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SEISMIC SOIL-STRUCTURE INTERACTION ANALYSIS USING AN EFFECTIVE SCALED BOUNDARY SPECTRAL ELEMENT APPROACH

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ABSTRACT

In this paper, an effective scaled boundary spectral element method is used to analyze seismic soil-structure interaction (SSSI) problems. Coefficient matrices are lumped by using Gauss-Lobatto-Legendre (GLL) quadrature and Lagrange interpolation functions. Required storage space of the computers can be reduced by using lumped coefficient matrices. In addition, a recursive algorithm is adapted to reduce computational effort of the seismic soil-structure interaction analysis. Adapted recursive algorithm can reduce computational effort of the original scaled boundary method (SBM) about 90%. Efficiency of the proposed method is displayed by solving some numerical examples. It is shown that accuracy of the semi local SBM depends on the selected cut off time step.

Keywords: Soil-structure interaction; spectral element; scaled boundary spectral element; recursive algorithm; acceleration unit impulse response matrix.

1. INTRODUCTION

Interacting between soil and structures has important effect on the response of the structural systems especially for seismic loading conditions. This effect magnifies for massive and stiff structures. For example, effects of the soil structure interaction on the structures such as gravity dams, power plants and bridges cannot be neglected. There are two main different methods to analyze soil structure interaction problems, direct method and sub-structure method [1]. In the direct method, soil and structure must be modeled in a single step and radiation damping condition of the unbounded soil media is enforced by using artificial boundary conditions [2]. Artificial boundary conditions are local in both time and space. This means that calculating of the artificial boundary condition for each degree of freedom is

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independent from other degree of freedoms. Also, artificial boundary conditions can be calculated in each time step without needing to previous time steps. Although the local methods are computationally efficient to use and simple to interpret, they cannot model radiation damping condition completely. In direct method, soil media must be modeled large enough.

Sub-structure method needs a global procedure to model radiation damping condition. These global methods are computationally expensive in comparison with local approaches [2]. Global approaches can able to model unbounded soil medium rigorously. In the substructure method, domain of the problem must be divided to two different sub-domains. The first sub-domain is named near field. Near field contains both structures (super-structures) and irregular surrounding soil media. Conventional finite element method (FEM) can be used to model near field. Finite element method can model complex geometries and nonlinear material behavior. FEM is an integration method based on the Rayleigh-Ritz or weighted residual approaches and can be used to solve governed partial differential equations. FEM cannot model unbounded mediums [3].

Second sub-domain is named far field. Far field contains unbounded soil media and it must enforce radiation damping condition on the near field. Domain discretization scheme of the sub-structure method is shown in Figure 1.

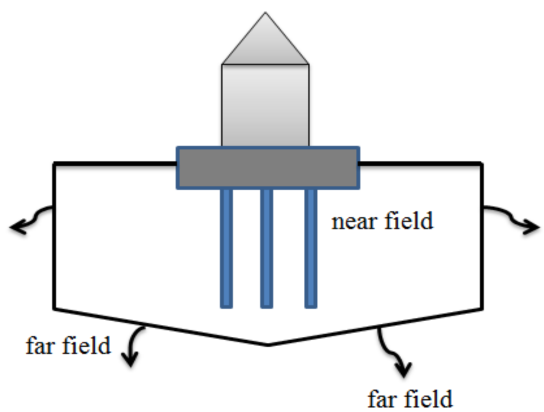


Figure 1. Domain discretization scheme of the sub-structure method.

Boundary element method (BEM) is another integration method. BEM can able to model bounded and unbounded mediums accurately. BEM is a strong numerical method with rigorous mathematical bases. Boundary element method has global formula therefore; it is more computationally expensive than local approaches. Unfortunately, BEM needs a fundamental solution to solve numerical problems. Getting these fundamental solutions is not a simple task and needs to a large computational effort. In addition, this powerful numerical method may be faced singular integrals. These singular integrals can enforce extra computational effort to the original BEM.

Based on the sub-structure procedure, coupled methods, e.g. FE-BE method, can be used to analyze soil structure interaction problems. In the coupled methods, a global method must be used to model unbounded soil media and a domain discretization method such as the finite element must be used to model near field soil media. Far field and near field are connected by a soil-structure interface boundary. Reactions of the unbounded soil media on the bounded near field of the system can be represented by a force-displacement (velocity or acceleration) relationship, which is global in both space and time [4].

Scaled boundary finite element method (SBFEM) is a novel semi analytical method that can be used to model bounded and unbounded domains. SBFEM reduces dimension of geometry by one [5]. Like FEM and BEM, SBFEM is a weighted residual based integration method. There are some different ways to construct scaled boundary methods. For example if test function and trial function are chosen same, as Galerkin weighted residual approach, scaled boundary finite element method is created. For another way, test function and trial function can be selected differently (for example in Petrov-Galerkin method) [6]. Point based scaled boundary finite element methods can be constructed if moving least square bases are used instead of conventional Lagrangian test or trial functions [7]. Iso-geometric scaled boundary finite element method can be developed by using b-spline curves as shape functions [8]. Spectral element approach is another way to improve scaled boundary method. Instead of conventional Gauss-Legendre (GL) quadrature, which is used in conventional finite element method, Gauss-Lobatto-Legendre (GLL) quadrature can be used to construct spectral elements. In spectral element method (SEM) quadrature points and element (field) nodes are coincided together [3]. By using this feature, some of the coefficient matrices can be lumped (e.g. mass matrix of bounded media in the finite element method). Lumped coefficient matrices can reduce required storage space in computers.

In this paper, the coupled GLL based spectral element method and scaled boundary spectral element method is used to model seismic soil structure interaction problems. As mentioned before, reaction of the far field on the near field can be represented in the form of a force-displacement (velocity or acceleration) relationship. Equations 1, 2 and 3 show these relationships. For the time domain dynamic problems, these relationships are in the form of convolution integrals.

$$r(t) = \int S^{\infty}(t-\tau)u(\tau)d\tau \tag{1}$$

$$r(t) = \int V^{\infty}(t-\tau)v(\tau)d\tau \tag{2}$$

$$r(t) = \int M^{\infty}(t-\tau)a(\tau)d\tau \tag{3}$$

In Eq. (1-3), $S^{\infty}(t)$, $V^{\infty}(t)$ and $M^{\infty}(t)$ are displacement, velocity and acceleration unit impulse response matrices, respectively and u, $\dot{u}$, $\ddot{u}$ are displacement, velocity and acceleration vectors of the near field and the far field interface boundary, respectively. A recursive algorithm was proposed to calculate this interaction force by Mohaseb [9]. Although this algorithm can reduce computational effort, it remains expensive. Mohaseb's algorithm (Eqs. (4, 5)) uses inverse Fourier transforms to link frequency and time domain formulas together.

$$\{r_b\}_n = \frac{1}{T}\sum_{j=-\infty}^{+\infty} S^{\infty}(\omega_j)\left(\int_0^t \{u_b(\tau)\}\exp(-i\omega_j\tau)d\tau\right)\exp(i\omega_j t) \tag{4}$$

$$\int_0^{t=n\Delta t} \{u_b(\tau)\}\exp(-i\omega_j\tau)d\tau = \int_0^{(n-1)\Delta t} \{u_b(\tau)\}\exp(-i\omega_j\tau)d\tau + \int_{(n-1)\Delta t}^{n\Delta t} \{u_b(\tau)\}\exp(-i\omega_j\tau)d\tau \tag{5}$$

Another recursive algorithm was proposed by Zhang [10] based on the linear and nonlinear behaviour of the acceleration unit impulse response matrix. This algorithm is completely in time domain. Lehmann [11] presented another recursive algorithm based on the Zhang's works in 2005. This new recursive algorithm is easy to implement and is fast to calculate interaction force. Lehmann's recursive algorithm reduces computational effort of the original SBM significantly. Unfortunately, Lehmann's recursive algorithm is formulated just for dynamic loading conditions. In this paper, Lehmann's recursive algorithm is adapted for seismic loading conditions and it is used in the scaled boundary spectral element method to calculate seismic interaction forces.

The outline of the paper is as follows: summary of the scaled boundary finite element method is presented in section 2. Spectral element approach is explained in section 3. Adapted recursive algorithm for the seismic soil structure interaction problems is presented in section 4. Benchmark examples are solved in section 5. Final section of the paper, section 6, is dedicated to present conclusions and summary.

2. SUMMARY OF THE SCALED BOUNDARY FINITE ELEMENT METHOD

Scaled boundary finite element method is a relatively novel, semi analytical approach, which can able to model bounded and unbounded domains accurately. SBFEM is an integration method like FEM and BEM. This novel semi analytical method reduces governed partial differential equations to sets of the ordinary differential equations. Scaled boundary finite element has four coefficient matrices (E^0, E^1, E^2, and M^0). E^0 is a positive definite symmetric matrix and just contains shape functions. This matrix can be calculated by using Eq. (6). E^1 contains both shape functions and their derivatives and can be calculated by using Eq. (7). E^2 is a non-symmetrical matrix and contains just derivatives of shape functions. This matrix can be calculated by using Eq. (8). M^0 is mass matrix of the unbounded media. As the mass matrix of the bounded mediums (in the finite element method) M^0 is symmetric and contains just the shape functions. This matrix can be calculated by using Eq. (9).

$$[E^0] = \int_{-1}^{+1} [B^1][D][B^1]\,|J|\,d\eta \tag{6}$$

$$[E^1] = \int_{-1}^{+1} [B^2][D][B^1]\,|J|\,d\eta \tag{7}$$

$$[E^2] = \int_{-1}^{+1} [B^2][D][B^2]\,|J|\,d\eta \tag{8}$$

$$[M^0] = \int_{-1}^{+1} [N]^T[\rho][N]\,|J|\,d\eta \tag{9}$$

In above equations, $[D]$ and $[\rho]$ are elasticity and density matrices respectively. $[B^1]$ contains shape functions and $[B^2]$ contains derivatives of the shape functions. For dynamic loading cases, equation of motion can be written as Eq. (10). In equation of motion, $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices respectively.

$$\begin{bmatrix} k_{ii} & k_{i\Gamma} \\ k_{\Gamma i} & k_{\Gamma\Gamma} \end{bmatrix}\begin{bmatrix} u_i(t) \\ u_\Gamma(t) \end{bmatrix}+\begin{bmatrix} C_{ii} & C_{i\Gamma} \\ C_{\Gamma i} & C_{\Gamma\Gamma} \end{bmatrix}\begin{bmatrix} v_i(t) \\ v_\Gamma(t) \end{bmatrix}+\begin{bmatrix} M_{ii} & M_{i\Gamma} \\ M_{\Gamma i} & M_{\Gamma\Gamma}+\gamma\Delta t M_0^\infty \end{bmatrix}\begin{bmatrix} a_i(t) \\ a_\Gamma(t) \end{bmatrix}=\begin{bmatrix} p_i(t) \\ p_\Gamma(t)-r_\Gamma(t) \end{bmatrix} \quad (10)$$

In Eq. (10), $\{a\}$, $\{v\}$ and $\{u\}$ are the acceleration, velocity and displacement vectors respectively. The subscript Γ indicates degree of freedoms of the nodes on the near and far field interface. The subscript i means the degree of freedoms on the remained nodes of the structure. In this equation, $\{r_\Gamma\}$ is interaction force vector and can be calculated by using Eq. (1-3) for dynamic loading cases.

Acceleration unit impulse response must be calculated before solving equation of motion (Eq. (10)). $M^\infty(t)$ can be obtained by solving Eq. (11). In Eq. (11) $H(t)$ is the Heaviside step function (for t < 0, H (t) = 0 and for t≥ 0, H (t) = 1). This equation can be solved by using time discretization methods such as constant acceleration Newmark's method.

$$\begin{aligned}&\int_0^t [M^\infty(t-\tau)][E^0]^{-1}[M^\infty(\tau)]d\tau + t\int_0^t [M^\infty(\tau)]d\tau + ([E^1][E^0]^{-1} - 1.5[I])\int_0^t\int_0^\tau [M^\infty(\tau')]d\tau' d\tau + \\ &\int_0^t\int_0^\tau [M^\infty(\tau')]d\tau' d\tau([E^0]^{-1}[E^1]^T - 1.5[I]) - \frac{t^3}{6}([E^2]-[E^1][E^0]^{-1}[E^1]^T)H(t) - t[M^0]H(t) = 0\end{aligned} \quad (11)$$

For seismic loading cases, seismic interaction force must be applied as a function of the seismic acceleration $\{a_\Gamma^g(t)\}$. Therefore, Eq. (3) must be altered as:

$$r_\Gamma(t) = \int M^\infty(t-\tau)(a_\Gamma^t(\tau) - a_\Gamma^g(\tau))d\tau \quad (12)$$

Superscripts t and g in Eq. (12) denote total value and ground value of the acceleration vector. Instead of $\{a_\Gamma^g(t)\}$, one can use the free field component of the seismic induced acceleration ($\{a_\Gamma^f(t)\}$). In this paper, input acceleration time history is used as the ground component of the seismic induced acceleration ($\{a_\Gamma^g(t)\}$).

3. SCALED BOUNDARY SPECTRAL ELEMENT METHOD

To solve equation of motion in the coupled FE-SBFE method, seven coefficient matrices must be calculated. $[M]$, $[C]$ and $[K]$ matrices of the bounded domain should be calculated by using the finite element method and $[E^0]$, $[E^1]$, $[E^2]$ and $[M^0]$ should be obtained from SBFEM. As mentioned before, $[M]$, $[E^0]$ and $[M^0]$ involve only shape functions and other equations involve derivatives of the shape functions. If integration points of a quadrature rule are coincided with the nodal locations of an element, no off diagonal terms are generated [3]. In this paper, this property is employed to lump $[M]$, $[E^0]$ and $[M^0]$.When Lagrange polynomials are used to construct shape functions and Gauss-Lobatto-Legendre (GLL) quadrature is used to integrate coefficient matrices then shape functions can display Kronecker delta function property in GLL (integration) nodes. Kronecker delta function property is described in Eq. (13).

$$\begin{cases} [N_{ij}] = 0 \ \ if \ \ i \neq j \\ [N_{ij}] = 1 \ \ if \ \ i = j \end{cases} \tag{13}$$

GLL nodes are roots of the derivatives of the Legendre polynomials [13], and can be calculated by using newton's iteration method. For a 2D element with $(p+1)^2$ nodes, degree of the Lagrangian shape function is equal to p. A twenty five nodded 2D spectral element is plotted in Figure 2.a. This element is used to model bounded medium in this paper. For unbounded medium, five nodded scaled boundary spectral elements are used. This type of element is shown in Figure 2.b.

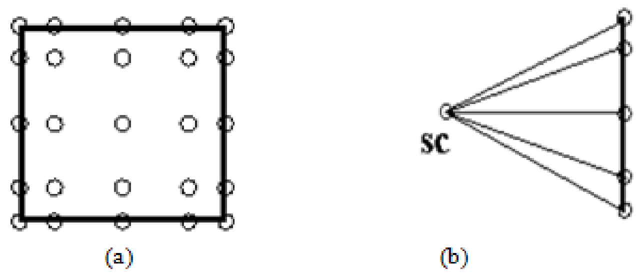


Figure 2. (a) A twenty five nodded 2D spectral element. (b) a five nodded scaled boundary spectral element.

Used 2D Lagrangian shape functions for both FEM and SEM are plotted in Figure 3. As this figure shows, both of the mentioned shape functions have Kronecker delta function property in the field nodes.

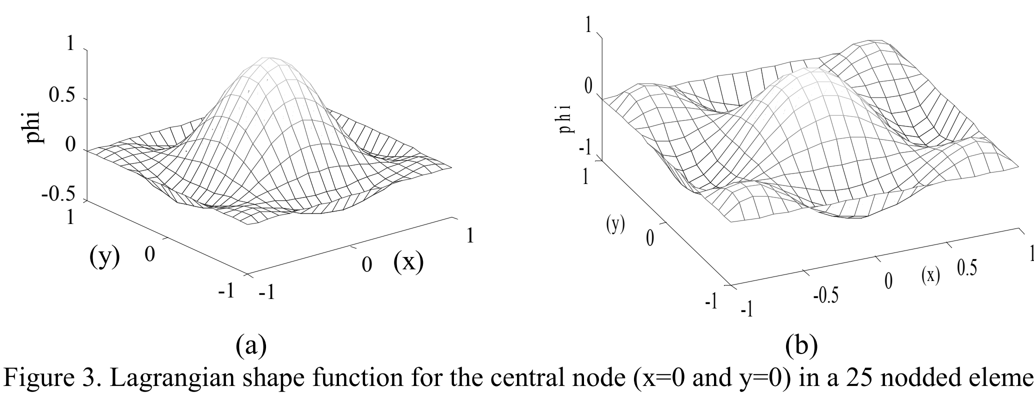


Figure 3. Lagrangian shape function for the central node (x=0 and y=0) in a 25 nodded element. (a) GLL nodes (b) with equally spaced nodes (Phi is the shape function)

Vu [12] and Bazyar [13] have used scaled boundary spectral element method with Lagrangian shape function and Gauss-Lobatto-Legendre quadrature for static and dynamic problems respectively. For dynamic problems, Bazyar solved some simple problems in frequency domain using the mentioned SEM.

Lumped coefficient matrices need small storage space in computers in comparison with full matrices. In addition, computational effort can be reduced to solve both Lyapunov system of equations and equation of motion by using lumped matrices. Diagonally lumped [E^0] and [M^0] matrices and blocky diagonal [E^1] matrix are shown in Figure 4.

(a) (b)

Figure 4. (a) Diagonally lumped form of [M^0] and [E^0] (b) Blocky diagonal lumped form of [E^2]

In addition to mentioned matrices, mass matrix of bounded media can be lumped by using spectral elements instead of conventional finite elements.

4. ADAPTED RECURSIVE ALGORITHM FOR SEISMIC CONDITIONS

An efficient, simple recursive algorithm which can reduce computational effort of the original SBFEM significantly, was proposed by Lehmann [11]. This algorithm can able to calculate soil-structure interaction force (Eq. (3)) efficiently. This algorithm is originally formulated for dynamic loading conditions. Lehmann's recursive algorithm is formulated based on the linear and nonlinear behavior of the acceleration unit impulse response matrix ([M^∞]) respect to time. This matrix can be discretised in respect to time as:

$$[M^\infty(t)] = \begin{cases} M_0^\infty & t \in [0, \Delta t] \\ M_1^\infty & t \in [\Delta t, 2\Delta t] \\ \vdots & \vdots \\ \vdots & \vdots \\ M_{n-1}^\infty & t \in [(n-1)\Delta t, n\Delta t] \end{cases} \tag{14}$$

For seismic loading cases, instead of Eq. (3), Eq. (12) must be used. Substituting Eq. (14) in Eq. (12), we will have:

$$r_n(t) = \sum_{j=1}^{n} M_{n-j+1}^\infty(t) \int_{t_{j-1}}^{t_j} a^t(\tau) - a^g(\tau) d\tau = \sum_{j=1}^{n} M_{n-j}^\infty \left(v_j - v_{j-1} - \frac{a_j^g + a_{j-1}^g}{2} \Delta t\right) \tag{15}$$

In Eq. (15), n is used to define current time step. This equation shows the global behavior of the acceleration-force relationship. To evaluate interacting force r in time t_n, all previous time steps are required. By using Newmark's implicit time integration scheme and separation the unknown acceleration vector a_n, interaction force can be formed as:

$$r_n = \gamma \Delta t M_0^\infty a_n + \sum_{j=1}^{n-1} M_{n-j+1}^\infty (v_j - v_{j-1} - \frac{a_j^g + a_{j-1}^g}{2}\Delta t) = \gamma \Delta t M_0^\infty a_n + \widetilde{r}_n \qquad (16)$$

In Eq. (16), $\widetilde{r_n}$ depends on the known velocities and accelerations at previous time steps ($t<t_n$) and is known at time station n. Diagonal entries of [M^∞ (t_1)], [M^∞ (t_2)],… behave linear from a certain time step (t_m). This linear behavior of the acceleration unit impulse response matrix is shown in Figure 5 respect to time.

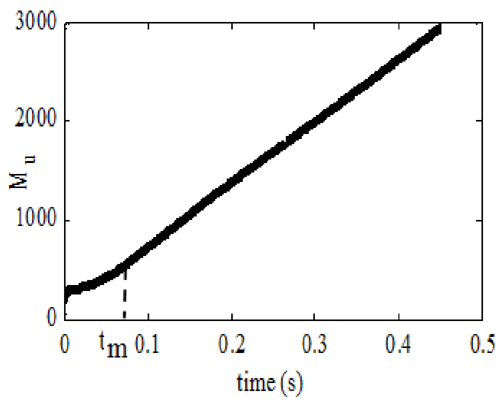


Figure 5. Behavior of a matrix entry on the main diagonal of [M^∞ (t) $=M_u$] respect to time

According to the linear behavior of acceleration unit impulse response matrix (from time step tm), this matrix can be formulated as a linear function of time. Eq. (17) shows this linear function.

$$M^\infty(t_i) = T^\infty t_i + C^\infty \qquad (17)$$

T^∞ is the slope of the linear part of M^∞ (t) and C^∞ is a constant matrix. Hence known interaction force term can be divided to two terms, linear and nonlinear. Eq. (18) shows known part of Eq. (16).

$$\widetilde{r} = \sum_{j=1}^{n+1-m} M_{n-j+1}^\infty (v_j - v_{j-1} - \frac{a_j^g + a_{j-1}^g}{2}\Delta t) + \sum_{j=n+2-m}^{n-1} M_{n-j+1}^\infty (v_j - v_{j-1} - \frac{a_j^g + a_{j-1}^g}{2}\Delta t) \qquad (18)$$

Linear part of Eq. (18) can be written as Eq. (20) by using Eq. (17, 19).

$$M_m^\infty = T_m + C^\infty \qquad (19)$$

$$r_n^{lin} = M_m^{\infty}(v_{n+1-m} - v_{n-m} - \frac{a_j^g + a_{j-1}^g}{2}\Delta t) + \sum_{j=1}^{n-m}(T^{\infty}(n-j+1) + C^{\infty})(v_j - v_{j-1} - \frac{a_j^g + a_{j-1}^g}{2}\Delta t) \tag{20}$$

By calculating differences between r_n^{lin} and r_{n-1}^{lin}, the recursive formula for linear part of the interaction force can be obtained. Eq. (21) shows this formula.

$$r_n^{lin} = r_{n-1}^{lin} + M_m^{\infty}({v^g}_{n+1-m} - {v^g}_{n-m} - \frac{a_{n+1-m}^g + a_{n+m}^g}{2}\Delta t) + T^{\infty}(v_1^g - v_0^g - \frac{(a_1^g + a_0^g)}{2}\Delta t) + T^{\infty}(v_2^g - v_1^g - \frac{(a_2^g + a_1^g)}{2}\Delta t) + ... \tag{21}$$

Eq. (21) can be written in the form of Eq. (22). This equation presents finial form of the adapted Lehmann's recursive algorithm for the linear part of the seismic interaction force.

$$r_n^{lin} = r_{n-1}^{lin} + M_m^{\infty}({v^g}_{n+1-m} - {v^g}_{n-m} - \frac{a_{n+1-m}^g + a_{n+m}^g}{2}\Delta t) + T^{\infty}(v_{n-m}^g - v_0^g) - T^{\infty}\sum_{j=1}^{n-m}\frac{a_j^g + a_{j-1}^g}{2}\Delta t \tag{22}$$

Relationship of the computational effort, which was proposed by Lehmann is still valid. Accuracy of the Lehmann's algorithm depends on the chosen cut off time (t_m). By increasing this time, accuracy of the answers gets better. Cut off times greater than the optimal cut off time have not significant effect on the accuracy.

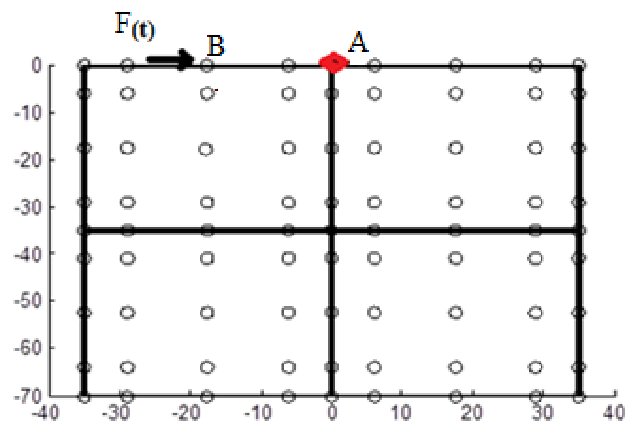
5. NUMERICAL EXAMPLES

5.1 Response of an elastic half space

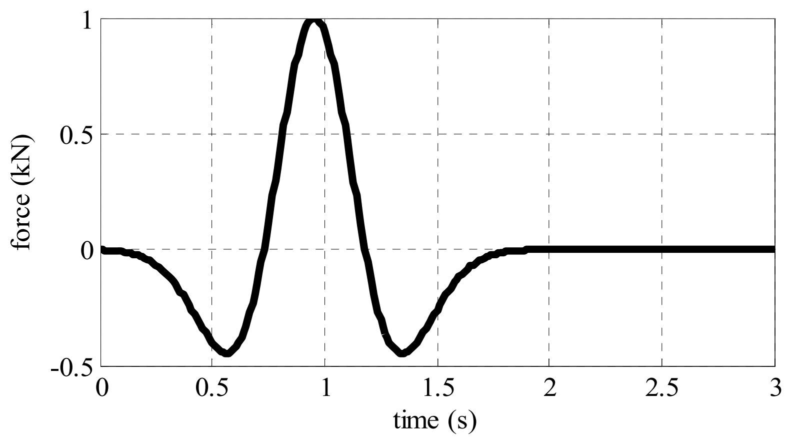
To ensure that the written code (in FORTRAN 90) for L. SE-SBSE (Lagrangian spectral element-scaled boundary spectral element) method has accurate answers, a simple example is solved first. This example has been solved by Estorff [14, 15]. Estorff used the coupled FE-BE method to solve it. In this example, elastic half space has a Young's modulus E=2.66×10^5 kN/m^2, Poisson's ratio ν =0.33 and a mass density ρ=2×10^3 Kg/m^3. As mentioned before, twenty five nodded spectral elements are used to discretize bounded media and five nodded scaled boundary spectral elements are used to discretize near field and far field interface boundary (Figure 6-a). A Ricker wavelet type horizontal dynamic load is applied to the point B. Ricker wavelet function can be obtained by using Eq. (23) [16].

$$\tau(t) = A_R (1 - 2(\frac{t - t_S}{t_0})^2) \exp(-(\frac{t - t_S}{t_0})^2) \tag{23}$$

In this example, A_R=1, t_0=1/π and t_s=3/π are selected. Figure 6-b shows this dynamic load function respect to time.



(a)



(b)

Figure 6. (a) SE-SBSE discretization of the considered system (b) Ricker wavelet type dynamic load function F(t).

As Figure 6.a shows, four spectral elements are used to discretize bounded media where six scaled boundary spectral elements are used to discretize near field-far field interface boundary. In this analysis time step is chosen equal to *dt*=.002*s*. Figure 7 shows obtained displacement time history of point A. As this figure shows, excellent agreement is obtained between used coupled spectral element-scaled boundary spectral element method and the coupled finite element-boundary element method. By solving this example, it is shown that written code leads to accurate coefficient matrices.

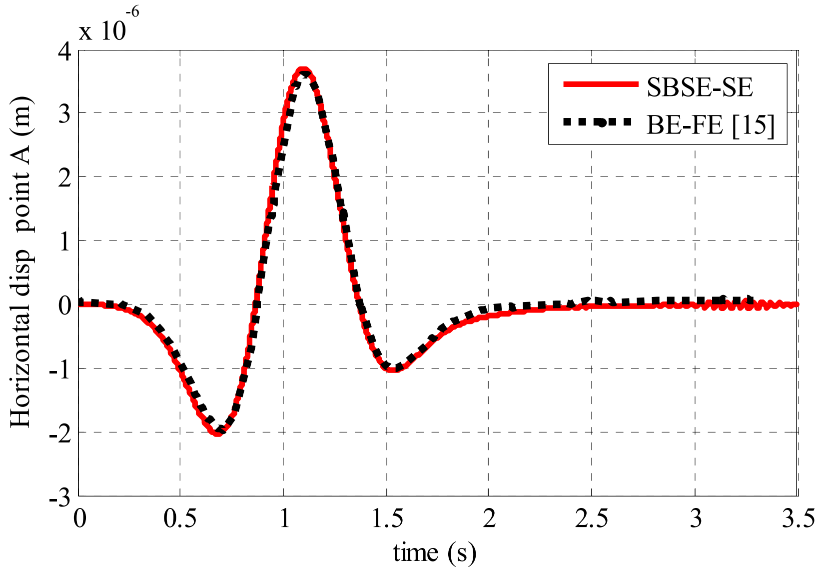


Figure 7. Displacement time history of point A caused by a horizontal dynamic load

5.2 Seismic soil structure interaction analysis

A frame-like structure on the elastic half space (as shown in Figure 8) is selected to show accuracy and efficiency of the adapted recursive algorithm. This example is solved originally by Bazyar [17] with the global SBFE method. Material properties of the super-structure are defined by the modulus of elasticity E=10^4 kN/m^2, Poisson's ratio v=.2 and mass density ρ=1Kg/m^3. Material properties of the half space are defined by the modulus of elasticity E=2400 kN/m^2, Poisson’s ratio v=.2 and mass density ρ=1 Kg/m^3.

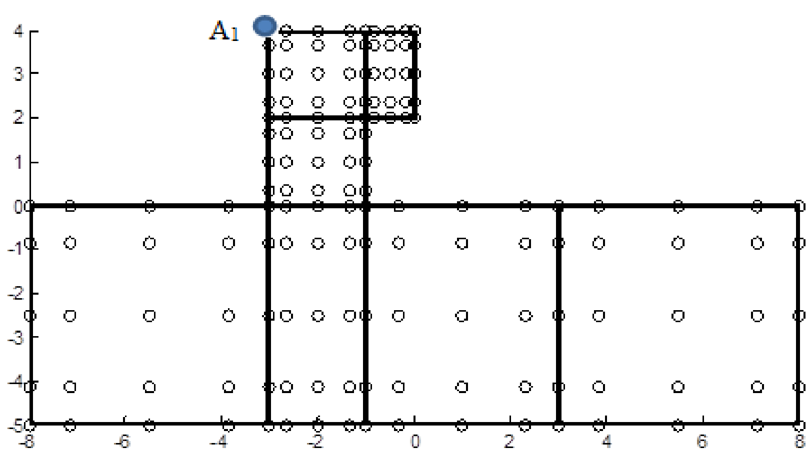


Figure 8. Used mesh for the frame-like structure on the flexible foundation

As shown in Figure 8, soil media is discretized by using four, twenty-five nodded spectral elements and far field-near field interface is discretized by using six scaled boundary spectral elements. Same as the reference paper ([17]), acceleration time history of the Tabas earthquake is selected as seismic input motion ($\{a_I^g\ (t)\}$). Used acceleration time history is shown in Figure 9.

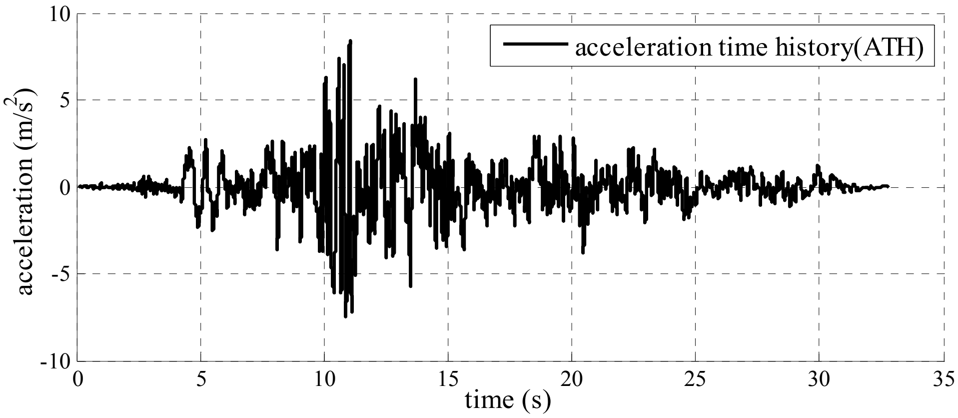


Figure 9. Acceleration time history of the Tabas earthquake

The horizontal displacement time history, which is caused by the seismic load at point A_1 is shown in Figure 10. As this figure shows excellent agreement is obtained between both of the global SBFE and the semi local SE-SBSE methods. In addition, this figure shows that radiation-damping condition is modeled accurately in the far and near field interface.

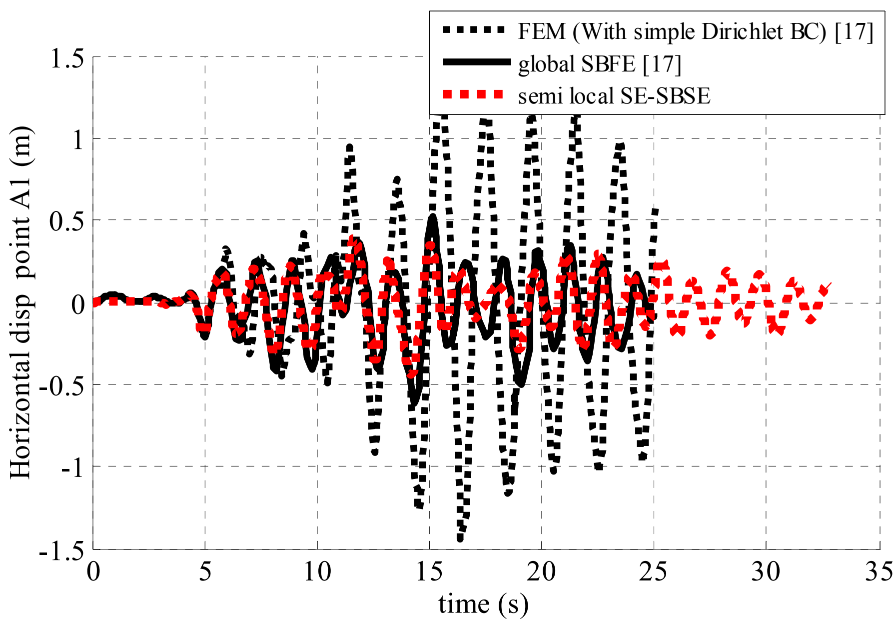


Figure 10. Horizontal displacement time history of point A_1

Cut off time step is selected equal to 700 (m_p =700). This cut of time step is sufficient to reduce computational effort and is sufficient to save accuracy of the obtained results. To show effects of the selected cut off time on the obtained answers, mentioned seismic soil structure interaction problem is solved again by using three different cut of time steps. Obtained displacement time histories are plotted in Figure 11.

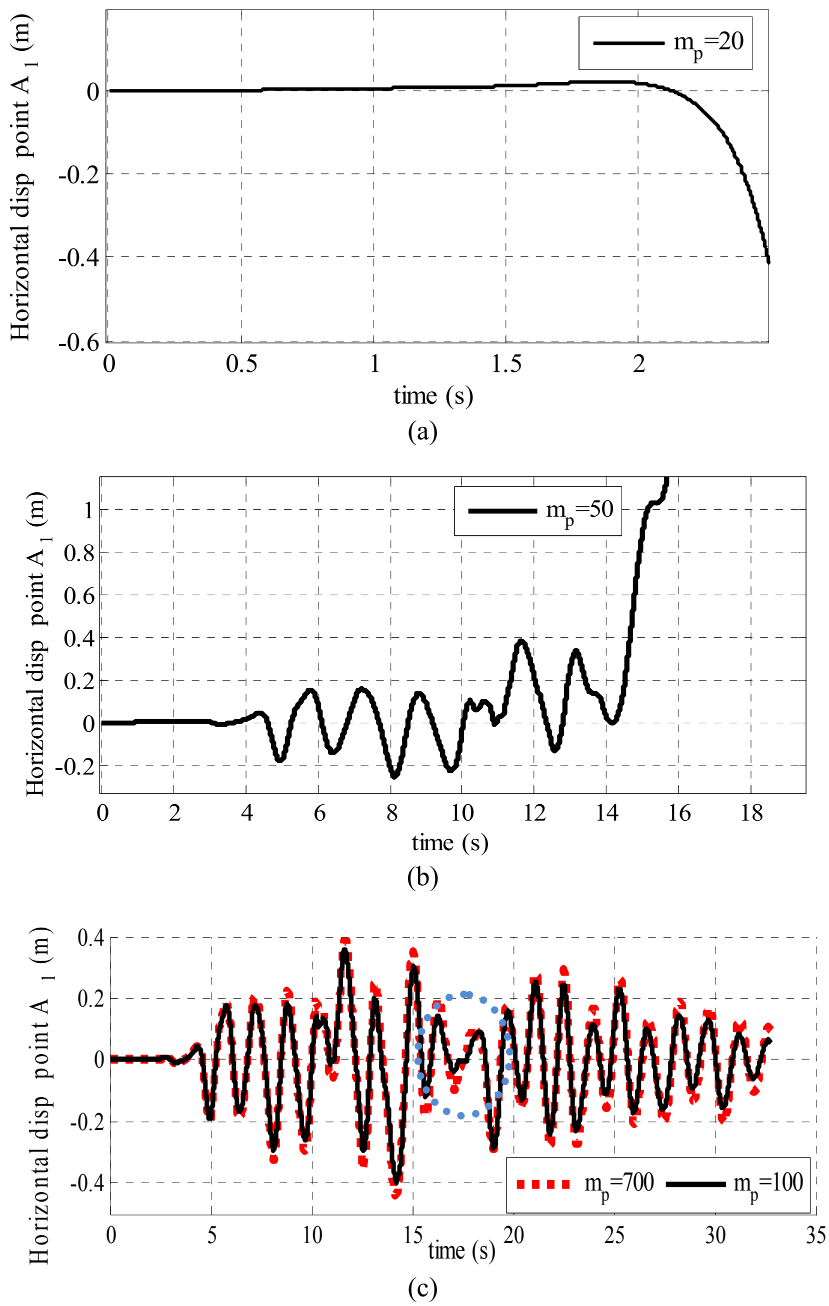


Figure 11. Effects of the selected cut of time step on the accuracy of the answers

In this example 52479 time steps (with dt =0.000625s) are used. Acceleration unit impulse response matrix calculated for m_p number of time steps directly where, adapted recursive algorithm is used to evaluate acceleration unit impulse response matrix and interaction force for the remained time steps. Lehmann's formula to calculate reduction of the necessary operations is valid yet (Eq. (24)).

$$O(\psi)=\frac{nm-.5m^2}{.5n^2}=\frac{m}{n}(2-\frac{m}{n}) \tag{24}$$

$O(\psi)$ is the remained computational efforts which are needed to analyze considered model, n is the total number of the time steps and m is equal to m_p. This equation shows that reduction of the necessary operations is independent of the *dof* (total degree of freedom in the used system) and only depends on the m/n therefore ψ in Eq. (24) is equal to m/n. By increasing m/n, further reduction in the necessary operations occurs. Figure 11 shows, when m_p=20, m_p=50 and m_p=100 are used as the cut off time steps of the discussed soil structure interaction problem, an oscillation will be occurred in the recorded displacements. This oscillation will be very little when m_p is selected equal to 100. Recorded displacement time histories with the cut of times m_p=100 and m_p=700 are compared in Figure 11.c and Figure 12. This oscillation can be reduced by increasing cut off time step from 100 to 700.

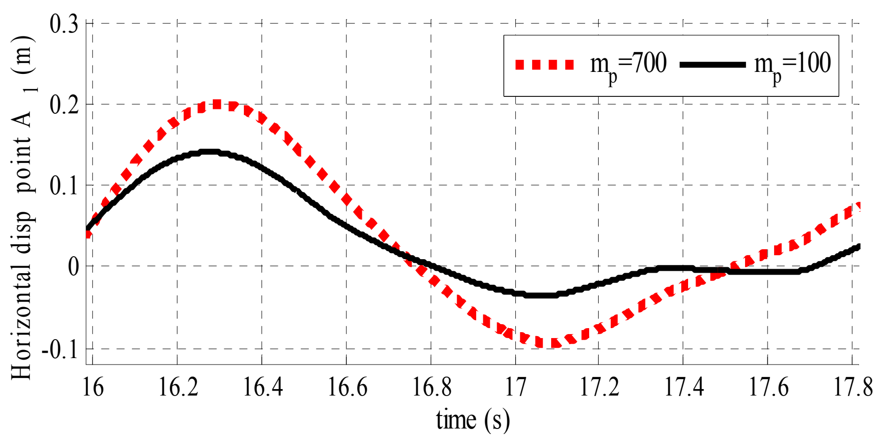


Figure 12. Comparison between recorded displacements with two different cut off times

6 CONCLUSION

Seismic soil structure interaction analysis can be very expensive if global methods are used to deal with this problem. Scaled boundary finite element method is a global method in both time and space. In this paper, a recursive algorithm that was proposed originally by Lehmann for dynamic loading cases is adapted to solve seismic induced SSI problems. It is shown that computational effort of the used SE-SBSE approach can be reduced significantly by using optimal cut off time step. In the used SE-SBSE method, high order elements with lumped coefficient matrices are used to solve governed partial differential equations. These lumped matrices make algebraic equations faster to solve. This is shown that, adapted method can able to model radiation-damping effect for SSSI problems accurately. In this paper, by using m_p=700 an excellent agreement between global SBFE approach and semi local SE-SBSE method was obtained. Computational effort was reduced nearby 97% without significant reduction in the accuracy. Hence, by using the proposed adapted approach, SE-SBSE method can be used with high efficiency in the practical seismic engineering problems.

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