

A NEW HYBRID META-HEURISTIC FOR OPTIMUM DESIGN OF FRAME STRUCTURES

A. Kaveh^{*a}, S. Talatahari^b and M.T. Alami^c

^aCentre of Excellence for Fundamental Studies in Structural Engineering, Iran University of
Science and Technology, Narmak, Tehran-16, Iran

^bMarand Faculty of Engineering, University of Tabriz, Tabriz, Iran

^cDepartment of Civil Engineering, University of Tabriz, Tabriz, Iran

Received: 10 October 2011, **Accepted:** 17 February 2012

ABSTRACT

Meta-heuristic methods provide powerful means to optimize frame structures and hybridizing these methods seems to be unavoidable to improve the properties of the algorithms. This paper provides a new hybrid advanced algorithm by using the abilities of heuristic particle swarm ant colony optimization (HPSACO) and a hybrid big bang–big crunch algorithm (HBB–BC). The advantages of the HPSACO and HBB–BC are combined to improve the performance of the resulted algorithm. In the present approach, there are three main steps as global searching step, local searching step and location controlling step. These steps all together improve the exploration and exploitation abilities of the algorithm. The proposed method is tested on frame structures from the literature. The results of the optimum design obtained by the present study are compared to those of some existing optimization methods to verify the suitability of the new method.

Keywords: Meta-heuristic optimization algorithms, Hybrid big bang–big crunch algorithm, Heuristic particle swarm ant colony optimization, Optimal design of frames

1. INTRODUCTION

In today's world, providing powerful algorithms to solve structural optimization problems forms an important research field due to the necessity of performing efficient designs. Meta-heuristics are known as one of the most powerful approaches in this area and hybridizing these methods seems to be unavoidable to improve the properties of the algorithms. This paper presents such an improved algorithm by hybridizing two recent advanced hybrid algorithms. These algorithms are heuristic particle swarm ant colony optimization (HPSACO) [1] and a hybrid big bang–big crunch (HBB–BC) optimization algorithm [2].

* E-mail address of the corresponding author: alikhavah@iust.ac.ir (A. Kaveh)

HPSACO was designed to improve the performance of the standard particle swarm optimizer (PSO). In fact, the main optimizer of HPSACO is the PSO and the advantages of other methods are added to improve the abilities of this method. Although the standard PSO often does eventually locate the desired solution, however, its practical use in solving engineering optimization problems is severely limited due to high computational cost of the slow convergence rate [3]. In addition it is known that PSO had difficulties in controlling the balance between exploration (global investigation of the search place) and exploitation (the fine search around a local optimum), [4].

The HBB-BC algorithm was designed to improve the standard BB-BC method. The standard BB-BC performs well in the exploitation, however it experiences some problems at the exploration stage. For example, if all of the candidates in the initial Big Bang are collected in a small part of search space, the BB-BC method may not find the optimum solution and with a high probability, it may be trapped in that subdomain [2]. HBB-BC adds the capacities of the PSO to the BB-BC where considers the combinational of the center of mass, the best position of each candidate and the best visited position of all candidates as an average point in the beginning of each Big Bang.

The new algorithm is obtained by collecting the good features of these two advantaged algorithms to increase the optimization abilities of the resultant algorithm. Finally some structures previously solved by meta-heuristic algorithms namely the HPSACO and HBB-BC are considered to investigate the capacity of the final algorithm in finding the optimum design of structures.

The remainder of this paper is organized as follows. Section 2 presents the formulation of the frame optimization problem. Section 3 reviews the HPSACO and HBB-BC as the previous hybrid algorithms. The new algorithm is presented in Section 4. Numerical study is provided in Section 5, and finally Section 6 concludes the paper.

2. FRAME OPTIMIZATION PROBLEMS

Optimal design of frame structures can be formulated as [1]

$$\begin{aligned} &\text{Find} && \mathbf{X} = [x_1, x_2, \dots, x_{ng}] \\ &\text{to minimize} && \text{fit}(\mathbf{X}) = f(\mathbf{X}) \times f_{\text{penalty}}(\mathbf{X}) \end{aligned} \quad (1)$$

subjected to the following constraints:

The maximum lateral displacement:

$$\mathbf{u}^\Delta = \frac{\Delta_T}{H} - R \geq 0 \quad (2)$$

The inter-story displacements:

$$\mathbf{u}_j^d = \frac{d_j}{h_j} - R_l \geq 0, \quad j = 1, 2, \dots, ns \quad (3)$$

where \mathbf{X} is the vector of design variables containing the cross-sectional areas of W sections; ng is the number of design variables or the number of member groups; $fit(\mathbf{X})$ is the fitness function; $f(\mathbf{X})$ is the cost function which is usually taken as the weight or volume of the structure; $f_{penalty}(\mathbf{X})$ is the penalty function which results from the violations of the constraints corresponding to the response of the structure; Δ_T is the maximum lateral displacement; H is the height of the frame structure; R is the maximum drift index; d_j is the inter-story drift; h_j is the story height of the j th floor; ns is the total number of stories; R_l is the inter-story drift index permitted by the code of the practice.

Using the AISC [5] for design, the permissible inter-story drift index is taken as 1/300 and from the LRFD interaction formula constraints (AISC, 2001, [5] Equation H1-1a,b), the violation is defined as

$$u_i^I = \frac{P_u}{2f_c P_n} + \left(\frac{M_{ux}}{f_b M_{nx}} + \frac{M_{uy}}{f_b M_{ny}} \right) - 1 \geq 0 \quad \text{For} \quad \frac{P_u}{f_c P_n} < 0.2 \quad (4)$$

$$u_i^I = \frac{P_u}{f_c P_n} + \frac{8}{9} \left(\frac{M_{ux}}{f_b M_{nx}} + \frac{M_{uy}}{f_b M_{ny}} \right) - 1 \geq 0 \quad \text{For} \quad \frac{P_u}{f_c P_n} \geq 0.2 \quad (5)$$

where P_u is the required strength (tension or compression); P_n is the nominal axial strength (tension or compression); f_c is the resistance factor ($f_c = 0.9$ for tension, $f_c = 0.85$ for compression); M_{ux} and M_{uy} are the required flexural strengths in the x and y directions; respectively; M_{nx} and M_{ny} are the nominal flexural strengths in the x and y directions, and f_b is the flexural resistance reduction factor ($f_b = 0.90$).

The cost function in the form of the weight of frame structure is expressed as

$$f(\mathbf{X}) = \sum_{i=1}^{ng} g_i \cdot x_i \cdot L_i \quad (6)$$

where g_i is the material density of member i ; L_i is the sum length of the members belonging to group i .

The penalty function is defined as

$$f_{penalty}(\mathbf{X}) = (1 + \mathbf{e}_1 \cdot \mathbf{u})^{e_2}, \quad \mathbf{u} = \sum_{i=1}^n \max[0, u_i] \quad (7)$$

where $n = nm + ns + 1$ represents the number of evaluated constraints for each individual design. Here, nm is the number of elements to control the interaction formula constraints (Eqs. (4), (5)), ns is the number of stories to check the inter-story drift constraint (Eq. (3)), and one is because of checking the total lateral displacement constraint. e_1 is set to 1, e_2 is selected in

a manner to decrease the penalties and reduce the cross-sectional areas. Thus, in the first steps of the search process, e_2 is set to 1.5 and it is ultimately increased to 3, [6].

3. A REVIEW OF TWO HYBRID META-HEURISTIC ALGORITHMS

In this section a brief review of the heuristic particle swarm ant colony optimization (HPSACO) and a hybrid Big Bang–Big Crunch optimization algorithm (HBB–BC) is presented.

3.1 Heuristic particle swarm ant colony optimization

The HPSACO algorithm applies particle swarm optimizer with passive congregation (PSOPC) for global optimization, while ant colony strategy (ACO) works as a local search, wherein, ants apply pheromone-guided mechanism to refine the positions found by particles in the PSOPC stage. In HPSACO, a simple pheromone-guided mechanism of ACO is proposed and employed for the local search.

The PSOPC stage involves a number of particles, which are initialized randomly in the feasible space. These particles fly through the search space and their positions are updated based on the best positions of individual particles, the best position among all particles in the search space, and the position of a particle selected randomly from the swarm in each iteration.

The update moves a particle by adding a velocity change, \mathbf{V}_i^{k+1} , to the current position \mathbf{X}_i^k as follows:

$$\mathbf{X}_i^{k+1} = \mathbf{X}_i^k + \mathbf{V}_i^{k+1} \quad (8)$$

$$\mathbf{V}_i^{k+1} = w\mathbf{V}_i^k + c_1r_1(\mathbf{P}_i^k - \mathbf{X}_i^k) + c_2r_2(\mathbf{P}_g^k - \mathbf{X}_i^k) + c_3r_3(\mathbf{R}_i^k - \mathbf{X}_i^k) \quad (9)$$

where w is an inertia weight to control the influence of the previous velocity; r_1 , r_2 and r_3 are three random numbers uniformly distributed in the range of (0,1); c_1 and c_2 are two acceleration constants; c_3 is the passive congregation coefficient; \mathbf{P}_i^k is the best position of the i th particle up to iteration k ; \mathbf{P}_g^k is the best position among all particles in the swarm up to iteration k ; and \mathbf{R}_i is a particle selected randomly from the swarm.

The ACO stage handles M ants equal to the number of particles in PSOPC, [1], and each ant generates a solution around \mathbf{P}_g^k which can be expressed as

$$\mathbf{Z}_i^k = N(\mathbf{P}_g^k, \mathbf{s}) \quad (10)$$

where, \mathbf{Z}_i^k is the solution constructed by ant i in the stage k ; $N(\mathbf{P}_g^k, \mathbf{s})$ denotes a random vector normally distributed with mean value \mathbf{P}_g^k and variance \mathbf{s} , where

$$\mathbf{s} = (X_{\max} - X_{\min}) \times \mathbf{h} \quad (11)$$

Here, h is the step size. ACO stage in the HPSACO algorithm works as a helping factor to guide the exploration and to increase the control in the exploitation [1].

Then, the value of the objective function for each ant, $fit(\mathbf{Z}_i^k)$, is computed and the current position of ant i , \mathbf{Z}_i^k , is replaced with the position \mathbf{X}_i^k , the current position of particle i in the swarm, if $fit(\mathbf{X}_i^k)$ is bigger than $fit(\mathbf{Z}_i^k)$ and current ant is in the feasible space.

In this algorithm, if a particle flies out of the variable boundaries, the solution cannot be used even if design constraints are satisfied. Here, the harmony search-based approach is employed to deal with this problem. According to this mechanism, any component of the solution vector (particle or ant) violating the variable boundaries can be regenerated randomly from \mathbf{P}_i^k as

$$x_i^{(k,j)} = \begin{cases} \text{w.p. HMCR} \implies \text{select a new value for a variable from } \mathbf{P}_i^k \\ \implies \text{w.p. } (1-\text{PAR}) \text{ do nothing} \\ \implies \text{w.p. PAR choose a neighboring value} \\ \text{w.p. } (1-\text{HMCR}) \implies \text{select a new value randomly} \end{cases} \quad (12)$$

where $x_i^{(k,j)}$ is the j th component of the particle i in iteration k ; The HMCR varying between 0 and 1 sets the rate of choosing a value in the new vector from the historic values stored in the \mathbf{P}_i^k , and $(1-\text{HMCR})$ sets the rate of randomly choosing one value from the possible range of values. The pitch adjusting process is performed only after a value is chosen from \mathbf{P}_i^k .

3.2 Hybrid Big Bang–Big Crunch optimization algorithm

The BB–BC method developed by Erol and Eksin [7] consists of two phases: a Big Bang phase, and a Big Crunch phase. In the Big Bang phase, candidate solutions are randomly distributed over the search space. The Big Crunch is a convergence operator that has many inputs but only one output, which is named as the *center of mass*. The term mass refers to the inverse of the fitness function value for the structures. The point representing the center of mass that is denoted by \mathbf{X}_c^k , is calculated according to

$$\mathbf{X}_c^k = \frac{\sum_{i=1}^M \frac{1}{fit(\mathbf{X}_i^k)} \cdot \mathbf{X}_i^k}{\sum_{i=1}^M \frac{1}{fit(\mathbf{X}_i^k)}} \quad (13)$$

where M is the population size in Big Bang phase.

After the Big Crunch phase, the algorithm creates the new solutions to be used as the Big Bang of the next iteration step, by using the previous knowledge. The hybrid BB–BC approach uses the center of mass, the best position of each candidate (\mathbf{P}_i^k) and the best global position (\mathbf{P}_g^k) to generate a new solution as

$$\mathbf{X}_i^{k+1} = a_2 \mathbf{X}_c^k + (1 - a_2)(a_3 \mathbf{P}_g^k + (1 - a_3) \mathbf{P}_i^k) + \frac{\mathbf{r}_i a_1 (x_{\max} - x_{\min})}{k + 1} \quad (14)$$

where \mathbf{r}_i is a random vector obtained from a standard normal distribution which changes for each candidate; a_1 is a parameter used for limiting the size of the search space; a_2 and a_3 are adjustable parameters controlling the influence of the global best and local best on the new position of the candidates, respectively. These successive explosion and contraction steps are carried out repeatedly until a stopping criterion has been met. A maximum number of iterations is utilized as a stopping criterion.

4. HEURISTIC BIG BANG–BIG CRUNCH PARTICLE SWARM ALGORITHM

The heuristic big bang–big crunch particle swarm algorithm (HBBPSO), a hybridized approach based on HPSACO and HBB–BC, is described in this section. This new algorithm follows the HPSACO levels and it can be considered as an improved HPSACO, however the positive characters of the HBB–BC is added to improve the final algorithm. In global searching level of the new algorithm instead of PSOPC, a methodology resulted by hybridizing the PSO and the big crunch level of the BB–BC is utilized. The local searching level employs the ACO stage of HPSACO and the big crunch level of the HBB–BC. Location controller is similar to HS strategy of HPSACO. The PAR parameter related to this level is updated according to some recent studies of the HS algorithm.

4.1 Global searching step

The main searching step is based on hybrid PSO and the BB–BC which is obtained by modifying the velocity formulation of the PSO algorithm by adding the term of the center of mass from the BB–BC algorithm. Often in order to enhance the searching abilities of the standard PSO algorithm, one or more additional terms are added to the velocity formula. For example Eq. (9) is obtained by adding \mathbf{R}_i (the location of a particle selected randomly from the swarm) to the standard PSO improving the searching performance of the algorithm because of increasing exploration of the algorithm. If one can utilize the effect of all other particles in the term of velocity (such as defined for charged system search [8,9]), more efficient PSO-based algorithm will be obtained. Since the center of mass point (\mathbf{X}_c^k) as defined in Eq. (13) is a good agent of all particles, it seems to be the best choice. Therefore the velocity formula is redefined as follows:

$$\mathbf{V}_i^{k+1} = w \mathbf{V}_i^k + c_1 r_1 (\mathbf{P}_i^k - \mathbf{X}_i^k) + c_2 r_2 (\mathbf{P}_g^k - \mathbf{X}_i^k) + c_3 r_3 (\mathbf{X}_c^k - \mathbf{X}_i^k) \quad (15)$$

4.2 Local searching step

After performing an iteration using the global searching, the position of the local best (\mathbf{P}_i^k) and the global best points (\mathbf{P}_g^k) are calculated. This level is used instead of the one defined for

the HPSACO in the ACO level (Eq. 10). In the new formulation, in addition to \mathbf{P}_g^k , other points such as \mathbf{P}_i^k and \mathbf{X}_c^k are used to generate new solutions of the local level. Therefore in this level, we will have

$$\mathbf{Z}_i^k = N(\mathbf{a}_2 \mathbf{X}_c^k + (1 - \mathbf{a}_2)(\mathbf{a}_3 \mathbf{P}_g^k + (1 - \mathbf{a}_3) \mathbf{P}_i^k), \mathbf{s}) \quad (16)$$

In fact this new relationship is obtained by combing the big bang level of HBB-BC defined by Eq. (14) and the ACO level of the HPSACO defined by Eq. (10). Because of using the information achieved by all particles instead of only \mathbf{P}_g^k , it is expected this new formula improves the exploitation ability of the algorithm.

4.3 Location controlling step

It is possible in both global search and local search levels, the particles move out of search space and therefore their locations must be corrected. The location correction level in the new algorithm is similar to the one defined for HPSACO, however the value of PAR is defined dynamically as follows [10]:

$$PAR^k = PAR_{\min} + \frac{PAR_{\max} - PAR_{\min}}{k_{\max}} k \quad (17)$$

where PAR^k is the value of PAR related to the k th iteration and PAR_{\max} and PAR_{\min} are the maximum and the minimum values for PAR and in this paper these are set to 0.99 and 0.35 according to the suggestion of Ref. [10].

Also, In Eq. (12), the definition of the term "choose a neighboring value" is as follows

$$x_i^{(k,j)} = x_i^{(k,j)} \pm r \cdot bw^k \quad (18)$$

where bw^k is defined as

$$bw^k = bw_{\max} \exp(c.k) \quad (19)$$

$$c = \frac{\ln(bw_{\min}/bw_{\max})}{k_{\max}}$$

In this paper, $bw_{\max} = 5$ and $bw_{\min} = 0.01$.

5. DESIGN EXAMPLES

This section presents the numerical example to evaluate the capability of the new algorithm in finding optimal design of steel structures. The examples contain one planar frame and one space frame. The final results are compared to the solutions of other methods to show the efficiency of the present approach.

5.1 Design of a 3-bay 24-story frame

Figure 1 shows the topology and the service loading conditions of a three-bay twenty four-story frame consisting of 168 members originally designed by Davison and Adams [11]. Camp and Bichon [12] utilized ant colony optimization (ACO), Degertekin [13] developed least-weight frame designs for this structure using a harmony search (HS) and the authors utilized HBB-BC [14] and imperialist competitive algorithms (ICA) [15] to optimize this frame.

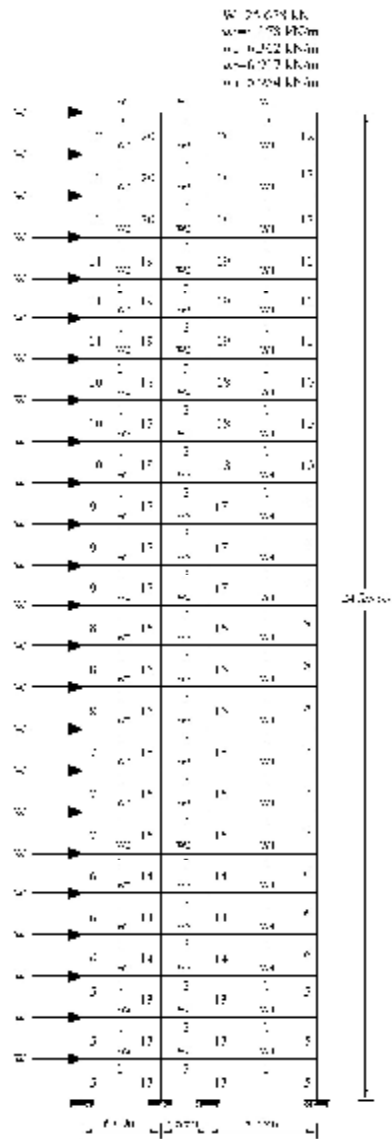


Figure 1. Topology of the three-bay twenty four-story frame

The frame is designed utilizing the LRFD specification and uses an inter-story drift displacement constraint. The modulus of elasticity $E=205\text{GPa}$ and a yield stress of $F_y=230.3\text{MPa}$. The detailed information is available in Ref. [14].

Table 1 lists the designs developed by: the HBBPSO, the HBB-BC algorithm [14], the ant colony algorithm [12], harmony search [13] and imperialist competitive algorithm [15]. The HBBPSO algorithm required 8,000 frame analyses to converge to a solution, while the 10,500 analyses were required by the HBB-BC, 15,500 analyses by the ACO, 13,924 analyses by the HS, 7,500 analyses for the ICA. Figure 2 compares the convergence histories for the HBBPSO and the HBB-BC algorithms. The HBBPSO algorithm can find the best results with 941.55kN which is 4.15%, 1.55%, 2.01% and 0.50% lighter than the results of the ACO, HS, HBB-BC and ICA, respectively.

Table 1: Optimal design comparison for the 3-bay 24-story frame

Element group	Optimal W-shaped sections				
	ACO [12]	HS [13]	HBB-BC [14]	ICA [15]	Present work
1	W30X90	W30X90	W30X90	W30X90	W30X90
2	W8X18	W10X22	W21X48	W21X50	W21X55
3	W24X55	W18X40	W18X46	W24X55	W21X48
4	W8X21	W12X16	W8X21	W8X28	W8X24
5	W14X145	W14X176	W14X176	W14X109	W14X176
6	W14X132	W14X176	W14X159	W14X159	W14X90
7	W14X132	W14X132	W14X109	W14X120	W14X99
8	W14X132	W14X109	W14X90	W14X90	W14X99
9	W14X68	W14X82	W14X82	W14X74	W14X74
10	W14X53	W14X74	W14X74	W14X68	W14X74
11	W14X43	W14X34	W14X38	W14X30	W14X38
12	W14X43	W14X22	W14X30	W14X38	W14X34
13	W14X145	W14X145	W14X159	W14X159	W14X145
14	W14X145	W14X132	W14X132	W14X132	W14X132
15	W14X120	W14X109	W14X109	W14X99	W14X109
16	W14X90	W14X82	W14X82	W14X82	W14X90
17	W14X90	W14X61	W14X68	W14X68	W14X74
18	W14X61	W14X48	W14X48	W14X48	W14X48
19	W14X30	W14X30	W14X34	W14X34	W14X38
20	W14X26	W14X22	W14X26	W14X22	W14X22
Weight (kN)	980.63	956.13	960.90	946.25	941.55
No. of required analyses	15,500	13,924	10,500	7,500	8,000

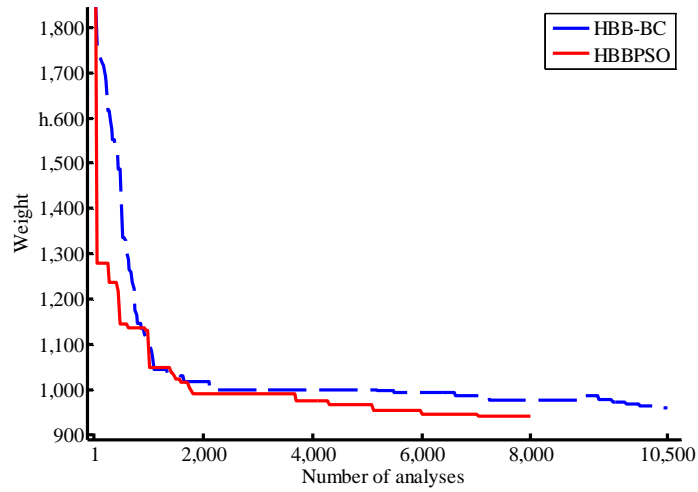


Figure 2. Comparison of the convergence history for the 2-bay 24-story planar frame

5.2 Design of a 290-member 10-story space frame

A 10-story space steel frame as the second example is considered as shown in Figure 3.

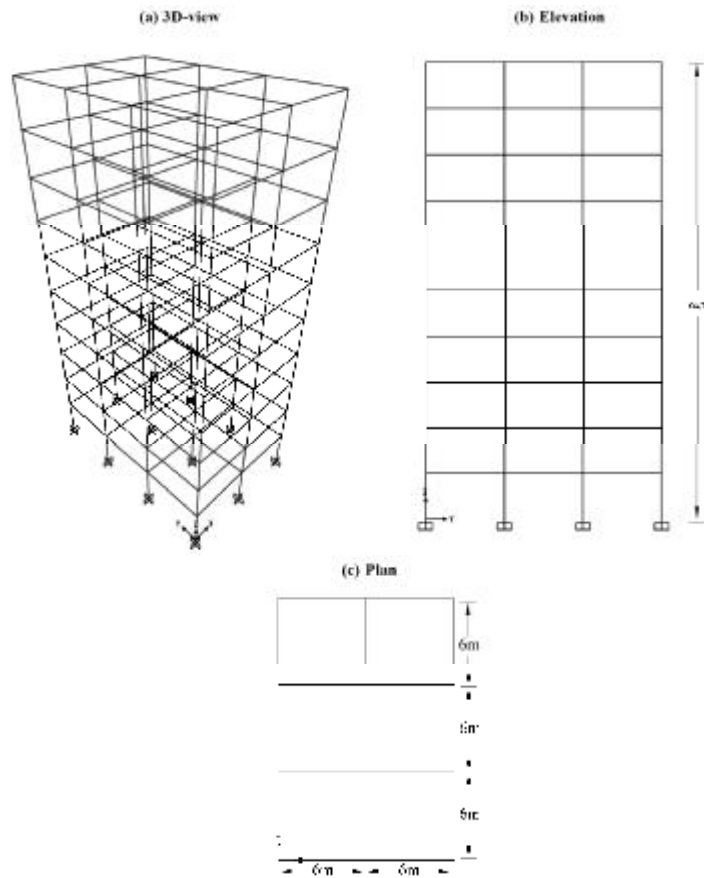


Figure 3. Topology of the 10-story space frame

The material properties have a modulus of elasticity $E = 200\text{GPa}$ and a yield stress of $f_y = 248.2\text{ MPa}$. The columns in each story are divided into three member groups as corner, inner and outer columns, whereas beams are divided into two groups as inner and outer beams. Each of the above five divisions includes the members having the same section in every three consequent stories and the last story thereafter. This results in a total of 20 distinct member groups as originally designed in [16].

The optimum design of the space frame described above is carried out using the HBBPSO, the HPSACO and the CSS in [16]. The optimum design attained by the HBBPSO method is 742.06 kN, while it is 760.64 kN, 799.43kN and 753.88kN for the HPSACO, HBB-BC and CSS methods, respectively [16]. The minimum weights as well as W-section designations obtained by these algorithms are provided in Table 2. The design history of the best run by each technique is shown in Figure 4 in which the maximum number of iterations is set to 14,000 analyses for the algorithms. This shows that the new algorithm can reach to better results compared to the other methods. For the design of the HBBPSO, the maximum value for the inter-story drift is 1.15cm which is less than the allowable limit (1.17 cm). The maximum value for the stress ratio is 96.66%.

Table 2: Optimal design comparison for the10-story space frame

Element group	Optimal W-shaped sections			
	HBB-BC [16]	HPSACO [16]	CSS [16]	Present work
1	W10X54	W14X61	W12X58	W14X61
2	W16X77	W16X77	W18X76	W18X76
3	W14X99	W12X79	W14X82	W18X86
4	W10X49	W16X57	W10X54	W16X50
5	W21X55	W18X55	W18X60	W24X62
6	W14X61	W18X71	W18X65	W21X68
7	W21X44	W8X24	W8X24	W14X30
8	W12X53	W14X43	W16X36	W14X38
9	W14X61	W21X50	W21X50	W21X48
10	W10X12	W5X16	W10X15	W10X12
11	W8X18	W8X28	W8X21	W12X22
12	W16X36	W16X31	W16X36	W18X40
13	W18X46	W21X44	W18X40	W18X40
14	W12X22	W10X19	W14X22	W10X19
15	W18X35	W18X35	W18X40	W16X36
16	W14X26	W12X26	W14X22	W14X26
17	W12X30	W16X31	W16X31	W14X26
18	W14X22	W10X15	W10X17	W10X19
19	W8X28	W14X22	W14X26	W10X26
20	W6X25	W8X28	W8X21	W10X19
Weight (kN)	799.43	760.64	753.88	742.06

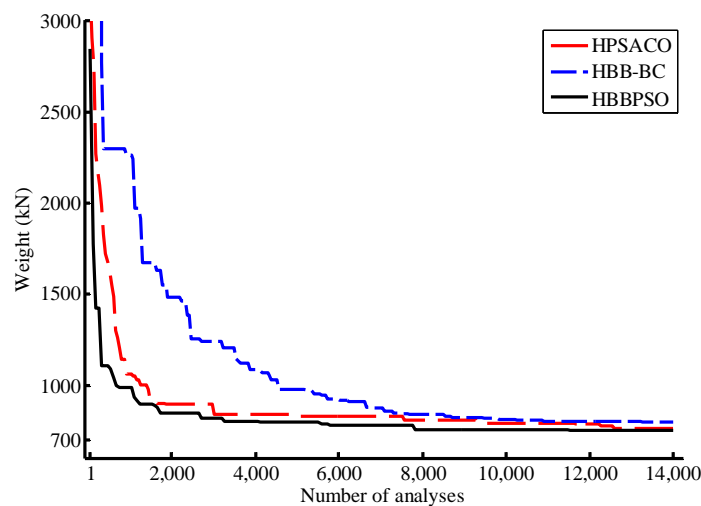


Figure 2. Comparison of the convergence history for the 10-story space frame

6. CONCLUSION REMARKS

The Heuristic Big bang–Big crunch Particle Swarm Optimization (HBBPSO) as a new hybrid meta-heuristic is developed for optimum design of frame structures. This new algorithm contains three steps:

- Global searching step where a hybrid PSO and BB–BC is utilized and the effect of all agents on the velocities of other particles is considered by using the center of mass, the main point obtained in big bang level of the BB–BC.
- Local searching step which is similar to the ACO level of the HPSACO while instead of using only the global best point to generate new solutions, the local best and the center of mass points are also utilized.
- Location controlling step where a modified harmony search algorithm is used to correct the location of particles flies to out of search space.

Compared to the HPSACO, this new algorithm does not require addition computational efforts. The extra computation corresponds to the center of mass point which must be calculated only one time for each iteration. Since for performing the global searching step calculating the global best, local best and center of mass points is necessary, in the next step (local searching step) this information can be utilized easily without additional calculation. In addition for the location controlling step, the local best points are utilized as the harmony memory and therefore this step can be utilized easily.

To investigate the efficiency of the new algorithm, two frame structures are selected. The first example is a benchmark planar frame solved by different meta-heuristic algorithm. Compared to these algorithms, the HBBPSO method can find better result and also it needs less number of iteration to convergence. For the second example, the HPSACO, the HBB–BC, the CSS in addition to HBBPSO are utilized to solve this problem. The HBBPSO can

reach a design which is 2.5% and 7.7% lighter than the results of the HPSACO and HBB-BC.

REFERENCES

1. Kaveh A, Talatahari S. Particle swarm optimizer, ant colony strategy and harmony search scheme hybridized for optimization of truss structures, *Computers and Structures*, Nos. 5-6, **87**(2009) 267–83.
2. Kaveh A, Talatahari S. Size optimization of space trusses using Big Bang–Big Crunch algorithm, *Computers and Structures*, Nos. 17-18, **87**(2009) 1129–40.
3. Smith S. The simplex method and evolutionary algorithms, *Proceedings of the IEEE International Conference on Evolutionary Computation*, (1998), 799–804.
4. Angeline P. Evolutionary optimization versus particle swarm optimization: philosophy and performance difference, *Proceeding of the Evolutionary Programming Conference*, San Diego, USA, 1998.
5. American Institute of Steel Construction (AISC), Manual of steel construction- Load resistance factor design, *3rd ed. Chicago, AISC*, 2001.
6. Kaveh A, Talatahari S. An improved ant colony optimization for design of planar steel frames, *Engineering Structures*, No. 3, **32**(2010) 864-73.
7. Erol OK, Eksin I. New optimization method: Big Bang–Big Crunch, *Advances in Engineering Software*, **37**(2006) 106–11.
8. Kaveh A, Talatahari S. A novel heuristic optimization method: charged system search, *Acta Mechanica*, Nos. 3-4, **213**(2010) 267–89.
9. Kaveh A, Talatahari S. Optimal design of skeletal structures via the charged system search algorithm, *Structural and Multidisciplinary Optimization*, No. 6, **41**(2010) 893-911.
10. Mahdavi M, Fesanghary M, Damangir E. An improved harmony search algorithm for solving optimization problems, *Applied Mathematics and Computation*, **188**(2007), 1567–79.
11. Davison JH, Adams PF. Stability of braced and unbraced frames, *Journal of the Structural Division, ASCE*, No. 2, **100**(1974) 319–34.
12. Camp CV, Bichon J. Design of steel frames using ant colony optimization, *Journal of Structural Engineering, ASCE*, **131**(2005) 369–79.
13. Degertekin SO. Optimum design of steel frames using harmony search algorithm, *Structural and Multidisciplinary Optimization*, **36**(2008) 393–401.
14. Kaveh A, Talatahari S. A discrete Big Bang–Big Crunch algorithm for optimal design of skeletal structures, *Asian Journal of Civil Engineering*, No. 1, **11**(2010) 103–22.
15. Kaveh A, Talatahari S. Optimum design of skeletal structures using imperialist competitive algorithm, *Computers and Structures*, **88**(2010) 1220–29.
16. Kaveh A, Talatahari S. Charged system search for optimal design of frame structures, *Applied Soft Computing*, No. 1, **12**(2012) 382-93.