SIGNIFICANCE OF EFFECTIVE NUMBER OF CYCLES IN ENDURANCE TIME ANALYSIS

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ABSTRACT

Endurance Time (ET) method is a dynamic analysis procedure using intensifying accelerograms which can be used to assess the structural performance at different excitation levels. In generation process of current ET accelerograms, the compatibility between acceleration spectrum produced by the ET accelerograms and acceleration spectrum associated with the real ground motions is applied, whereas the duration consistency is not directly considered. It is generally accepted that the strong-motion duration can strongly influence the response of structures which have stiffness or strength degrading characteristics. In this study, the number of cycles that a structure should resist due to a ground motion is considered as a parameter related to shaking characteristics of that ground motion. In addition, the number of cycles of a structure when is subjected to the ET records and the real ground motions are compared. This study further determines how the ET records should be modified to have the best duration consistency with the real ground motions. Two methods are proposed to achieve this objective. It is shown that determining a specific target time for the ET records alone is not a suitable approach to achieve the best duration consistency.

Keywords: Effective number of cycles; endurance time method; strong-motion duration; rainflow counting; response history analysis

1. INTRODUCTION

Earthquake ground motion essentially is an elaborated phenomenon and thus there is no single parameter which can completely characterize the whole characteristics of ground motion. These characteristics consist of amplitude, frequency content, and strong-motion duration. There are a number of situations in which the seismic response of a structure depends not only on the maximum amplitude of the motion but also its duration. Many authors have pointed out that the effective number of cycles of the ground motion is a more robust representative of shaking characteristics of the motion. Strong-motion duration can
have a significant effect on inelastic deformational and energy dissipation demand, especially in structures which are weak and short period [1]. Therefore, when dynamic analysis is employed, duration of the strong motions used in dynamic analysis should be consistent with the design scenario. The extent of impact of duration in damage depends on many factors. Many researchers stated that the number of cycles of a motion is an important factor in aspect of the seismic design as well as damage assessment of structures. Measures of effective number of cycles of motion are more likely to convey useful indications of shaking influence on the response of structures and soil because they also provide information on the amplitude of motion in a more explicit manner compared to most definitions of duration [2]. Different approaches are proposed to determine the effective number of cycles of motion; nevertheless, there is no universally accepted approach. Nearly all different definitions are reviewed and employed in this article.

In the earthquake engineering, the dynamic analysis is recognized as a method which can incorporate nearly all sorts of material and geometry. As a result of these advantages, the tendency to use the dynamic analysis is soaring more rapidly compared to the past; however, there are still several obstacles that prevent the prevailing application of this method such as the time required to analyze a structure by this method. The ET Method is a dynamic analysis procedure that uses the specially designed intensifying accelerograms. In this methodology, accelerograms which are employed as dynamic loading; is the predominant parameter which appreciably affects the results analysis. In this article, effective number of cycles of current ET accelerograms are compared to those associated with the real ground motions; afterwards, it is investigated that how could we reduce this discrepancy and subsequently enhance the accuracy of the ET accelerograms.

2. REFERENCE GROUND MOTION SET

Non-linear dynamic analysis is becoming the most frequently used procedure for seismic assessment of structure responses. At any time when this procedure is exploited, the selection of ground motions as a dynamic loading is a momentous consideration because it can strongly influence the response of structures. This potential influence of the selection process; is not considered in the current design codes. Most contemporary seismic codes, such as ASCE standards 7-05 [3], describe relatively similar procedure for selection of seismic input motions. Seismic motions can be represented by real, artificial, or even simulated records, while a number of important seismological parameters, such as magnitude, distance, and local site conditions, should reflect in local seismic scenario. This study utilizes the far-field record set for non-linear dynamic analysis which is proposed by FEMA695 code. These records are considered to be applicable to structures located at different sites with different ground motion hazard functions, site and source conditions.

3. MAKE UP OF ET EXCITATION FUNCTIONS

Intensifying accelerograms used in ET analysis are generated to make meaningful correspondence between the response of a structure at a particular time in the ET analysis and the average of response to ground motions [4-7]. These ground motions are selected on the
basis of criteria such as magnitude (M)-distance (R) pairs which should represent the seismicity of a particular site at certain hazard level. The response spectra of an ET accelerogram increase by time; moreover, this acceleration function at each time window can be attributed to a particular hazard level regarding the acceleration spectra produced by the ET accelerogram at corresponding time window. Nevertheless, the current ET records are matched at one hazard level (i.e. DBE hazard level) at a particular time called target time. For other times, the produced spectrum by ET excitation functions varies linearly as [7]:

\[ S_{ac}(T,t) = \frac{t}{t_{target}} S_{ac}(T) \]  

(1)

Where \( S_{ac}(T) \) is the target spectrum, \( S_{ac}(T, t) \) is the spectrum to be produced at time (t) by the ET excitation functions, and \( t_{target} \) is target time.

Displacement spectrum is also highly important consideration for characterizing a dynamic excitation. Target displacement spectrum can be defined as a function of acceleration spectrum as [4, 7]:

\[ S_{uc}(T,t) = \frac{t}{t_{target}} S_{uc}(T) \times \frac{T^2}{4\pi^2} \]  

(2)

Where, \( S_{uc}(T,t) \) is the target displacement spectrum to be induced at time (t) by the ET excitation functions.

In the second generation of the ET excitation functions, the concept of response spectrum and numerical optimization were introduced and numerically significant results were achieved [8]. By extending the range of period of vibration into very long periods, records in this generation produced highly reasonable estimates in non-linear range of behavior as well [9]. In third generation, non-linear response spectra were included in the optimization procedure [6, 11]. This study uses four series of ET accelerograms which are presented in Table 1.

<table>
<thead>
<tr>
<th>Series</th>
<th>Target spectrum</th>
<th>Long periods included</th>
<th>Non-linear optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETA20a</td>
<td>code spectrum (standard 2800)*</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>ETA20e</td>
<td>average of several recorded motion on stiff soil</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>ETA40g</td>
<td>code spectrum (ASCE standard)</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>ETA20en</td>
<td>average of several recorded motion on stiff soil</td>
<td>-</td>
<td>Y</td>
</tr>
</tbody>
</table>

*Iranian national code
4. REVIEW OF DIFFERENT APPROACHES FOR DETERMINING NUMBER OF CYCLES OF MOTION

In spite of the fact that there are many different cycle-counting definitions in use, all cycle-counting approaches can be divided into five generic groups, as following [2]:

- Peak counting
- Level crossing counting
- Range counting
- Indirect estimation
- Definitions based on structural response.

A number of these definitions count cycle ranges (from peak to trough), whereas others count cycle amplitudes (from the zero baseline to the peak). The first one is classified as range-counting.

A cycle counting definition is classified as peak counting if it counts the number of peaks in the strong-motion [2]. A number of these definitions only count the largest peaks between zero crossings, whereas others count all peaks.

Range-counting are commonly used for assessment of fatigue damage [2]. The most popular range-counting method is the rain-flow counting. It counts both high-and-low frequency cycles in broad-banded signals [2].

Malhotra [12] used the method of half-cycles to accumulate damage caused by individual cycles. The damage expression (equation (3)) can be re-written as:

\[ D = C \sum_{i=1}^{2m} u_i^c \]

Where, \( u_i \) is the amplitude of \( i \)-th half-cycle, \( u_{\text{max}} \) is the largest amplitude of half cycles, and \( tn \) is the total number of cycles. \( C \) and \( c \) are application-dependent damage coefficients: \( c \) is a linear scale factor and \( c \) determines the relative importance of different amplitude cycles.

Moreover, Malhotra [12] divided the cumulative damage \( D \) by damage caused by the largest full-cycle amplitude \( u_{\text{max}} \) to obtain the equivalent number of cycles (of amplitude \( u_{\text{max}} \)) which causes the same damage to a structure compared to the entire deformation history, i.e.

\[ N_{cy} = \frac{1}{2} \sum_{i=1}^{m} \left( \frac{u_i}{u_{\text{max}}} \right)^c \]

values of \( C=2, C=1 \) are adopted for this study.

One of the significant differences among different definitions is if they are calculated using the absolute amplitude of the cycles or employing levels relative to the cycle with largest amplitude (\( u_{\text{max}} \)). Damage parameter given in equation (3) is typical of an absolute measure; moreover, the definition of the effective number of cycles given in equation (4) is typical of relative measure of the number of cycles in the motion [2].
In this study, peak counting uses zero crossing, peak counting uses non-zero crossing, and rainflow counting which is completely described by ASTM [13], are employed. In all cases, damage parameter given in equation (3) and the effective number of cycles given in equation (4), are applied to quantify the number of cycles in a motion which has cycles with different amplitudes.

5. COMPARISON OF NUMBER OF CYCLES OF ET RECORDS AND REAL GROUND MOTIONS

Since acceleration response of a SDOF system due to a motion is not necessarily similar to the acceleration of the imposed motion, thus it can be concluded that the number of cycles which a structure should resist is essentially different from the number of cycles of input motion and significantly depends on period of the structure. Figure 1 shows that the effective number of cycles which structures with different periods should resist is significantly different from the number of cycles of input motion.

![Figure 1. The effective number of cycles spectrum using rain-flow counting and absolute measure (Northridge 1994)](image)

Figure 1 displays the effective number of cycles which structures with periods about 0.5 sec to 1 sec should resist due to 1994 northridge earthquake motion, exceed about 6 times of effective number of cycles of that motion. The effective number of cycles for real ground motions are computed. Figure 2 shows the dispersion the effective number of cycles spectrum for the real ground motions. It should be noted that the records are scaled so that they produce acceleration spectrum equal to the ones associated with 2800 code (Iranian building code) at corresponding period.
As a result, the number of cycles spectrum should be considered instead of using the effective number of cycles of the input motion. For the sake of simplicity, the area under the effective number of cycles spectrum, i.e., hereafter $N_c$-spectrum, is considered as a parameter representing shaking characteristics of a motion.

The ET records at each time window can be considered as a single motion for instance, 10 sec window or 20 sec window of a ET record are two separate motions; therefore, an ET record inherently is not a single motion and hence its $N_c$-spectrum varies against time. It is noteworthy that in each time window, the ET records are scaled as the aforementioned manner. Moreover, the target time is defined as the time at which $N_c$-spectrum of the ET records would be consistent with the real ground motions.

Figure 2. Effective number of cycles spectrum of the real ground motions using rainflow counting and relative measure

Figure 3. Determination of target time for ETA20a series using rainflow counting
Target time for different series of ET accelerograms and different cycle-counting approach are presented in Table 2.

Table 2. Target time for different series of ET accelerogram and different cycle-counting approach

<table>
<thead>
<tr>
<th>Series</th>
<th>Zero crossing (abs*)</th>
<th>Non zero crossing (abs)</th>
<th>Rain-flow counting (abs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETA20a</td>
<td>14.77</td>
<td>14.76</td>
<td>16.76</td>
</tr>
<tr>
<td>ETA20e</td>
<td>13.55</td>
<td>13.37</td>
<td>14.79</td>
</tr>
<tr>
<td>ETA40g</td>
<td>15.63</td>
<td>15.80</td>
<td>17.45</td>
</tr>
<tr>
<td>ETA20en</td>
<td>14.48</td>
<td>14.41</td>
<td>15.94</td>
</tr>
</tbody>
</table>

* abs refers the absolute measure

It is revealed that the target time about 15 sec is approximately appropriate for all different series and all different cycle-counting methods.

Accordingly, the time at which acceleration spectrum produced by an ET record approaches those associated with the real ground motions at MCE hazard level, i.e. hereafter $t_{MCE}$, should be determined. The ET records are scaled so that they have the target time equal to 15 sec (target time which is calculated in pervious section). This procedure is depicted in Figure 4.

![Figure 4. To determine MCE time for ETA20a series using rainflow counting](image-url)
Table 3 presents $t_{MCE}$ for different ET series and different cycle-counting approaches.

<table>
<thead>
<tr>
<th>Series</th>
<th>Zero crossing (abs)</th>
<th>Non zero crossing (abs)</th>
<th>Rainflow counting (abs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETA20a</td>
<td>19.67</td>
<td>19.73</td>
<td>20.56</td>
</tr>
<tr>
<td>ETA20e</td>
<td>19.52</td>
<td>19.45</td>
<td>20.19</td>
</tr>
<tr>
<td>ETA40g</td>
<td>20.79</td>
<td>20.85</td>
<td>21.39</td>
</tr>
<tr>
<td>ETA20en</td>
<td>19.85</td>
<td>19.82</td>
<td>20.44</td>
</tr>
</tbody>
</table>

It is noteworthy that those cycle-counting definitions which use relative measure are not dependent on how a motion is scaled. It means that for a motion whether it is scaled or not, their corresponding effective number of cycles spectrum would be identical. In this part, relative definition is used. The real ground motions which are considered as the reference motion in this study, are associated with MCE hazard level. The time at which the real ground motions and the ET records have the consistent $N_{c-spectrum}$ is referred as $t_{MCE}$. Figure 5 illustrates this procedure:

![Figure 5](image_url)

Figure 5. Determination of MCE time for ETA20a using rainflow counting in a relative manner

Table 4 represents $t_{MCE}$ for different series of the ET records and different cycle-counting definitions using relative measure.
Table 4: MCE time for ET records using relative cycle-counting definitions

<table>
<thead>
<tr>
<th>Series</th>
<th>Zero crossing (rel)</th>
<th>Non zero crossing (rel)</th>
<th>Rain-flow counting (rel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETA20a</td>
<td>21.45</td>
<td>21.51</td>
<td>23.64</td>
</tr>
<tr>
<td>ETA20e</td>
<td>19.37</td>
<td>16.57</td>
<td>20.62</td>
</tr>
<tr>
<td>ETA40g</td>
<td>22.72</td>
<td>22.31</td>
<td>25.50</td>
</tr>
<tr>
<td>ETA20en</td>
<td>20.18</td>
<td>17.81</td>
<td>21.83</td>
</tr>
</tbody>
</table>

It is shown that $t_{MCE}=20\text{sec}$ is approximately acceptable for different ET series and different cycle counting definitions. This is in agreement with $t_{MCE}$ calculated when absolute cycle-counting definitions are used.

These proposed target times and $t_{MCE}$, either absolute cycle counting or relative cycle counting is used, guarantees that $N_{\text{c-spectrum}}$ of the ET records and the real ground motions would be consistent at corresponding hazard level; nevertheless, considering this parameter does not provide any excessive information about the number of cycles which a structure should resist. Figure 6 demonstrates the number of cycles spectrum associated with the ET records at the proposed target time and the real ground motions. The discrepancy between these spectra is noticeable, whereas they are consistent in $N_{\text{c-spectrum}}$.

![Figure 6. Comparison between the number of cycles spectrum (rain-flow counting) at proposed target time](image-url)
6. CONCLUSIONS

ET method is a dynamic analysis procedure using intensifying accelerograms which can be used to assess the structural performance at different excitation levels. This study compares the number of cycles that structures experience when subjected to the ET excitations and the real ground motions. $N_{c}$-spectrum is defined as area under the effective number of cycles spectrum. This parameter is calculated for the real ground motions and the ET records in DBE and MCE hazard levels. In order to achieve the best duration consistency between the ET records and the real ground motions, the times which an ET record produces hazard levels correspond to DBE level and MCE level are determined to have consistent $N_{c}$-spectrum compared to the real ground motions at these levels. $T_{\text{target}}$ equal to 15 sec and $T_{\text{MCE}}$ equals to 20 sec satisfies approximately the aforementioned conditions. Moreover, $T_{\text{MCE}}$ is determined using relative measure of cycles number. When relative measure is employed, $T_{\text{MCE}}$ equal to 20 sec is calculated which is compatible with the one computed using absolute measure. To sum up, it is observed that determining a target time by itself does not guarantee the best duration consistency because there is a noticeable discrepancy between the effective number of cycles spectrum of the ET records and the real ground motions, whereas they have consistent $N_{c}$-spectrum at the proposed target time. It may be possible to consider the effective number of cycles directly in the equation which are used to generate the ET accelerograms. Another approach that can be used is a trial and error approach to generate a pool of ET accelerograms and select those which have the highest consistent number of cycles spectrum.

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REFERENCES

3. ASCE. Minimum design load for building and other structures ASCE standard No.007-05, American society of civil engineers, 2006.


