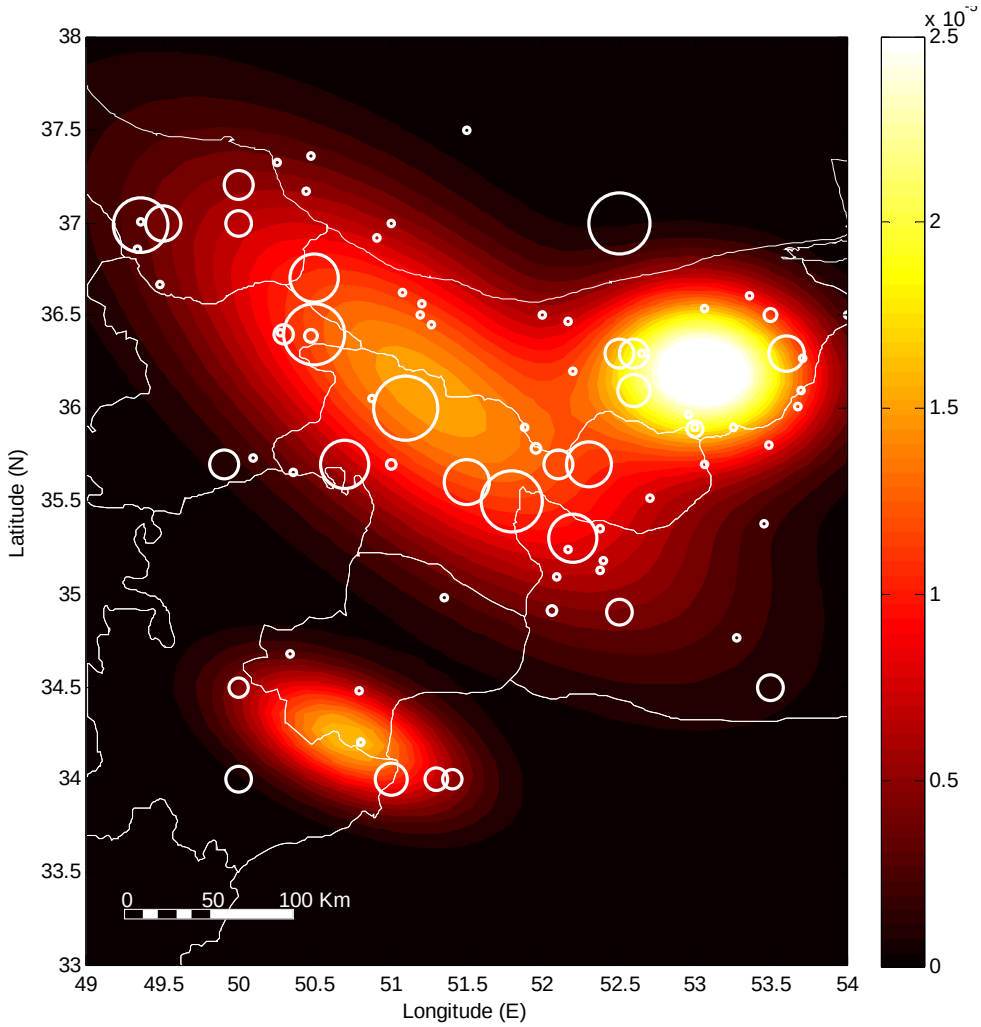

Figure 4. Epicentral distribution: convex combination of three bivariate Gaussian distributions

Joint distribution of f_{H,X_e,Y_e} is evaluated for five values of H (PGAH) through Equation (2) and the related distributions are shown in Figure 5. Each of these distributions defines a point on the hazard density curve by using Eq. (1). Note that the grey patch in the contour maps is the metropolitan Tehran area. The presented procedure in Figure 5 is repeated for different scales and site conditions. Corresponding cumulative hazard density functions are displayed in Figure 6.

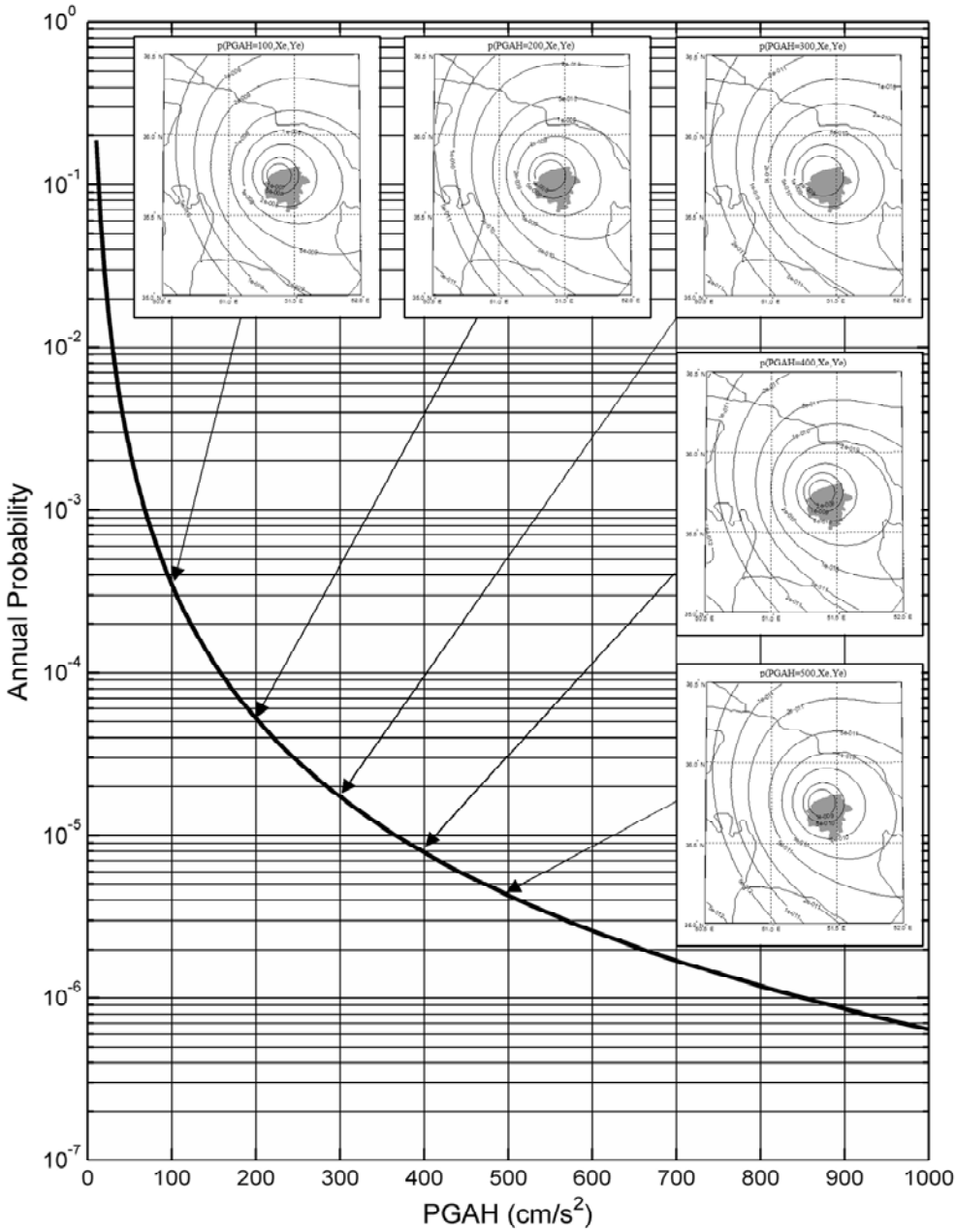

Figure 5. Evaluation of hazard density function by using f_{H,X_e,Y_e} for PGAH (rock)

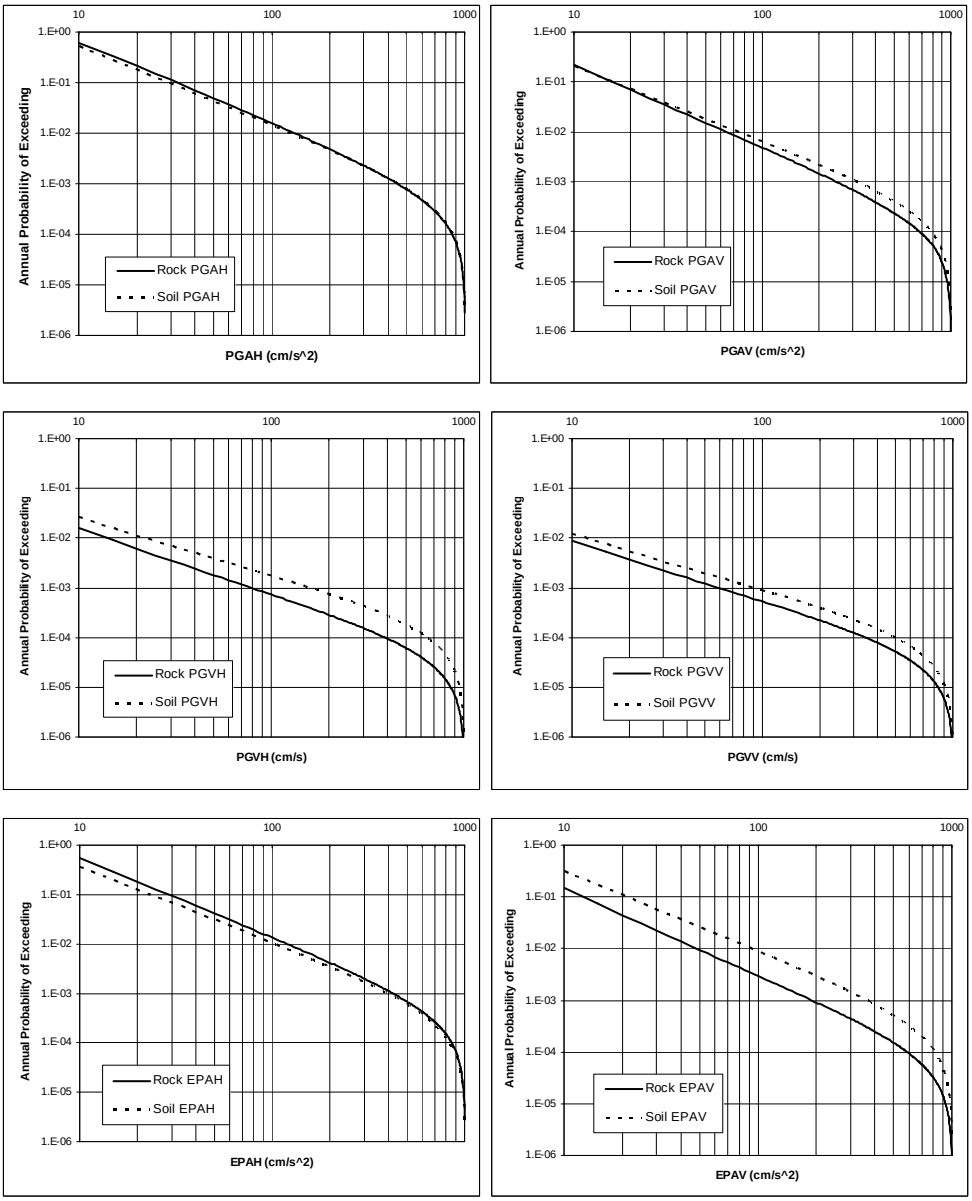

Figure 6. Cumulative hazard density functions for different scales and site conditions

For the annual exceeding probability of 0.002 (or %10 in 50 years), PGAH is obtained about 325 gal or 0.33g. While a previous study by Boland Payeh Co. estimates the value of 0.49g for this parameter at the site [11], the Iranian seismic code proposes 0.35g for the same hazard level in Tehran city, which is closer to the result of this paper [15]. The difference between these values may be justified by the different applied attenuation relationships, seismic sources and seismicity parameters.

Table 4: PGAH values on the Milad tower site for annual exceeding probability of 0.002

Boland Payeh Co. [11]	0.49 g
Iranian seismic code (for Tehran city) [15]	0.35 g
Estimation of current study	0.33 g

5. CONCLUSION

In this paper, a new method for using deterministic attenuation relationships in the probabilistic seismic hazard analysis is presented. While PSHA requires the attenuation relationships to be in the probabilistic form, analyzer must make some assumption for application of the deterministic attenuation models. In other words, a standard deviation is commonly assumed for the deterministic attenuation relationship before it can be used in the probabilistic analysis.

In order to avoid such assumption in the PSHA, a method is proposed which employs deterministic attenuation relationships directly. Required uncertainty for the attenuation of strong ground motion is obtained through the magnitude density function by using the probability theory. The relationships are elaborated for any arbitrary epicentral distribution and hazard density function is finally obtained.

The method is tested for the site of Milad tower in Tehran city and the results (annual probability of exceeding) are in agreement with a previous study of the site and Iranian code of practice for seismic resistant design of buildings. Comparison of the results shows that using magnitude uncertainty for deterministic attenuation relationships leads to expected values of strong ground motion parameter.

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