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Technical Note

IMPROVED METHOD OF ESTIMATING DEFLECTION IN PRESTRESSED STEEL I-BEAMS

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ABSTRACT

Prestressing steel has been popular in the recent past, due to the developments in the field of anti-corrosive coatings. The literature substantiates the application of technique of prestressing to steel structures both in safety and economy point of view. However, for all design calculations, the maximum allowable span for a given load carrying capacity is based on maximum deflection which is calculated by principle of superposition (considering the effect of prestress and total load individually). This paper proposes a method of arriving at expression for deflection of simply supported, prestressed homogenous steel I-beams calculated by considering the combined effect of prestressing and total load. A straight tendon configuration with an eccentric prestressing force is considered for study.

Keywords: Deflection; prestressed steel; superposition; pure bending; load carrying capacity; simply supported; homogenous.

1. INTRODUCTION

1.1 General

The necessity for economy in steel in view of large quantities needed for construction and rehabilitation of various steel structures prompted the requirement of saving steel. This is possible by using more reinforced concrete and prestressed concrete than steel. But for long spans where prestressed concrete is not viable, prestressed steel is the ideal solution. The concept of prestressing steel is not a new frontier in the field of Civil Engineering. But it is being widely considered only in the recent past, despite a long and successful history of

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prestressing concrete members thanks to the advancements made in the field of anti corrosive coatings.

1.2 Basic concept of prestressing

The term 'prestressed steel' means application of a pre-determined concentric or eccentric force to a steel member so that the state of stress in the member resulting from this force and from any other anticipated external loading will be restricted to certain specified limit. This is done by inducing opposite stresses in a structure before it is put to its actual use by application of external forces. These forces are controlled in magnitude and direction to counter act the developed stresses in beam due to working loads.

It is very difficult to establish a bond between the prestressing tendon and a steel beam. Hence external prestressing technique is adopted for prestressing of steel beams.

1.3 Advantages of prestressed steel beams

Lighter and slender members are possible by use of prestressed steel when compared with un-prestressed steel beams. The whole cross-section is effective in prestressed steel beams. Prestressing of steel beams enhances its elastic range of the material and makes it more ductile when compared with un-prestressed steel beams. The economic advantage of any prestressed steel structure when compared with the corresponding un-prestressed steel structure is about 15% taking into account the cost of cables and the technology of prestressing. Prestressing of steel beams improves fatigue life of a structure and hence the technique is recommended for dynamically loaded structures. The external prestressing technique can be applied to improve the load carrying capacity of existing un-prestressed steel beams which is not always possible in Reinforced Cement Concrete beams. We can temporarily improve the load carrying capacity of a beam steadily by stage-by-stage prestressing to acquire the required load carrying capacity. Prestressed steel beams are ideally suitable against prestressed concrete beams for temporary bridge construction because the former can be reused. Standard components required for prestressed steel beams can be manufactured in factories and hence a lot of time is saved. Prestressing remarkably reduces ultimate deflection and hence high span depth ratios can be achieved. Prestressing steel girders increase the level of the stress at which the beam starts to buckle. The total loss of tensile prestress in cables is small in case of prestressed steel structures when compared with prestressed concrete structures. Tendons of External prestressed beams can easily be designed to be replaceable and restressable without major cost implications. Generally the webs can be made thinner resulting in a comparatively lighter structure thereby facilitating execution.

1.4 Disadvantages of prestressed steel beams

Longer spans are not preferred to avoid possible bulking. Externally prestressed members can easily be damaged with minor equipment. For certain cross-sections and construction procedures, the handling of the tensioning devices is difficult. The availability of builders and engineers experienced with the technique of external prestressing of steel is scanty. Initial equipment cost is very high. Prestressing tendons are brittle as high strength steel is used. The tendons are more exposed to environmental influences, fire, aggressive chemicals etc. The deviators and anchor plates which are delicate components, have to be placed very

accurately. As the tendons are not bonded to the member (or bonded only at particular points) the ultimate strength is developed in the ultimate design resulting in a higher prestressing steel consumption. Usually cross-section cannot be fully utilised. Therefore a greater depth or additional prestressing is required.

1.5 Applications of technique of prestressing steel

Depending on the type of structure and its future working conditions, prestress may be applied during the erection or at the manufacturing plant. Prestressing can be applied in single or multiple stages. In utilising the material, the greater effect may be obtained in multistep prestressing. This prestressing is possible only if the load is constant. The sequence of steps used to create prestressing depends on the type of structure or loading and is subject to its effect on the economies being sought from prestressing. Prestressing is used in the design of new structures as well as for the existing ones. Reinforcing of an existing structure by prestressing results in an increase in its load carrying capacity and stiffness with minimal consumption of additional material. Considering its structural use, prestressing has been successfully applied in the design of new structures such as girders, frames, arches, trusses, buildings, towers, masts and bridges as well as to strengthen old bridges.

Some of the typical applications of technique of prestressed steel where external tendons are feasible, practical and economical are as follows.

1. Trusses: Prestressed trusses are used in industrial buildings and in the roofs of the boilers.
2. Bridges: Many large metal bridges have been built recently with the application of prestressing (bridge girders prestressed by tendons)
3. Sheet Structures and Wall Structures: These are prestressed so that they can take the compressive load effectively.
4. Masts and Towers: Prestressed steel is used in Masts and Towers to increase the rigidity of the structure.

2. LITERATURE REVIEW

Coff [2] in the United States proposed a 250 foot span prestressed steel plate girder bridge**.** Finn and Needham [4] performed an extensive testing program for a 90-foot span prestressed steel truss. Petrov [9, 10] examined the parameters of prestressed steel beams for designing with cables all along the beam length. Ferjencik [3] and Tochacek and Amrhein [14] described progress in prestressed steel design in Czechoslovakia. Research began in 1960 and actual design specifications were adopted as a result of that research. Ferjencik developed a catalog of applications of prestressing including applying it to girders and trusses. Tochacek and Mehta [15] pointed out that the safety factor for the portions of prestressed steel structures subjected to a range of both tension and compression can be reduced by 20 % under a working stress design. Belenya [1] conducted tests on prestressed homogenous beam. Kalburgi [6] developed the equations for finding the optimum prestressed steel I-section in non-dimensional parameters. Troitsky et al. [8] made a study on prestressed – steel continuous – span girders. Russel and Syder [12] made a study on prestressed steel girders for single span bridges. Raman Singh et al. [11] made a case study

on prestressed steel bridge. He suggested measures that can be taken in case of steel bridges for strengthening compression member. Guptha [5] made a study on prestressed steel structures. Ravindra et al. [13] developed a procedure for Computerised Design of Prestressed steel beams.

The concept of prestressing steel beams is being adopted in the design and analysis of different bridges all over the world since long and more significantly in the recent past, there are many assumptions being considered for the same. It can be observed from the study of literature that the maximum allowable span for a given load carrying capacity is based on the maximum deflection, which is calculated by the principle of superposition (considering the effect of prestress and total load individually). However, it will be different from the deflection calculated by considering the combined effect of prestressing and total load. This aspect affects the economy of the structure and compels the design engineer to adopt a higher factor of safety. Hence, the objective of this work is to formulate an expression for evaluating deflection by considering the combined effect of prestressing and total load on the deflection and compare it with that by considering the effect of prestress and total load individually.

3. ESTIMATION OF DEFLECTION IN PRESTRESSED BEAMS

3.1 Importance of control of Deflection

Deflection forms an important criterion for safety of any structure. Suitable control on deflection is very essential for the following reasons.

1. Excessive sagging of any structural members is unsightly and may sometimes render the floor unsuitable for use by causing damage to finishes, partitions and associated structures.
2. Large deflections under dynamic effects and under the influence of variable loads may cause discomfort to users.

3.2 Effect of Tendon Profile on Deflection

In the most of the cases of prestressed beams and tendons are located with eccentricities towards the soffit of beams to counteract the sagging bending moments due to transverse loads. Consequently the prestressed beams deflect upwards (camber) on the application of prestress. Since the bending moment is the product of the prestressing force and eccentricity, the tendon profile itself will represent the shape of the bending moment diagram.

3.3 Expressions for Estimation of Deflection

The method of computing deflections of simply supported eccentrically prestressed beam with straight cable in two different approaches will be discussed in the following sections

1. Combined Effect of Prestress and External Load on Deflection

Consider a simply supported beam of span ‘l’, Flexural rigidity EI subjected to a total uniformly distributed load of w/unit length and prestressed by a force ‘P’ at an eccentricity ‘e’. Let the deflection of the beam at a distance ‘x’ from left support be ‘y’. (Figure 1) Then bending moment at any section at a distance of ‘x’ from left support is given by

$$M_x = \left(\frac{w \times l \times x}{2}\right) - \left(\frac{w \times x^2}{2}\right) - P(e - y) \tag{1}$$

Hence from the theory of pure bending

$$E \times I_x \times \frac{d^2 y}{dx^2} = -\left(\frac{w \times l \times x}{2}\right) + \left(\frac{w \times x^2}{2}\right) + P(e - y) \tag{2}$$

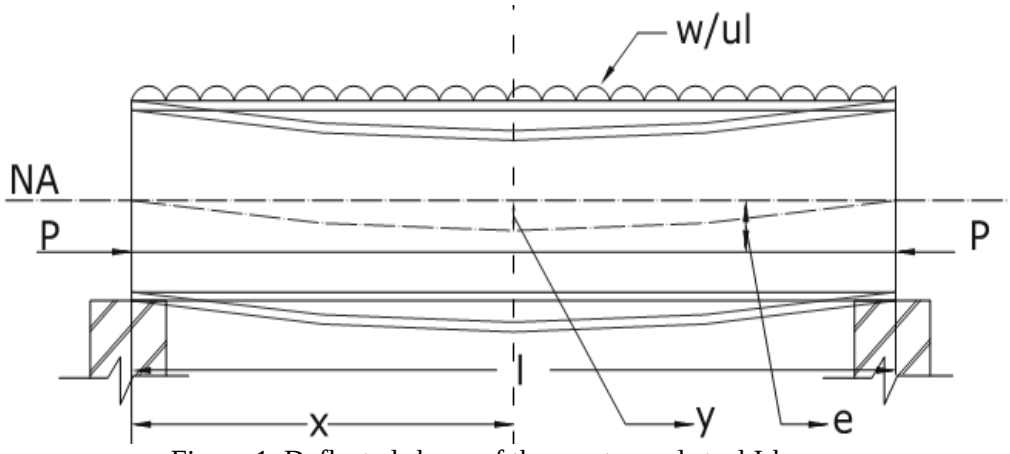

Figure 1. Deflected shape of the prestressed steel I-beam

Solving Eq. (2),

$$y = \left(\frac{w \times E \times I_x}{P^2} - e\right) \times \left\{\frac{Cos\left(\sqrt{\frac{P}{E \times I_x}} \times \left(x - \frac{l}{2}\right)\right)}{Cos\left(\sqrt{\frac{P}{E \times I_x}} \times \frac{l}{2}\right)} - 1\right\} + \left(\frac{w \times x}{2 \times P}\right) \times (x - l) \tag{3}$$

This is the generalised expression for finding the deflection of a simply supported eccentrically prestressed beam considering the combined effect of prestress and total load.

Maximum deflection by symmetry occurs at x = l/2.

Hence,

$$y_{\max} = \left(\frac{w \times E \times I_x}{P^2} - e\right) \times \left\{Sec\left(\sqrt{\frac{P}{E \times I_x}} \times \frac{l}{2}\right) - 1\right\} - \left(\frac{w \times l^2}{8 \times P}\right) \tag{4}$$

2. Deflection by Principle of Superposition

(By considering the effect of prestress and total load individually)

a. Upward deflection due to prestressing force.

Consider a simply supported beam of span 'l', prestressed by a force 'P', at an eccentricity 'e'. Then bending moment at any section on the beam is

$M_x = - P \times e$

Hence from the theory of pure bending

$$E \times I_x \times \frac{d^2 y}{dx^2} = (P \times e) \tag{5}$$

Solving Eq. (5),

$$y = \left(\frac{P \times e \times x^2}{2 \times E \times I_x}\right) - \left(\frac{P \times e \times l \times x}{2 \times E \times I_x}\right) \tag{6}$$

This is the expression to calculate the upward deflection i.e., camber due to the eccentric prestressing force alone.

b. Downward deflection due to self-weight and live load.

Consider a simply supported beam of span 'l', Flexural rigidity EI, which is subjected to a total uniformly distributed load of w/unit length. Then bending moment at any section, which is at a distance 'x' from the left end is given by

$$M_x = \left(\frac{w \times l \times x}{2}\right) - \left(\frac{w \times x^2}{2}\right) \tag{7}$$

Solving

$$y = -\left(\frac{w \times l \times x^3}{12 \times E \times I_x}\right) + \left(\frac{w \times x^4}{24 \times E \times I_x}\right) + \left(\frac{w \times l^3 \times x}{24 \times E \times I_x}\right) \tag{8}$$

This is the deflection equation of a simply supported beam subjected to uniformly distributed load alone. The generalized expression for net deflection due to prestressing force and self-weight of the beam is obtained by algebraic sum of Equations (17) and (24).

$$y = -\left(\frac{w \times l \times x^3}{12 \times E \times I_x}\right) + \left(\frac{w \times x^4}{24 \times E \times I_x}\right) + \left(\frac{w \times l^3 \times x}{24 \times E \times I_x}\right) + \left(\frac{P \times e \times x^2}{2 \times E \times I_x}\right) - \left(\frac{P \times e \times l \times x}{2 \times E \times I_x}\right) \tag{9}$$

By symmetry, the maximum deflection by symmetry occurs at $x = \left(\frac{l}{2}\right)$.

$$\therefore y_{\max} = \left(\frac{5 \times w \times l^4}{384 \times E \times I_x}\right) - \left(\frac{P \times e \times l^2}{8 \times E \times I_x}\right) \tag{10}$$

4. CONCLUSION

The following conclusions can be drawn from this study.

1. For a simply supported prestressed steel I-beam, the generalised expression for deflection of beam by considering the combined effect of prestress and total load is given by

$$y = \left(\frac{(Lcc + dl) \times E \times I_x}{P^2} - e \right) \times \left\{ \frac{Cos\left(\sqrt{\frac{P}{E \times I_x}} \times \left(x - \frac{l}{2} \right) \right)}{Cos\left(\sqrt{\frac{P}{E \times I_x}} \times \frac{l}{2} \right)} - 1 \right\} + \left(\frac{(Lcc + dl) \times x}{2 \times P} \right) \times (x - l)$$

3. Maximum deflection by symmetry occurs at center of the span is given by

$$y_{\max} = \left(\frac{(Lcc + dl) \times E \times I_x}{P^2} - e \right) \times \left\{ Sec\left(\sqrt{\frac{P}{E \times I_x}} \times \frac{l}{2} \right) - 1 \right\} - \left(\frac{(Lcc + dl) \times l^2}{8 \times P} \right)$$

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