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NONLINEAR DYNAMIC ANALYSIS OF REINFORCED CONCRETE SLABS SUBJECTED TO BLAST LOADING

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ABSTRACT

A numerical investigation has been carried out to examine the behavior of reinforced concrete slabs subjected to uniform blast loading. The aim of this work is to determine the effects of various parameters on the results. Finite element simulations were performed in the non linear dynamic range using an elasto-plastic damage model. The main parameters considered are: the negative phase of blast loading, time duration, equivalent weight of TNT, stand-off distance of the explosive, and slab dimensions. Numerical modeling has been performed using ABAQUS/Explicit. The results obtained in terms of displacements and propagation of damage show that the above parameters influence considerably the nonlinear dynamic behavior of reinforced concrete slabs under uniform blast loading.

Keywords: Blast loading; reinforced concrete slabs; elasto-plastic damage model; negative phase; time duration; equivalent weight of TNT; explosive distance; slab dimensions

1. INTRODUCTION

Structures may experience blast loads due to intentional or accidental activities. Such loads may cause severe damage or collapse due to their high intensity and dynamic nature. The analysis of the dynamic response of reinforced concrete slabs under blast loading is complicated due to the fact that the impulsive load caused by an explosion is highly nonlinear and occurs in an extremely short duration. Reinforced concrete slabs are among the most common structural elements. In spite of the large number of concrete slabs used, the effects of their details on their behavior under blast loading are not always properly taken into account. Since explosive tests are dangerous and costly and their reproducibility is not always ensured and the results of such tests always present uncertainties, the recourse to numerical methods is unavoidable.

This problem can be tackled in several ways. The approach that can describe accurately the behavior of concrete slabs is through numerical simulations, usually using the finite element method. These analyses have been used by a number of investigators, Mosalam and

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Mosallam [1], Krauthammer and Altenberg [2] and Pan and Watson [3]. An alternative approach, which involves several simplifications and assumptions and which is rather simple has been adopted by many researchers, Low and Hao [4, 5], Morison [6]. It consists of using stiffness of equivalent single degree of freedom structural systems.

The main objective of this paper is to conduct a parametric study using the finite element method in order to assess the effect of certain parameters on the behavior of reinforced concrete slabs subjected to blast loading.

2. EXPLOSION AND BLAST PHENOMENA

2.1 The explosion process

In general, an explosion is the result of a sudden and rapid release of a large amount of energy and is characterized by a physical or chemical change in the material. This occurs under sudden change of stored potential energy into mechanical work with creation of a blast wave and a powerful sound. Conventional explosives such as trinitrotoluene (TNT) depend on the rearrangement of their atoms for the energy while nuclear explosions result from the release of energy building protons and neutrons within the atomic nuclei. For flammable materials, the energy is mainly derived from the chemical reaction. Explosive materials may be classified according to their physical state: solids, liquids and gases. Solid explosives are primarily high explosives for which the blast effects are best known Beshara [7].

The blast wave is generated when the atmosphere surrounding the explosion is forcibly pushed back by the hot gases produced from the explosion source. This wave moves outward from the central part only a fraction of second after the explosion occurs. The front of the wave called the shock front, is like a wall of highly compressed air and has an overpressure much greater than that in the region behind. This peak overpressure decreases rapidly as the shock is propagated outward. After short time the pressure behind the front may drop below the ambient pressure. During such a negative phase, a partial vacuum is created and air is sucked in.

The pressure time history of a blast wave can be illustrated with a general shape as in Figure 1. The illustration is an idealization for an explosion in free air. The pressure time history is divided into a positive phase and a negative phase. In the positive phase, maximum overpressure, P_s^+, is developed instantaneously and decays to atmospheric pressure, P_0, in the time, T^+. For the negative phase, the maximum negative pressure, Ps^-, has much lower amplitude than the positive overpressure. The duration of the negative phase, T^-, is much longer compared to the positive duration, T^+. The positive phase is more relevant in studies of blast wave effects on structures because of the high amplitude of the overpressure and the concentrated impulse i^+, which is the area under the positive phase of the pressure-time curve. However some studies have shown that the effects of the negative phase cannot be always neglected. The pressure time-history can be approximated by the following exponential form, Bulson [8].

$$P(t) = P_0 + P_s^+ \left(1 - \frac{t}{T^+}\right) e^{-bt/T^+} \tag{1}$$

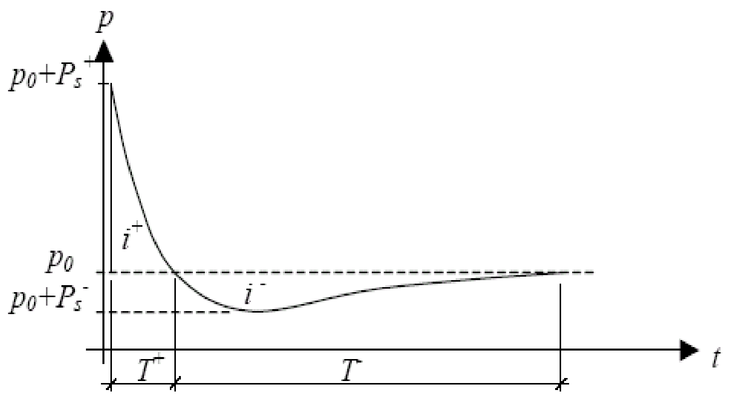


Figure 1. Pressure-time history from a blast (exponential form)

where P(t) is the overpressure at time t.

By choosing different values for b, many distinct curves can be obtained. The peak of the pressure depends on the stand off distance of the charge and the weight of the explosive. In addition, if the peak pressure, the positive impulse and the positive duration are known, then b can be determined and the pressure-time history curve will be known.

Equation (1) is often replaced by a simplified triangular curve, Eq. (2), Figure 2, see Bulson [8].

$$P(t) = P_{max}\left(1 - \frac{t}{T^+}\right) \tag{2}$$

The parameter **b** is decisive to the extension of the negative phase in the exponential distribution of equation (1). If the parameter **b** is less than one, the negative phase is important, while for **b** larger than one the negative phase becomes less significant.

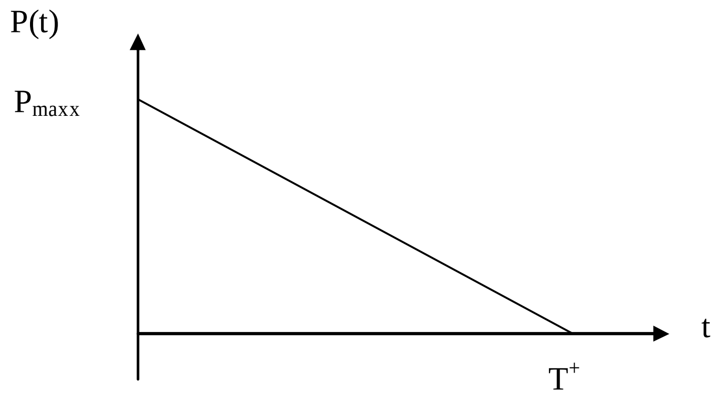
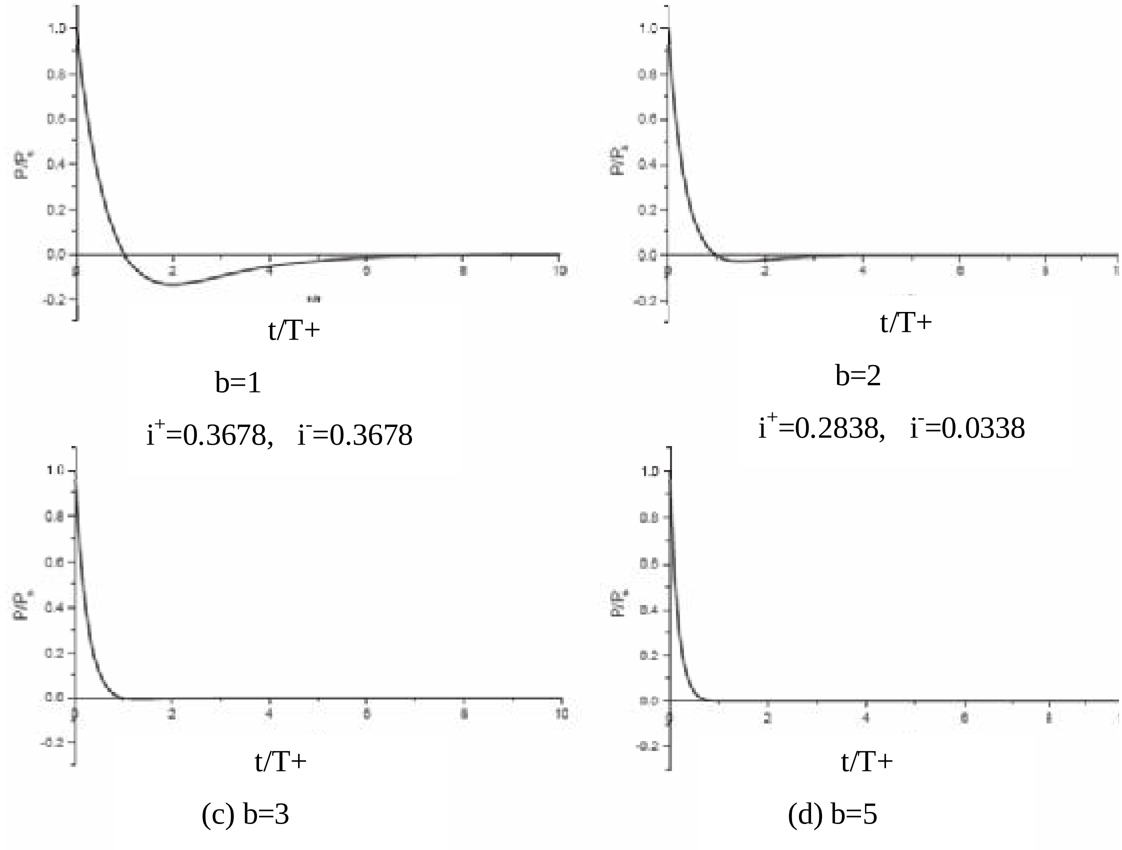


Figure 2. Simplified triangular form

The parameter **b** is decisive to the extension of the negative phase in the exponential distribution of equation (1). If the parameter **b** is less than one, the negative phase is important, while for **b** larger than one the negative phase becomes less significant.

Baker [9] provided a graph plotting **b** with respect to the scaled distance **Z**, where it is shown that the parameter **b** varies between 0.1 and 10. Figure 3 illustrates how the shape of the exponential distribution depends on **b** and gives the corresponding values of the positive and negative areas of the pressure. The values **b**=1, 2 and 3 have been used, with shapes shown in figures 3a-3c respectively. The value **b**=1 is a reasonable average corresponding to the area of negative pressure being equal to the one of the positive pressure. The exponential form as well as the value **b** =1 are well supported by experimental work of Jacinto, et al. [10]



t/T+

b=1

$i^{+}=0.3678$, $i^{-}=0.3678$

t/T+

b=2

$i^{+}=0.2838$, $i^{-}=0.0338$

t/T+

(c) b=3

$i^{+}=0.2277$, $i^{-}=0.0055$

t/T+

(d) b=5

$i^{+}=0.1602$, $i^{-}=0.0002$

Figure 3. Exponential distribution of a blast pressure depending on the value of **b**

Conventional high explosives tend to produce different magnitudes of peak pressure. As a result, the environments produced by these chemicals will be different from each other. In order to establish a basis for comparison, various explosives are compared to equivalent TNT values with the pressure range for different chemicals.

A scaling parameter is introduced, first noted by Hopkinson; see Bulson [8]. With the parameter Z in Eq. (3), it is possible to calculate the effect of a detonated explosion, conventional or nuclear, as long as the equivalent weight of charge in TNT is known:

$$Z = \frac{R}{W^{1/3}} \tag{3}$$

where R is the distance from the detonation and W is the equivalent weight of TNT. In table 1 some values of the overpressure in MPa are shown.

Table 1: Reflected overpressures with different combinations of W-R [11]

R\W	100Kg TNT	500Kg TNT	1 ton TNT	2 ton TNT
1 m	165.8	345.5	464.5	602.9
2.5 m	34.2	89.4	130.8	188.4
5 m	6.65	24.8	39.5	60.19
10 m	.85	4.25	8.15	14.7
15 m	.27	1.25	2.53	5.01
20 m	.14	.54	1.06	2.13
25 m	.09	.29	.55	1.08
30 m	.06	.19	.33	.63

The peak pressure and the duration of an explosion are normally related to the scaled distance, Z(in $m/kg^{1/3}$), where $Z=R/W^{1/3}$ (R and W are respectively the distance from charge center in meters and the equivalent TNT charge weight in kg). Empirical equations and graphs have been proposed aiming to predicting blast loading from an explosion at different scaled distances. Naumyenko and Petrovsky [12] were one of the earliest to suggest an empirical equation to relate the peak pressure with scaled distance, which is of the form:

$$P_{max} = \frac{6.7}{Z^3} + 1 \qquad bar \qquad for\ P_{max} \geq 10 bars \tag{4a}$$

$$P_{\max} = \frac{0.975}{Z} + \frac{1.455}{Z^2} + \frac{5.85}{Z^3} - 0.019 \qquad bar \quad for\ 0.1 bar \leq P_{\max} < 10 bars \tag{4b}$$

Brode [13] proposed something very similar:

$$P_{max} = \frac{107}{Z^3} - 1 \qquad kp/cm^2 \qquad for\ Z \leq 1 \tag{5a}$$

$$P_{max} = \frac{0.76}{Z} + \frac{2.55}{Z^2} + \frac{6.5}{Z^3} \qquad kp/cm^2 \qquad for\ 1 \leq Z \leq 15 \tag{5b}$$

Henrych [14] divided the peak pressure into three ranges according to the scaled distance as below:

$$P_{max} = \frac{14.072}{Z} + \frac{5.540}{Z^2} - \frac{0.357}{Z^3} + \frac{0.00625}{Z^4} \quad bar \quad for \; 0.05 \le Z < 0.3 \tag{6a}$$

$$P_{max} = \frac{6.194}{Z} - \frac{0.326}{Z^2} + \frac{2.132}{Z^3} \quad bar \quad for \; 0.3 \le Z \le 1 \tag{6b}$$

$$P_{max} = \frac{0.662}{Z} + \frac{4.05}{Z^2} + \frac{3.288}{Z^3} \quad bar \quad for \; 1.0 \le Z \le 10 \tag{6c}$$

Mills [15] gave an estimation of the form:

$$P_{max} = \frac{1772}{Z^3} - \frac{114}{Z^2} + \frac{108}{Z} \quad kPa \tag{7}$$

The peak pressure predicted can be converted to peak reflected pressure which results from the reflection that occurs when the front of an air blast wave strikes the face of a structure. The equation proposed by Mills [15] is:

$$Pr_{max} = \frac{2P_{max}\left(710 + 4P_{max}\right)}{710 + P_{max}} \quad kPa \tag{8}$$

Furthermore, there are also graphs that enable pressure estimation given a scaled distance, for example, by US Air Force Manual AFM 88-82 [16], Technical Manual TM5-855-1 [17] , and a consolidated software Conwep (see Hyde [18]) which based its estimation on the manual TM5-855-1 [19] . All of these give similar estimations of peak reflected pressures with some variations. However, at very small distance and very large scaled distance, the predicted peak reflected pressures from these empirical relations could differ by about 10 times. This is because there are many uncertainties involved in a blasting test, and an explosion process is very unstable and difficult to be repeated.

3. MATERIAL MODELS

3.1 Concrete model

Pure elastic damage models or pure plastic constitutive models are not totally satisfactory to describe the behavior of concrete. Thus, a more rational approach would be a coupled elasto-plastic damage model. There are several methods for coupling plasticity and damage in a single constitutive relation. In one method the damage growth is a function of plastic strains while the other approach uses the concept of effective stress. In this work, the latter method is adopted.

3.1.1 Plasticity

Plasticity is governed by the following classical equations where the effective stress is substituted to the applied stress:

$$\dot{\varepsilon} = \dot{\varepsilon}^{e} + \dot{\varepsilon}^{p} \tag{9}$$

$$\sigma^{f} = E\varepsilon^{e} \tag{10}$$

$$\dot{\varepsilon}^{p} = \dot{\lambda} m\left(\sigma^{f}, k\right) \tag{11}$$

$$\dot{k} = \dot{\lambda} h\left(\sigma^{f}, k\right) \tag{12}$$

where E is the elastic stiffness , m is the flow vector, k is a set of internal variables, h is the plastic modulus and σ^{f} the effective stress. The dot denotes time derivatives. The plastic multiplier $\dot{\lambda}$ is given by the loading-unloading criterion (Kuhn-Tucker form):

$$F\left(\sigma^{f}, k\right) \leq 0, \quad \dot{\lambda} \geq 0., \quad F\left(\sigma^{f}, k\right)\dot{\lambda} = 0. \tag{13}$$

where F is the plastic yield function defined in the effective stress space. In this model, a non-associated plasticity rule is used resulting into a non-symmetric set of equations.

3.1.2 Damage

The stress-strain relations are governed by scalar damaged elasticity

$$\sigma = (1-d)D_{0}^{el} : \left(\varepsilon - \varepsilon^{pl}\right) = D^{el} : \left(\varepsilon - \varepsilon^{pl}\right) \tag{14}$$

Where D_0^{el} is the initial undamaged elastic stiffness; $D^{el} = (1-D)D_0^{el}$ is the degraded elastic stiffness; and d is the scalar stiffness degradation variable, which can take values in the range from zero (undamaged material) to one (fully damaged material).

Within the context of the scalar-damage theory, the stiffness degradation is isotropic and characterized by a single degradation variable, d . The effective stress is classically defined as:

$$\bar{\sigma} = D_{0}^{el} : \left(\varepsilon - \varepsilon^{pl}\right) \tag{15}$$

The Cauchy stress is related to the effective stress through the scalar degradation relation:

$$\sigma = (1-d)\bar{\sigma} \tag{16}$$

The reduction of the elastic modulus is given in terms of the scalar degradation variable, d , as

$$E = (1-d)E_{0} \tag{17}$$

Where E_0 is the initial (undamaged) modulus of the material.

The degraded response of concrete is characterized by two independent uniaxial damage variables, d_t and d_c which are assumed to be functions of the plastic strains:

$$
\begin{aligned}
d_t &= d_t\left(\tilde{e}_t^{pl}\right) \qquad \left(0 \le d_t \le 1\right) \\
d_c &= d_c\left(\tilde{e}_c^{pl}\right) \qquad \left(0 \le d_c \le 1\right)
\end{aligned}
\tag{18}
$$

where the subscripts *c* and *t* stand for compression and traction respectively

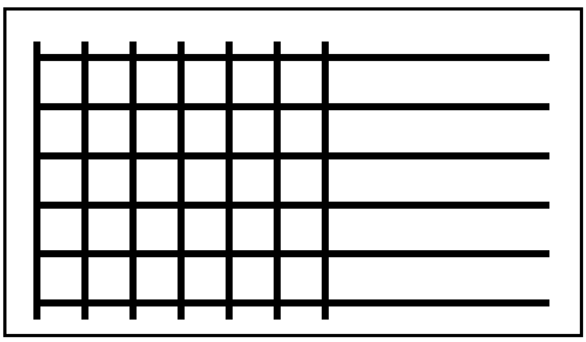
The uni-axial degradation variables are increasing functions of the equivalent plastic strains. The stress-strain relations under uni-axial tension and compression loading are given by:

$$
\begin{aligned}
s_t &= \left(1 - d_t\right) E_0 \left(e_t - \tilde{e}_t^{pl}\right) \\
s_c &= \left(1 - d_c\right) E_0 \left(e_t - \tilde{e}_c^{pl}\right)
\end{aligned}
\tag{19}
$$

4. MODELLING ASPECTS

4.1 Description of the slab

The dimensions of the plate are 12.0×10.0×0.90m



Concrete
E= 34000 MPa
$f_c = 32.8\,MPa$
$f_t = 3.28 MPa$
$n = 0.20$
steel ratio $r = 3\%$

Steel
$E = 210000\,MPa$
$n = 0.3$
$f = 400\,MPa$

Figure 4. Geometry of the slab and material properties

4.2 Finite element model

Finite element analysis is performed on simply supported slabs using the general purpose finite element code ABAQUS/EXPLICIT [20] which can incorporate non linear geometry, strain rate sensitivity, and thermal effects.

ABAQUS offers an element library for a wide range of geometric models. In the present study, the fourth node shell element S4R with reduced integration and hourglass control was used to model the geometry of the slabs. Because of symmetry, only a quarter of the slab was modeled. The reinforcement is modeled as smeared and perfect bond is assumed.

4.3 Dynamic analysis

The dynamic equilibrium equations have been integrated with the central difference scheme with automatic time stepping. The loading function shown in Figure 2 is scaled such that P_{max} = 0.1 MPa. The positive time durations of the blast wave, T^{+}, used in this study are equal to 1ms, 2 ms, 10 ms and 20 ms. The time integration is carried long enough to capture both the forced vibration as well as the free vibration afterwards.

5. RESULTATS AND DISCUSSIONS

5.1 Influence of slab dimensions

A series of simulations is run for slabs with different span lengths. To ensure that the total force acting on the slabs under the same pressure loading is the same, the area of the slab is maintained constant and equal to 120m². The actual dimensions of the slabs used in this simulation are shown in Table 2.

Table 2: Slab dimensions

B(m)	L(m)	B/L
10	12	0.83
8	15	0.53
6	20	0.30
5	24	0.20

The other parameters remain the same. Since the flexural rigidity of a beam depends on I/L^4, where I is the moment of inertia. increasing L and reducing B results in a lower flexural rigidity and a larger allowable deflection.Hence for slabs with the same surface area, the larger the span length is, the better is its capacity to resist blast loading. From Figure 5, we can see that for one way slabs (B/L < 0.40) and hence having a larger span length L but a smaller width B, the displacement at the centre of the slab is smaller than that of two ways slabs (B/L> 0.4). Thus, a larger span length leads to smaller displacements and this fact is confirmed by Figure 6 which shows the spread of damage as a function of B/L. These results are similar to those obtained from the study of two slabs with different dimensions Kadid [21].

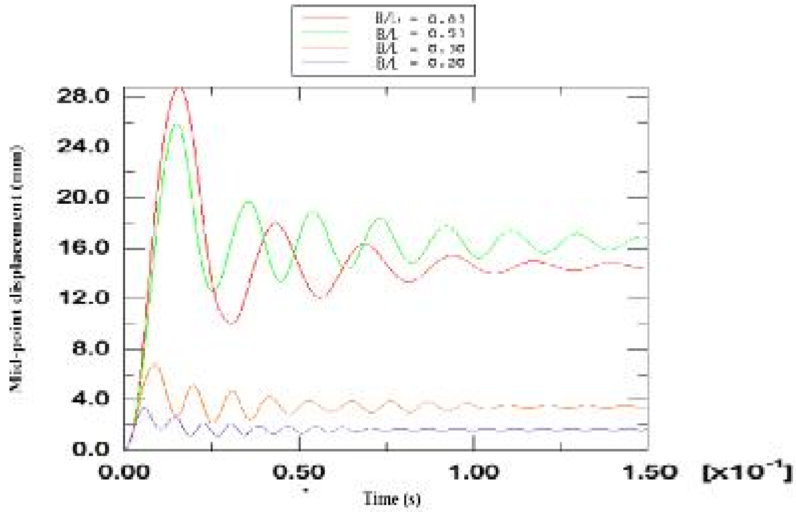
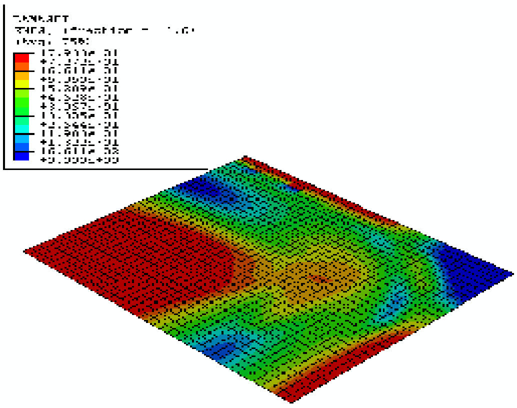
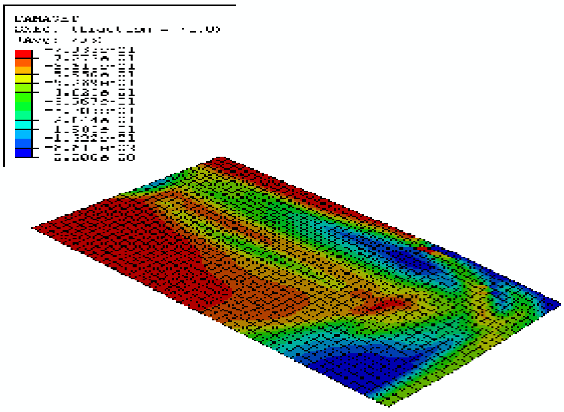


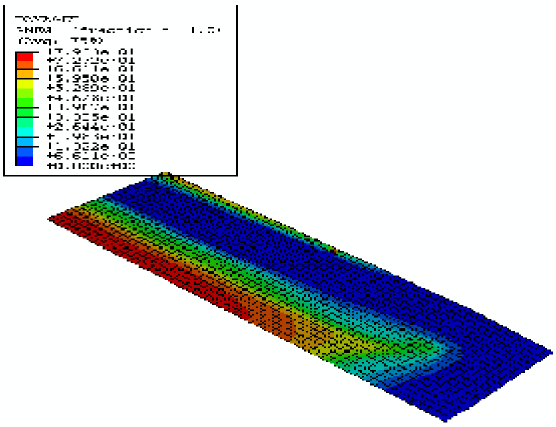
Figure 5. Influence of slab dimensions T^{+} = 20 ms



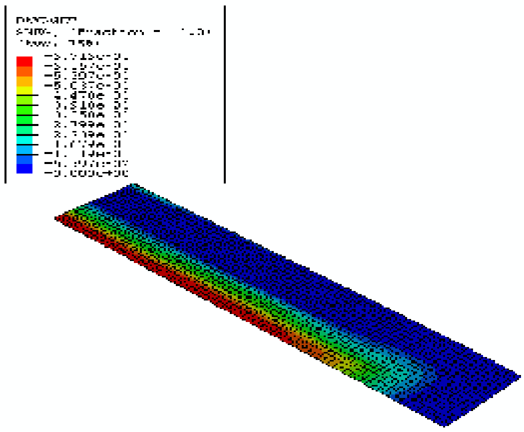
B/L = 0.83



B/L = 0.53



B/L = 0.30



B/L = 0.21

Figure 6. Tensile damage depending on slab dimensions T^{+} = 20 ms

5.2 Influence of time duration

Increasing time duration by factors of 2, 10 and 20 results in an increase in the mid-point displacement by a factor of 2.09, 6.45 and 11.01, respectively (Figure 7).

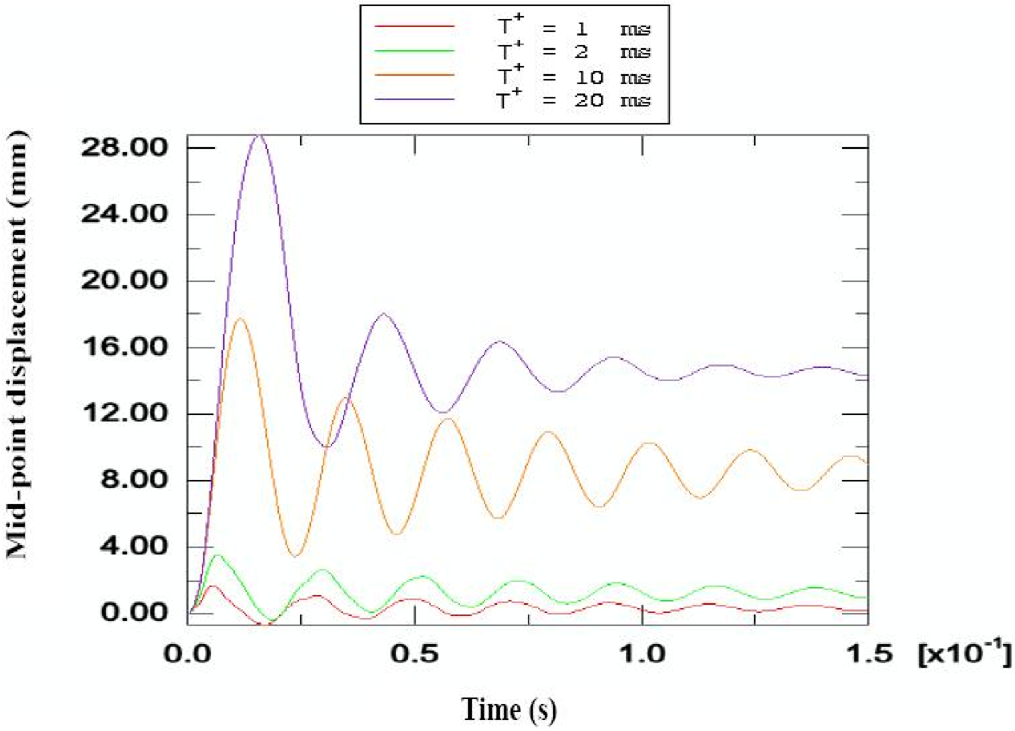


Figure 7. Influence of time duration (L=12m, B= 10 m)

5.3 Influence of the stand-off distance and the equivalent TNT weight of the explosive

Here, we have tried to study the effects of the stand-off distance and weight of the explosive on the dynamic response of the slab. The time duration is equal to 2ms. From Figure 8, it is evident that the stand-off distance of the explosion is a critical parameter that must be considered. Indeed, for an equivalent charge of 100Kg TNT reducing the distance of the explosion from 10m to 5m results in an increase in the mid-point displacement by a factor of 8.62. This factor is equal to 52.93 when the distance of the explosion is reduced from 20m to 5m. For an explosive of 500Kg equivalent TNT, these ratios are equal to 8.28 and 71.66 respectively, Figure 9. This shows clearly that the distance of the explosion is a capital factor that can drastically change the response of a reinforced concrete slab subjected to blast loading and the observation of the evolution of tensile damage in the slab confirms it, Figure 10.

Concerning the effect of the explosive charge, it can be seen from Figures 11 and 12 that increasing the explosive charge from 100Kg to 2000Kg for a stand-off distance of 10m results in increase by a factor of 20.62 in the mid-displacement. For a distance equal to 15m, this factor is equal to 11.56. Thus, the value of the explosive charge has got an important effect but less pronounced than that of the stand-off distance as confirmed by the evolution of tensile damage shown in Figure 13.

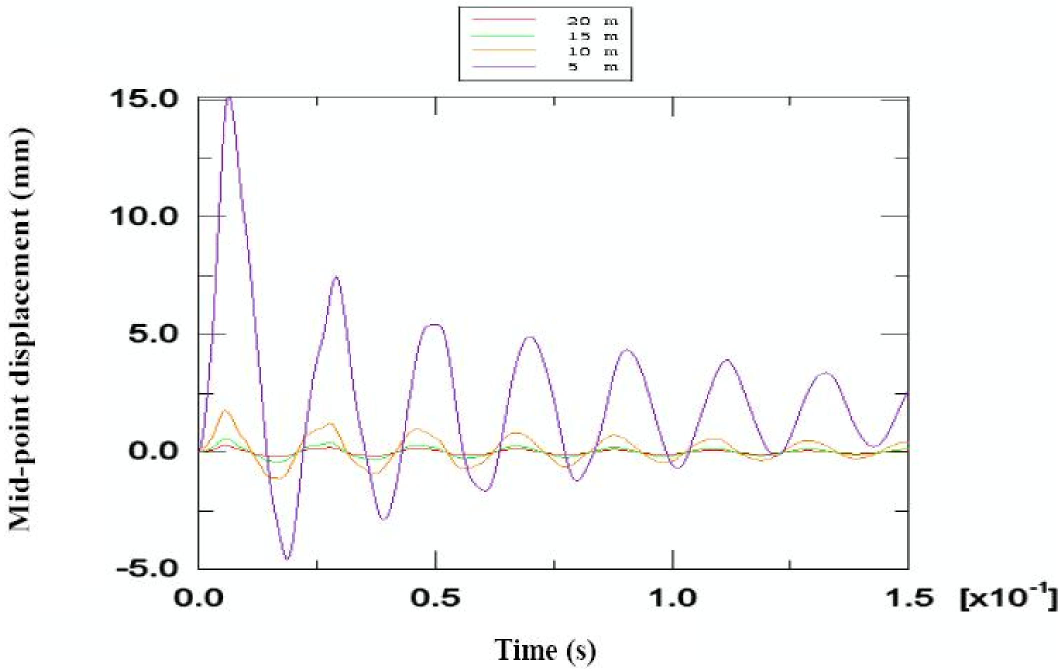


Figure 8. Influence of the stand-off distance from the explosion for a charge of 100Kg TNT (L=12m, B= 10m)

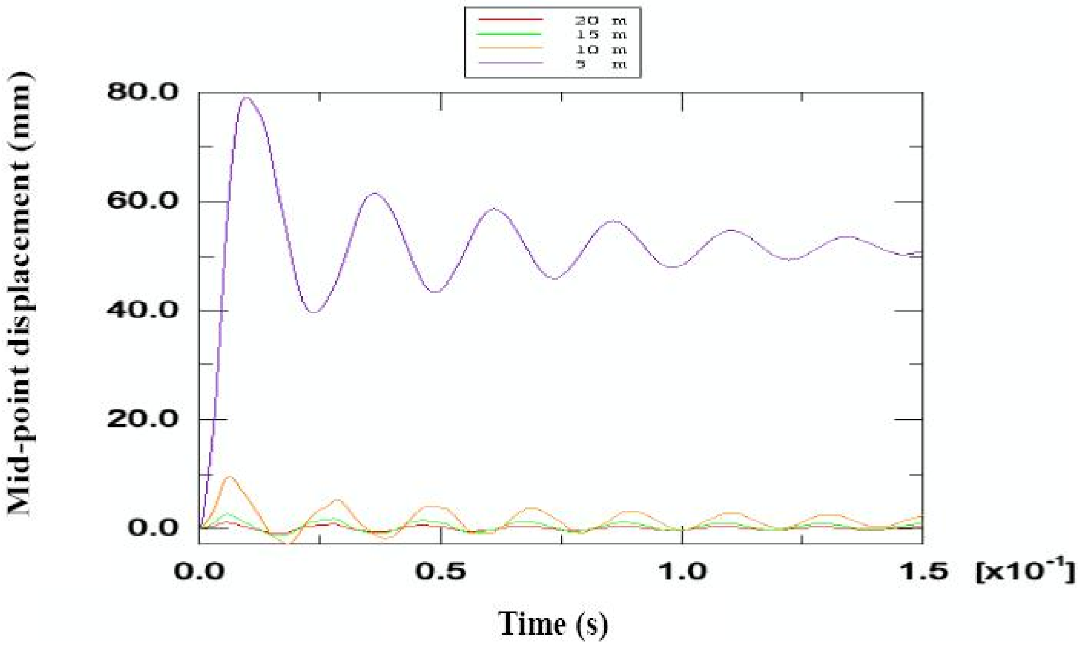


Figure 9. Influence of the stand-off distance from the explosion for a charge of 500 Kg TNT (L=12m, B= 10m)

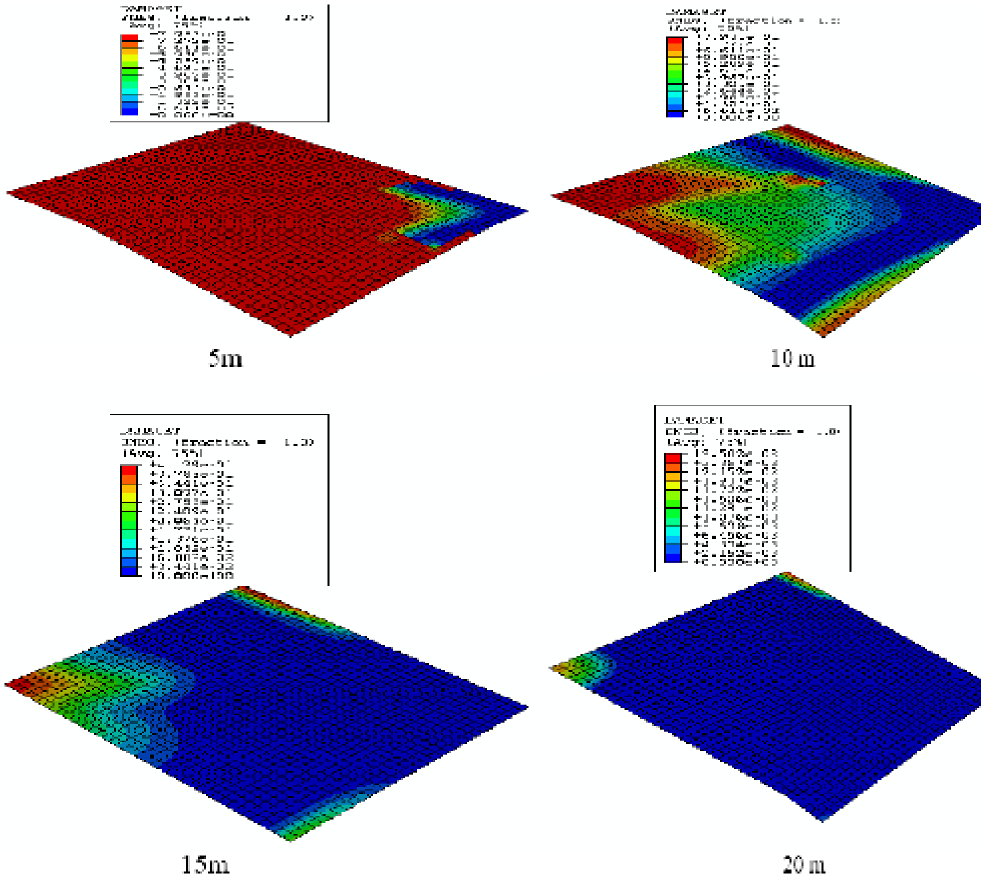


Figure 10. Tensile damage depending on the stand-off distance for a charge of 500kg TNT (L=12m, B=10m)

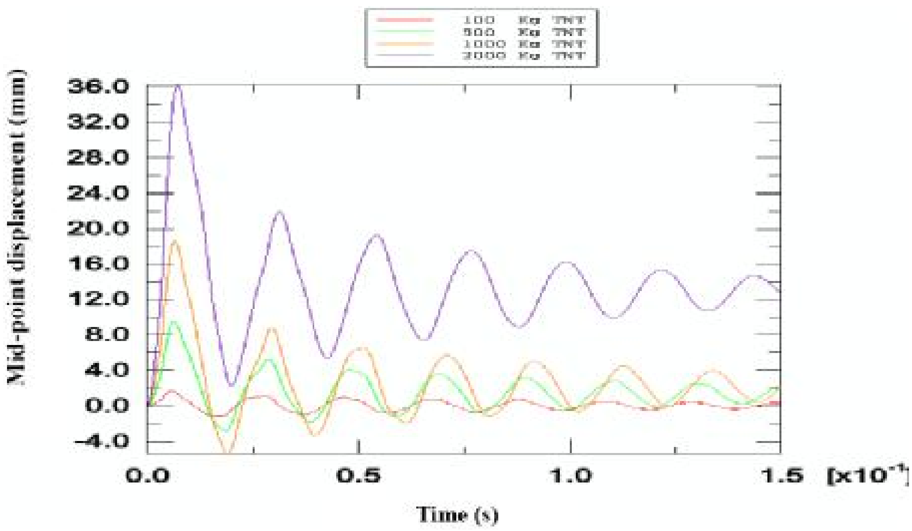


Figure 11. Influence of the weight of the explosive for a stand-off distance of 10m (L=12m, B=10m)

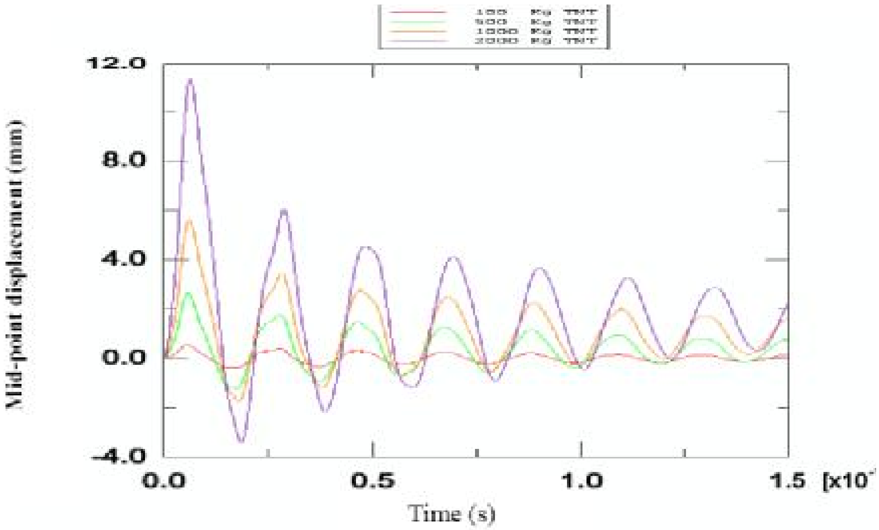


Figure 12. Influence of the weight of the explosive for a stand-off distance of 15m (L=12m, B= 10m)

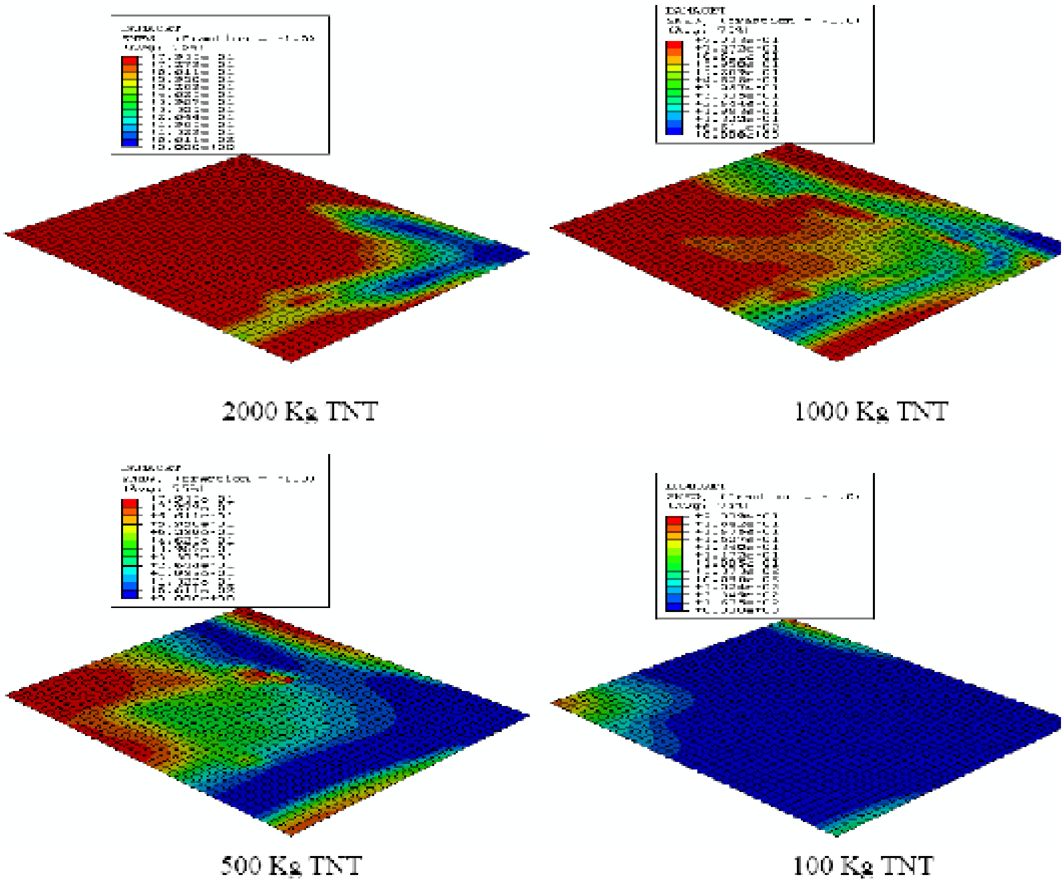


Figure 13. Tensile damage depending on the weight in kg TNT for a distance of 10m (L=12m, B=10m)

5.4 Influence of the negative phase

The graphs of the exponential form of the overpressure for different values of b and time duration t_d are shown in Figure 14. It can be observed that for b=0.1, the area of the negative phase is very important especially when the value of the time duration, t_d decreases. For b=1, the area of the negative phase is equal to that of the positive phase.

From structural dynamics point of view, taking into account the influence of the negative phase is equivalent to studying the structure in forced vibrations in the negative excitation with initial conditions from the positive excitation. From Figure 15, it can be observed that during the residual vibration era, an important displacement opposite to that of the positive excitation occurs and which corresponds to a drag for **b** = 0.1. These results are further confirmed by the observation of spread of tensile damage in the slab, Figure 16.

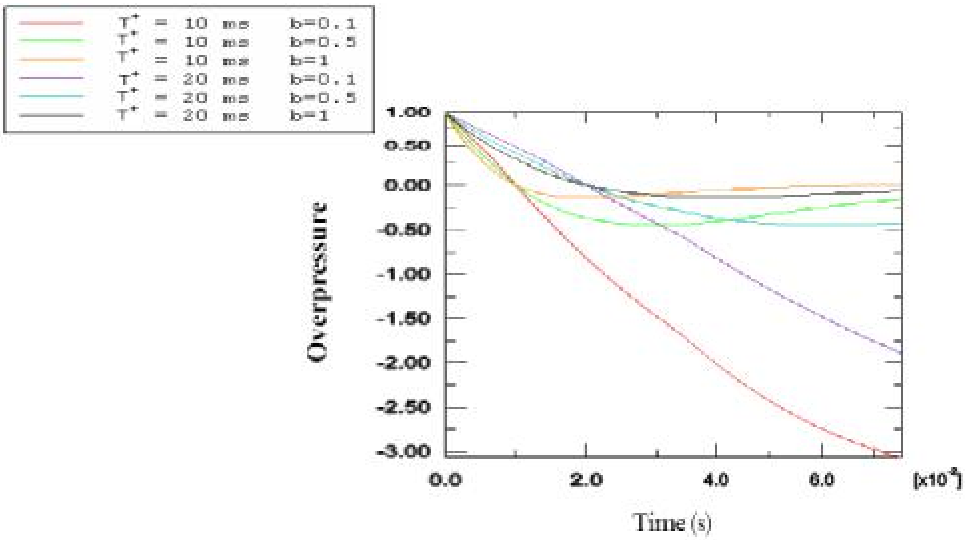


Figure 14. Negative phase area depending on the values of b and T^+

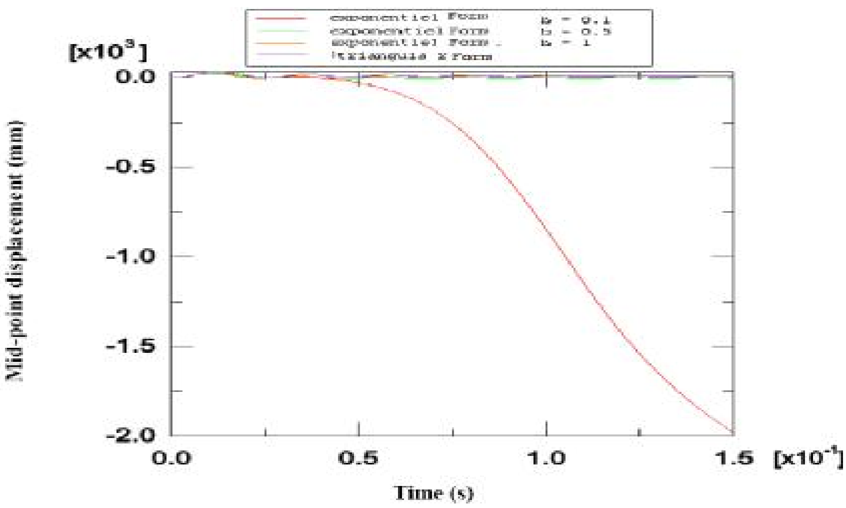


Figure 15. Influence of the negative phase (L=12m, B= 10m)

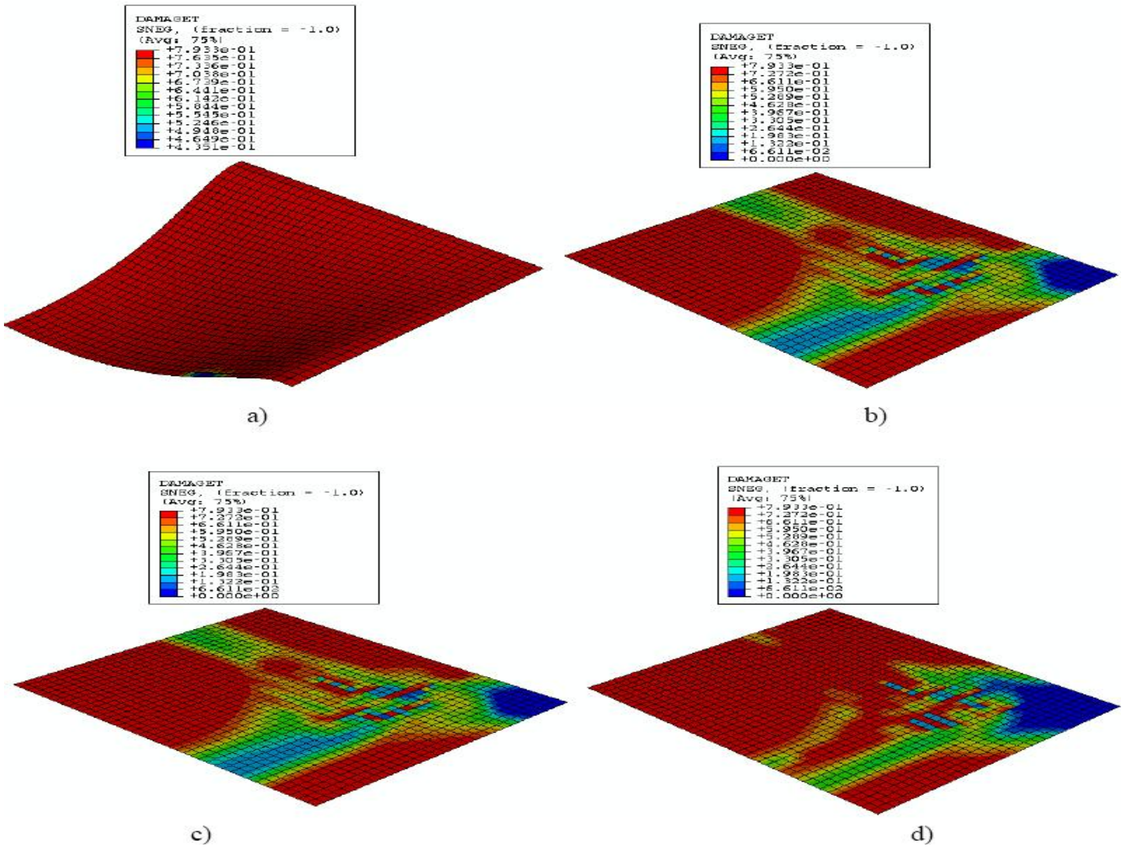


Figure 16. Evolution of tensile damage during the negative phase a) b=0.1 2) b=0.5 c) b=1 d) triangular form (L=12m, B= 10 m)

6. CONCLUSIONS

From the non linear dynamic finite element analyses carried out to examine the behavior of reinforced concrete slabs under blast loading, the following conclusions can be drawn:

i. The time duration is an important parameter since it has an influence on the response of reinforced concrete slabs.

ii. The inclusion of the negative phase of blast loading (which is often neglected) affects consequently the response of reinforced concrete slabs since the displacement in the opposite direction of the positive duration of blast loading can very excessive and will result in the collapse of the slabs especially when the value of the parameter **b** is close to 0.1.

iii. Slab dimensions are an important parameter that has to be considered when designing reinforced concrete slabs against blast loading. The results obtained indicate that slabs having one dimension much larger than the other have a better behavior than that of slabs having close dimensions in the two directions.

iv. The stand-off distance and the equivalent weight in TNT of the explosive are critical parameters that should be taken into account when analyzing reinforced concrete slabs under blast loading.

v. The plastic-damage plasticity model of concrete coupled with the explicit method is an efficient scheme for predicting the evolution of damage in reinforced concrete slabs.

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