NEWMARK IMPLICIT TIME INTEGRAL FOR APPLICATION IN PSEUDO-DYNAMIC TESTING – EXPERIMENTAL VERIFICATION

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ABSTRACT

Conventional experimental techniques like, quasi-static, effective force and shake-table techniques are generally being adopted to evaluate the seismic response of a civil engineering structure under earthquake loading. Among these techniques, shake-table technique has a merit over the other techniques due the fact that it realistically simulates all the three basic dynamic force parameters namely inertial, damping and elastic forces in the test structure. However this technique needs a sophisticated shake-table driven by servo hydraulic actuators with excellent control electronics. In the absence of such an expensive shake-table, it is possible to simulate the three dynamic force parameters using a static actuator through application of an equivalent pseudo-dynamic force system by computation of inertial forces in the back-ground. Such a hybrid Pseudo-dynamic (PsD) technique needs a specialized algorithm based on an appropriate mathematical model for the off-line time integration and computation of inertial forces such that the dynamic displacements/forces are applied statically through static actuators. Restoring forces offered by the structure are experimentally measured on-line at each time step and reflects the actual in-elastic and energy dissipation characteristics of the tested structure. The paper presents the mathematical formulation of a ‘predictor-corrector’ method using Newmark implicit relations and its implementation in PsD technique for seismic response evaluation of structures. In the proposed method displacement iterations are made in the corrector phase in achieving the implicit displacement which is an improvement over the conventional method where displacement iterations are made to achieve the explicit displacement resulting in lesser accuracy. To experimentally verify this improved PsD technique, the seismic response quantities including base shear, roof displacement and energy dissipation of a model steel frame structure subjected to a simulated earthquake predicted using PsD technique were calibrated with seismic responses evaluated using standard shake-table technique. The paper also presents the causes for the deviation in the predicted seismic responses using PsD

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technique and highlights its merits and demerits over shake-table technique.

**Keywords:** Seismic response evaluation; earthquake engineering, pseudo-dynamic technique; shake-table technique; newmark implicit relations; predictor-corrector method

1. SEISMIC RESPONSE EVALUATION TECHNIQUES

Seismic response evaluation is an iterative process involving alternate stages of experimentation and computation. Several experimental techniques (Kausel [1]) are conventionally used to evaluate the seismic response of structures and structural systems. These include quasi-static, effective force and shake-table techniques. In a quasi-static technique the test structure is subjected to slowly changing prescribed forces or deformations by means of hydraulic actuators. Inertial forces within the structure are not considered in this technique. The purpose of this elementary test technique is to observe the material behavior of the structural elements, components, or joints when they are subjected to cycles of loading and unloading during a seismic event. The effective force technique is based on applying dynamic forces to a test structure that is anchored rigidly to an immobile ground; these forces are proportional to the prescribed ground acceleration and the local masses. The deflections measured in this test correspond to the motions of the structural points relative to the ground that would have been observed had the structure been subjected at its base to the actual earthquake. The shake-table technique is the most realistic experimental technique for evaluating the seismic response of structures subjected to simulated earthquake motions. This technique is used towards developing and validating new design and construction methodologies with improved seismic resistance and also for bench-marking new analytical tools and software. Essentially in shake-table technique, the three basic dynamic forces namely, inertial, elastic and damping forces are induced in the tested structure. Such a pure experimental seismic response evaluation of structures necessitates the use of sophisticated and expensive dynamic actuators and control systems. Also, it is difficult to design large shake-tables capable of reproducing actual ground motions, particularly when simulating multi-axial earthquakes. Among the reasons limiting the simulation of realistic effects are the deformability and inertia of the shake-table, its characteristic modes of vibration, the devices needed to carry the dead load of test structure and overturning moments without impeding the table’s motion, the friction of the bearings, the physical capabilities of the hydraulic actuators, and to a lesser extent the limitations in the control devices.

The pseudo-dynamic (PsD) technique resembles the quasi-static technique in that it also consists in applying slowly varying cyclic forces to the test structure. However, during testing, the motions and deformations observed in the test structure are used to infer the inertial forces that the structure would have been exposed to the actual earthquake; this information is then fed back into a control engine so as to determine and adjust the effective dynamic displacements that must be applied onto the structure. These pseudo-dynamic forces are typically accomplished by means of actuators pushing against a large reaction wall. This alternate seismic response evaluation technique is picking up in the recent years and it is essential for to-day’s needs of growing India with enhanced seismic risk. This technique has the advantage of testing large and tall structures with center of mass well above the base,
which normally can not be tested on a shake-table for evaluating their seismic performance. As this technique involves application of dynamic forces in an equivalent static mean through static actuators, close monitoring of the structural behavior including crack initiation, crack growth and stiffness degradation becomes possible. The draw back in such a hybrid technique is the lack of simulation of strain rate effects which may not be critical under seismic loads. Also the technique is time consuming due to its iterative nature.

2. PSEUDO-DYNAMIC TECHNIQUE

PsD technique is a combined computational and experimental technique for evaluating dynamic systems originally proposed by Takanashi et al. [2]. The technique relies on modeling inertial and damping forces computationally, while the nonlinear restoring forces are measured experimentally. The basic dynamic equilibrium equation of a single degree of freedom (SDOF) system can generally be expressed as

\[ ma + cv + kd = f \quad \text{(or)} \quad ma + cv + r = f \]  

(1)

Where, \( m \), \( c \) and \( k \) are mass, damping and stiffness values and \( d \), \( r \) and \( f \) are the displacement, internal restoring force and applied force respectively. For a linearly elastic system the internal restoring force vector \( r \) is equal to the product \( k.d \). The PsD technique (Hilbert et al. [3]) excludes numerical damping \( c \), since it is assumed that damping is implicitly taken into account in the experiment on reading the actual restoring forces which are directly measured from the test structure. Also, it is assumed that that \( r(x) \) is the only source of nonlinearity which can be obtained accurate enough through experimental measurements. The PsD technique uniquely utilizes both computational and experimental terms to form the equation of motion (Eq. (1)). The response is obtained by discretising time and calculating it in a step-by-step manner. A time stepping formulation computes a displacement step which is subsequently imposed on the structure by means of computer controlled servo-hydraulic actuators. Once the structure has been deformed, the resulting restoring forces are measured. Based on these restoring forces and the current damping and applied forces, the resulting new acceleration is calculated. A new displacement step can then be calculated, and the next step has thus commenced.

In comparison to shaking table technique, there are some important differences. As the PsD technique is carried out in a step-by-step fashion, it is clear that it is unrealistic to be able to progress the test in real time. Furthermore, as inertial effects are modeled computationally, such forces should not exist in the physical model. The time scale of a typical test is therefore expanded in magnitude which has both beneficial and adverse effects. The fact that the structure is displaced slowly (and can even be stopped) provides a good opportunity for inspection and any detailed readings to be taken; however, the strain rate effects on material response are neglected.

2.1 Mathematical formulation of PsD technique

Several time stepping algorithms including centre difference, Wilson-\( \theta \) and Newmark-\( \beta \) algorithms, have been proposed for application in pseudo-dynamic technique (Buonopane
The majority of these formulations are explicit due to the fact that the nonlinear structural restoring forces at the end of any time step are unknown and displacement iterations in pseudo-dynamic test are undesirable as these might result in partial unloading (Shing and Manivannan [8]). Although implicit techniques have the advantage of being unconditionally stable, the duration of the time steps still has to be limited for accuracy purposes due to rapid changes in both loading and stiffness.

2.2 PsD formulation based on Newmark explicit relations

The Newmark explicit relations were originally proposed by Chang et al. [9] for implementation in PsD technique. The basic Newmark implicit relations after excluding numerical damping are given as

\[ ma_{n+1} + r_{n+1} = f_{n+1} \]  
\[ v_{n+1} = v_n + (1 - \gamma)\Delta t a_n + \gamma \Delta t a_{n+1} \]  
\[ d_{n+1} = d_n + \Delta t v_n + \left( 1 - \beta \right) (\Delta t)^2 a_n + \beta (\Delta t)^2 a_{n+1} \]

In the above, Eq. (2) is the system equilibrium equation and Eqs. (3) and (4) are respectively the velocity and displacement predictor equations. The terms \( f_{n+1} \) and \( r_{n+1} \) are respectively the numerically assumed external force (earthquake force) and the internal restoring force referred at time \((n+1)\Delta t\) and \( ma_{n+1}, v_{n+1} \) and \( d_{n+1} \) are respectively the structure inertial force, velocity and displacement, referred to the same time step. In Eq. (2), the system mass and external forces are defined numerically, while the internal restoring force \( r_{n+1} \) is experimentally measured from a load cell fitted in the servo hydraulic actuator.

The constants \( \gamma \) and \( \beta \) are the same as included in the generalized Newmark technique. If a value \( \beta = 0.25 \) and \( \gamma = 0.5 \) is adopted, we get the Newmark constant acceleration version (Newmark 1959)[10]. Close examination of displacement predictor Eq. (4) reveals that the displacement predictor equation takes implicit form due to the presence of the unknown term \( a_{n+1} \) for predicting the displacement step to progress in pseudo-dynamic testing. In order to overcome this difficulty of implicit conditions Chang et al. [9] proposed \( \beta = 0 \), so that the implicit displacement predictor equation reduces into explicit form as

\[ d_{n+1} = d_n + \Delta t v_n + \frac{(\Delta t)^2 a_n}{2} \]

This new reduced Newmark explicit form of displacement predictor equation is used for predicting the displacement step to progress in PsD tests. This approximation results in amplitude errors leading to numerical damping and conditional stability (Shing and Mahin [11]). By reducing the time step it is possible to improve the accuracy of PsD technique. With reduced time step, PsD can be a fair approximate technique for predicting the seismic
response of simple structural systems. However for complicated systems the technique needs further improvement which can be achieved by adopting the original Newmark implicit relations without omitting the $\beta$ term. This improvement made by including $\beta$ term in the displacement predictor equation results in unconditional stability and zero numerical damping leading to near zero amplitude errors in PsD technique.

2.3 Improved PsD formulation based on Newmark implicit relations

In order to implement the Newmark implicit relations in pseudo-dynamic technique, a ‘Predictor-Corrector’ method is adopted in the present study. The proposed method eliminates the difficulty of unknown restoring force $r_{n+1}$ in the beginning of the time step. The method excludes damping, since it is assumed that damping is implicitly taken into account in the experiment on reading the actual restoring forces which are measured from the test structure. For a SDOF system, the algorithm consists in solving iteratively the discrete system of Newmark implicit equations, Eqs. (2, 3 and 4). To implement the algorithm in a pseudo-dynamic formulation, it is necessary to reformulate Eq. (4). Rearranging Eq. (2) in terms $a_{n+1}$, we get

$$a_{n+1} = m^{-1}(f_{n+1} - r_{n+1})$$  \hspace{1cm} (6)

Substituting Eq. (6) in Eq. (4), the expression for the displacements can be rewritten as

$$d_{n+1} = \hat{d}_{n+1} - Br_{n+1}$$  \hspace{1cm} (7)

Where,

$$\hat{d}_{n+1} = d_n + \Delta t v_n + \left(\frac{1}{2} - \beta\right)\Delta t^2 a_n + \beta\Delta t^2 m^{-1} f_{n+1}$$  \hspace{1cm} (8)

Eq. (8) represents the explicit part of the displacement $d_{n+1}$, because all terms in the equation can be calculated with the available information from the previous time step. The constant $B$ is defined as:

$$B = m^{-1} \beta \Delta t^2$$  \hspace{1cm} (9)

Eq. 7 represents the implicit form for the displacement, as it can be noticed that the last term on the right hand side cannot be calculated at time $(n + 1)\Delta t$. In fact, as the test structure is displaced by actuators, the restoring forces also change continuously, while $r_{n+1}$ is unknown at the beginning of the time step.

2.4 Predictor-Corrector method for PsD technique

This proposed method splits each displacement step into a ‘Predictor Phase’ and a ‘Corrector Phase’ and solves the problem of unknown restoring force iteratively, through a corrective driving parameter (Combescurc and Pegon [6]) measured experimentally from the test structure as the test progresses. Figures 1a & 1b respectively show the predictor phase and corrector phase in the present PsD scheme adopting predictor-corrector method. In order to solve the problem of unknown restoring force, the method performs the following steps:
1. In the initial ‘Predictor Phase’ (Figure 1a), the predictor displacement \( \hat{d}_{n+1} \) (i.e., the explicit displacement) is computed numerically using Eq. (8). Then this numerically computed displacement vector is imposed on to the test structure through servo hydraulic actuators.

2. In the subsequent ‘Corrector Phase’ (Figure 1b), as soon as the test structure is displaced by the actuator to achieve the desired displacement \( d_{n+1} \), an intermediate restoring force vector \( r \) and displacement \( x \) change continuously.

3. Meanwhile, the actuator controller is continuously monitoring a ‘Corrective Driving Parameter’ called \( e \) which is given by:

\[
\begin{align*}
\text{e} &= \hat{d}_{n+1} - (x + Br) \\
&= (\hat{B}_n x_n) + \hat{d}_{n+1} - (x + Br)
\end{align*}
\]

4. After the end of the time step, the corrective parameter \( e \) should vanish \( (e = 0) \); then, the measured displacement \( x \) given by:

\[
\begin{align*}
\text{x} &= \hat{d}_{n+1} - Br_{n+1}
\end{align*}
\]

matches \( d_{n+1} \) corresponding to the desired actual displacement vector.

5. With the use of Eqs. (2) and (3) for calculation of actual system acceleration and velocity vectors, the completion of the actual structural kinetic state is achieved; then, the entire data are sent to the actuator control unit, and the procedure is repeated from step 1 for the next displacement increment \( d_{n+2} \).

Figure 1a. Predictor phase of the displacement step in PsD testing
3. EXPERIMENTAL VERIFICATION OF THE IMPROVED PSEUDO-DYNAMIC TECHNIQUE

The accuracy of improved PsD technique presented is verified through calibrating the seismic responses predicted using PsD technique with test results obtained using shake-table technique. For conducting PsD and shake table tests three scaled synthetic earthquake inputs (magnitudes ±0.05g, ±0.1g and ±0.15g) in the form of spectrum compatible acceleration time history matching the design acceleration response spectrum specified in [12] for Design Basis Earthquake (DBE), Zone IV, Medium Soil conditions are used. Figure 2 shows a typical earthquake input record (magnitude ±0.1g; duration 20.48 seconds) adapted in the present study. The test structure selected for the experimental verification is a single-bay three-storey steel frame having an overall height of 2.7m and plan dimensions of 0.93mx0.93m. The basic dynamic characteristics namely natural frequency, mode shape and associated damping values of the test structure were initially evaluated using shake-table technique. The first three natural frequencies (translation modes) were found to be respectively 5.75 Hz, 20.25 Hz and 38.50 Hz and the associated damping values were 2.27%, 1.69% and 1.12% respectively.
3.1 PsD test performed on the test structure

In the PsD test a single degree of freedom (SDOF) idealization is adapted using a displacement controlled servo hydraulic actuator (+50kN load capacity; ±50mm stroke; ±1.0g acceleration capacity; Saginomiya make) displacing the test structure at the roof level. The LVDT built inside the actuator measures the applied displacement (roof displacement) and the load cell fixed at the end of the piston measures the reactive force (base shear) generated by the test structure. Figure 3 shows the photographic view of the PsD test arrangement made on the test structure.
The selected earthquake input record (Figure 3) of duration 20.48 seconds consists of 4096 data points at a sample interval of 0.005 second and this input requires 4096 displacement steps to complete. In each displacement step the method iterates in the corrector phase until the corrective driving parameter $e$ becomes either zero or insignificant. A target value of zero for $e$ at each displacement step may likely to result in large number of displacement iterations leading to very long test duration. Hence in the present study a target value of 0.05 (5% of maximum implicit displacement) was adapted for $e$. With this target value, the PsD technique took about 22,118 seconds (6.144 hours) to complete all the 4096 data points in the selected earthquake input record of 20.48 seconds duration. This means the test duration is expanded in real time by 1080 folds due to the iterative nature of the PsD technique. However stopping the test process at an intermediate stage and restarting the test process again was possible. This allows close monitoring of the material behaviour like monitoring propagation of crack trajectories at any point of time in the middle of the test, which becomes a major advantage in PsD technique. Figure 4 shows the traces of seismic responses evaluated at an intermediate stage in the middle of the PsD test. Figure 5 shows the complete record of seismic response quantities (roof displacement and base shear) of the test structure evaluated using PsD technique. Similarly, Figure 6 shows the corresponding base shear vs. roof displacement (hysteretic) curve of the test structure.

Figure 4. Traces of seismic responses of the test structure evaluated at an intermediate stage in the middle of the PsD test
Figure 5. Seismic responses (roof displacement & base shear) of the test structure evaluated using PsD technique

Figure 6. Base shear vs. roof displacement (hysteretic) curve of the test structure evaluated using PsD technique
3.2 Shake-table test performed on the test structure

For the shake-table test, the same synthetic earthquake input in the form of a spectrum compatible acceleration time history as that used for PsD test is used. Figure 7 shows the photographic view of the shake table test arrangement made on test structure. The shake-table used in the present study is a unidirectional shake-table of size 2mx2m driven by a displacement controlled servo hydraulic actuator (±100kN load capacity; ±150mm stroke; ±1.0g acceleration capacity; Instron make). Since the shake-table is driven by a displacement controlled actuator, a displacement time history derived from the spectrum compatible acceleration time history was used.

![Figure 7. View of the shake-table test arrangement made on the test structure for seismic response evaluation](image)

Acceleration and displacement responses were measured respectively using accelerometers (B&K type 4507 accelerometers coupled with B&K type Nexus 2693-014 signal conditioners) and LVDTs (HBM type W-100TS LVDTs coupled with HBM type MVD2555 signal conditioners) at every floor level (three in the present study) and simultaneously acquired on a 16 Channel Data Acquisition System (Dewtron type DEWE-2010) where post processing of the measured response signals were done. In the post processing stage the displacement responses were numerically multiplied with the total column stiffness to arrive at the elastic forces generated in the test structure. Similarly, by numerical multiplication of acceleration responses with floor mass, the inertial forces generated in the test structure were arrived. Using these inertial and elastic forces, the base shear forces were arrived. Inter-storey displacement responses of the test structure were obtained as the difference in the displacement responses of the successive floors. Finally by plotting the base shear force against inter-storey displacement response, the energy dissipation hysteretic curves were arrived for every floor. From the area of the energy dissipation hysteretic curve the energy dissipation ability of the test structure was evaluated. Figure 8 shows the seismic responses (roof displacement & base shear) of the test structure evaluated using shake-table technique. Similarly, Figure 9 shows the corresponding base shear vs. roof displacement (hysteretic) curve of the test structure.
Figure 8. Seismic responses (roof displacement & base shear) of the test structure evaluated using shake-table technique

Figure 9. Base shear vs. roof displacement (hysteretic) curve of the test structure evaluated using shake-table technique
3.3 Observations and discussions

Table 1 shows the comparison of seismic response quantities of the test structure obtained using PsD and shake-table techniques. Typical graphical comparison of the seismic responses evaluated using PsD and shake-table techniques are shown in Figures 10 & 11. These figures show the blow-up view of seismic responses for 3 second duration (between 9 second to 12 second) in the middle of the record. In these figures it is important to note that the time axis is different for PsD test and shake-table test. For shake-table test it represents the real time, whereas for PsD test it represents the expanded time. Hence for comparison purpose, seismic responses evaluated using shake-table test at every sample interval and seismic responses evaluated using PsD test at corresponding data point are plotted together in the same graph. Figure 12 shows typical graphical comparison of base shear vs. roof displacement curves representing the energy dissipation capacity of the test structure evaluated using PsD and shake-table techniques.

From Table 1 and Figures 10 & 11 it is observed that the improved PsD technique presented estimates the roof displacement and base shear fairly well. However, the energy dissipation characteristics are observed to be marginally underestimated, which is evident from Figure 12. The smoothness of the seismic response curves evaluated from PsD test shows the absence of higher mode participation due to SDOF idealization which is also evident from Figures 10 & 11. This error phenomenon observed in underestimating the seismic responses could be due to the absence of rate effect simulation and the inability to capture visco-elastic effects into the tests, which are the drawbacks of PsD technique. Increased degrees of freedom using additional actuators could improve the accuracy of the seismic response prediction leading to reduced errors.

<table>
<thead>
<tr>
<th>Technique adapted</th>
<th>Pseudo-dynamic technique</th>
<th>Shake table technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input magnitude</td>
<td>±0.05g ±0.10g ±0.15g</td>
<td>±0.05g ±0.10g ±0.15g</td>
</tr>
<tr>
<td>Base shear (N)</td>
<td>+609.6 -611.8 +1225.2 -1229.5 +1839.9 -1841.6</td>
<td>+620.65 -1237.3 +1858.5</td>
</tr>
<tr>
<td>Roof Displacement (mm)</td>
<td>+1.20 -1.71 +2.45 -3.38 +3.71 -5.11</td>
<td>+1.24 -1.75 +2.59 -3.49</td>
</tr>
<tr>
<td>Energy dissipation per cycle (N-mm)</td>
<td>302.45 580.95 880.52</td>
<td>334.67 656.87 1013.58</td>
</tr>
</tbody>
</table>
Figure 10. Typical comparison of roof displacement of the test structure evaluated using PsD and shake-table techniques

Figure 11. Typical comparison of base shear of the test structure evaluated using PsD and shake-table techniques
Figure 12. Typical comparison of energy dissipation capacity of the test structure evaluated using PsD and shake-table techniques

4. SUMMARY AND CONCLUSIONS

The genesis and mathematical formulation of pseudo-dynamic (PsD) technique for experimental seismic performance evaluation of structures are presented in the paper. An improved PsD test scheme adopting predictor-corrector method using Newmark implicit relations is presented. The improved scheme makes iterations in the corrector phase at every displacement step in achieving the actual implicit displacement. The presented PsD test scheme was verified through calibrating the pseudo-dynamically evaluated seismic responses of a single-bay three storey steel frame structure with shake-table test results. The proposed PsD test scheme is found to estimate the base shear force and roof displacement.
fairly well and marginally underestimate energy dissipation capacity, which were attributed to the absence of higher mode participation, rate effect simulation and inability to capture visco-elastic effects. Hence in the absence of shake-table technique which is costlier, the PsD technique will be a viable alternative for seismic performance evaluation of structures.

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