EXPLORING THE CONCEPT OF NOVATION

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ABSTRACT

‘Configuration processing’ is a branch of knowledge that deals with the concepts and software necessary for generation and processing of geometric configurations. ‘Formex algebra’ and its programming language ‘Formian’ provide a convenient environment for configuration processing of all kinds. Architects and structural engineers working with structural forms are among experts in many different disciplines that benefit from configuration processing concepts and tools.

The objective of the present work is to explore the capabilities of a particular configuration processing concept that is referred to as ‘novation’. The concept of novation is implemented as a ‘function’ in Formian. This function is an effective configuration processing tool. In particular, the function provides a powerful conceptual aid for creation of freeform configurations.

In this paper, the emphasis is on the practical considerations and guidance for processing of forms, rather than involvement in details of the mathematical theory. The paper contains many examples providing an overall view of the capabilities of the novation function.

In using the concept of novation for processing of configurations, the following main parameters are used for the control of the operation:

(1) The overall guide for the formation of the configuration is provided by specification of movements of points on or around a given configuration. The novation function will then cause the configuration to be shaped in ‘conformity’ (harmony) with the specified movements (relocations). For example, the simple grid of Sketch (a), below, can be formed into the shape of Sketch (b) by specifying the relocations indicated by the arrows.

(2) There are a number of choices for the way in which conformity, in the context of novation, may be interpreted. This choice of the ‘interpretation of conformity’ is a key parameter in creating the required shape. For example, a change of the style of conformity for Sketch (b) will change the configuration of the grid into Sketch (c).

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1. BACKGROUND

The concept of novation was introduced for the first time, in August 1997 during the International Colloquium on Structural Morphology of the IASS (International Association for Shell and Spatial Structures), University of Nottingham, UK, see Nooshin et al. [2]. The principal ideas of the concept have remained the same, although applications to many problems, as well as additional conformity rules, have been employed in the recent years, see Moghimi [1] and Nooshin et al. [3].

The concept of novation is normally used in a ‘formex environment’ with Formian as the programming language. The rudiments of formex algebra were evolved in early 1970’s. However, a suitable basic textbook for formex algebra and Formian is a recent book, see Nooshin et al. [4]. Part C of this book is devoted to ‘Formex Configuration Processing’. There will be many references to this book in the present paper and, for convenience, the abbreviation FCP (acronym for Formex Configuration Processing) will be used for reference.

2. FUNDAMENTALS OF THE CONCEPT OF NOVATION

The concept of ‘novation’ concerns ‘reshaping’ of geometric configurations consisting of ‘elements’ connected together at ‘nodes’. The concept may be applied to modify the form of a configuration in a required manner. The process may be carried out any number of times to achieve the desired effects.

The idea of novation is of general applicability to geometric forms related to any discipline. However, in the present paper, explanations regarding the particulars of the concept of novation are exemplified using various ‘structural forms’.

To begin with, consider a simple structural form consisting of 32 identical straight line elements that are connected together along a straight line, as shown in Fig 1a. Each of these elements has a ‘node’ at each of its ends. In Fig 1, the nodes are represented by little dots.
EXPLORING THE CONCEPT OF NOVATION

The process of novational reshaping of a geometric form is guided by specifying one or more movements or ‘relocations’ of points (the term ‘translocation’ may also be used synonymously). For instance, let it be required to ‘relocate’ (move) the starting points of the vectors in Fig 1b to their end points (relocate points A to points B).

In applying the concept of novation, in most cases, only a few relocations are specified, but the bulk of the body of the configuration assumes its shape by following the ‘trend’ of the specified relocations. Thus, the reshaping mainly takes place by the nodal points of the configuration moving in ways that are in ‘conformity’ (harmony) with the specified relocations. The question then is: how to interpret conformity?

For example, Figs 1c, 1d and 1e illustrate three different ways of interpreting conformity with respect to the specified relocations of Fig 1b. Theoretically, there are many possibilities for ‘conformity rules’, but the rules illustrated in Figs 1c, 1d and 1e are the ones that are, at the present, implemented as parts of the ‘novation function’ in Formian.

- The shape assumed by the ‘line form’ in Fig 1c is seen to be similar to the shape of a rope (chain, cable, membrane) which is supported at the specified relocated points. Indeed, in most applications, a good model for visualising the manner of shaping of the conformity rule of Fig 1c is to assume that the form is made from such ‘soft material’ as ropes and membranes, supported at the relocated points. This conformity rule is thus referred to as the ‘soft conformity rule’. A more general model for the visualisation of the effects of the soft conformity rule will be introduced later in Section 5.
- The rule used in Fig 1d is referred to as the ‘linear conformity rule’. It works in a way that the parts of the form tend to assume somewhat ‘linear’ orientations, and again the relocated points act as supports. Actually, the overall shape created by the linear conformity...
The rule used in Fig 1e is referred to as the ‘supple conformity rule’. A good way of visualising how it works is to imagine that the form is made from a flexible springy metal, with the relocated points acting as supports. With the supple conformity rule, there will be no ‘sharp corners’ and the overall shape will be ‘smooth and curvy’.

Each of these conformity rules has its own mathematical formulation. These details are explained in the Appendix. Also, a reader interested in the mathematical foundations of the concept of novation can refer to Nooshin et al. [2].

The summary of Fig 1 is presented in Line (a) of Fig 2. The remaining lines of Fig 2 show similar operations with variations on the types of relocation:

Line (b) of Fig 2 shows that the relocations need not necessarily follow a regular or symmetric pattern.

Line (c) of Fig 2 shows that the relocations need not necessarily be in the vertical direction and may have any orientation.

Line (d) of Fig 2 shows that a relocation may specify a ‘zero movement’, effectively, specifying that a point must remain unmoved. This ‘zero relocation’ is graphically represented by a ‘little circle’ as shown at the point indicated by S in line (d) of Fig 2. Actually, specification of ‘zero movements’ implying points that should remain ‘fixed in position’ is frequently required in applying the concept of novation.

Line (e) of Fig 2 shows that a relocation may be specified at a point which is not on the configuration (like point T in Line (e) of Fig 2). It should be noted that when the point of application of a relocation is not on the configuration, then the resulting form loses the sharp apex which is characteristic of both soft and linear conformity rules (at positions indicated by P in line (e) of Fig 2).
3. A FORMIAN PROGRAM

To show a sample of Formian instructions for applying the concept of novation, consider a program for the generation of the forms of Figs 1c, 1d and 1e, as follows:

(*) Generating the initial configuration, consisting of 32 line elements, one metre long each (*)
Con = rin(1,32,1)[[0,0,0; 1,0,0];

(*) Generating the lists of starting points and end points of the relocations (*)
ListA = {[0,0,0], [8,0,0], [16,0,0], [24,0,0], [32,0,0]};
ListB = {[0,0,-1], [8,0,3], [16,0,1], [24,0,3], [32,0,-1]};

(*) Generating the novated configuration (*)
NovCon = nov(mode, C, ListA, ListB)|Con;

The above Formian program (scheme) consists of four lines of Formian code, terminated by semi-colons, and three lines of comments, enclosed in ‘floret symbols’, that is, the compound symbol (*). The first line of code generates the simple line configuration of Fig 1a. The second and third lines of code specify the starting points and the end points of the relocations. The last line of code uses the ‘novation function’ to apply the relocations. The argument of the novation function is the ‘initially generated configuration’ represented by the ‘formex variable Con’ and the resulting novated configuration is represented by the formex variable ‘NovCon’. The novation function, represented by ‘nov’, has four parameters. The last two of these parameters are represented by formex variables ListA and ListB that specify the relocations. For detailed information about the programming language Formian, see Nooshin et al. [4]. Also, for detailed information about the concept of novation, see Nooshin et al. [2] and Moghimi [1].

The first parameter of the novation function, in the above Formian program, is ‘mode’. This parameter can assume one of the four values 1, 2, 3 or 4, with the following implications:

• Mode 1: represents sharp novation,
• Mode 2: represents soft novation,
• Mode 3: represents linear novation in Formian-K (supple novation in Formian-2) and
• Mode 4: represents supple novation in Formian-K (linear novation in Formian-2).

The ‘sharp novation’, represented by mode = 1, works by applying the specified relocations without any considerations regarding the conformity rules. For example, with mode = 1, the above Formian program will produce the configuration shown in Fig 3.

Figure 3. An example of the application of sharp novation

In general, sharp novation is a useful configuration processing tool, employed whenever it is required to change the positions of a group of nodes. An example of the application of
sharp novation may be seen in FCP, Section 2.15.

In relation to the novation function in the above Formian program, it is also necessary to explain the role of parameter C. This is a ‘control parameter’ whose value ranges from zero to infinity. It mostly affects the soft novation (mode 2) and the larger it is the more ‘local’ the novational effects will be. Actually, being ‘more local’ can also be interpreted as being ‘softer’.

For example, using the above Formian program, the forms of Fig 1c for four different values of parameter C are shown in Fig 4. The value used for the control parameter C of the soft novated forms of Figs 1 and 2 was 3. More information about the effects of parameter C can be found in the Appendix.

4. NOVATING A SURFACE

Fig 5a shows a view of a planar configuration consisting of ‘plane surface elements’ with each element being 1 m by 1m, with four corner nodes. This configuration may, among other possibilities, represent a ‘finite element mesh’. The configuration is to be subjected to soft novation with the four corners of the configuration being ‘held in position’ (having relocation of zero) and with the central node being relocated vertically by 4 m. In Fig 5a, nodes with fixed positions are indicated by little circles and the central relocation of 4 m is indicated by a vector. Assuming a control parameter of C = 2, the resulting surface is as shown in Fig 5b.

A Formian program for this operation is given below:

(*) Creating the planar mesh configuration with 1 m by 1 m elements (*)
Mesh = rinid(16,16,1,1)[[0,0,0; 0,1,0; 1,1,0; 1,0,0];

(*) Specifying the initial and end points of the relocations (*)
ListA = {{0,0,0}, [0,16,0], [16,16,0], [16,0,0], [8,8,0]};
ListB = {{0,0,0}, [0,16,0], [16,16,0], [16,0,0], [8,8,4]};
Creating the novated form (*)
NovMesh = nov(2,2,ListA,ListB)|Mesh;

The same program can be used to create novated forms corresponding to the ‘linear’ and ‘supple’ conformity rules by inserting appropriate values for the ‘mode’ of the novation function in the last line of the program. The results are shown in the top row of the configurations in Fig 6.

The middle row of configurations in Fig 6 is obtained in a similar way, except that instead of a single central relocation, four vertical 4 m relocations are used. Also, the configurations in the bottom row are obtained using a single off centre vertical 4 m relocation.

Comparison of the novated forms in Fig 6 shows the following points:
There are similarities between the forms created by the soft conformity rule and those created by the linear conformity rule. Although, with control parameter C larger than 2, the differences between the results of the soft and linear conformity rules become more noticeable. Actually, with large values of C (say, C > 20), the results of the ‘soft novation’ approach those of the sharp novation and with small values of C (say, C < 0.1), the soft and linear conformity rules produce similar results. Theoretically, the results of the soft novation with C equal to infinity will be identical to those of the sharp novation and the results with C equal to zero will be identical to those of the ‘linear novation’.

Now, concentrating on the soft and supple novations, three surfaces with different boundary shapes are considered in Fig 7. The configuration of Fig 7a is an 18 by 18 mesh.
with 1 m by 1 m square elements and with a central hole. For the process of novation, the four corner nodes and all the nodes around the central hole are fixed in position and four vertical relocations of 4 m are applied. In Fig 7a, fixed positions are indicated by little circles and relocations are indicated by vectors. The resulting form with the soft conformity rule is shown in Fig 7b and that with the supple conformity rule is shown in Fig 7c.

The next boundary variation is shown in Fig 7d, where the corner regions of the mesh are removed and the nodes indicated by little circles at the corners are fixed in position. Also, 5 relocations of 3 m are applied at the positions shown. The application of the novation function with soft conformity rule will produce the configuration of Fig 7e and with the supple conformity rule the resulting configuration will be as shown in Fig 7f.

Finally, the boundary shape of Fig 7g is considered. Here, the mesh has an L-shape with the points indicated by little circles being held in position. The relocations, whose values are indicated in the figure, are applied at the midpoints of three elements. As a consequence of the relocated points being outside the nodes (at the middle of elements), the soft novation looses its characteristic sharpness at the peaks (Fig 7h).

Figure 7. Some novated surfaces with different boundary conditions

5. NOVATING A BODY
This section is devoted to the presentation of examples illustrating how a body responds to novation when relocations are applied all around it. Consider the arrangement shown in Fig 8. The elliptic form shown consists of 80 line elements connected together at 80 nodal
points. The length of the major axis of the ellipse is twice that of its minor axis. Here, the elliptic form is intended to represent a general geometric configuration consisting of elements and nodes.

![Figure 8. An elliptic configuration consisting of 80 line elements connected together at 80 nodal points](image)

Fig 9 shows examples of the elliptic body of Fig 8 being novated using the soft conformity rule. In Fig 9a the novated form is obtained by applying two pairs of opposite relocations. One of the pairs of relocations acts inwards and the other pair acts outwards. The magnitudes of the relocation vectors are proportional to their lengths. Also, the form of the body before novation is shown by dotted line.

The novated form in Fig 9b is obtained by applying three relocations that are directed outwards. Also, the novated form of Fig 9c is obtained by two inwards relocations and by fixing the position of the lower middle point of the body, indicated by a little circle.

![Figure 9. Some novated forms obtained using the soft conformity rule](image)

Fig 10 shows some novations of the elliptic body of Fig 8, using the supple conformity rule. In Fig 10a, five nodal points of the elliptic body have relocations. Three of these nodal points have been assigned ‘zero relocations’, which is another way of saying that their positions have been fixed. Also, one nodal point has an inward horizontal relocation and another nodal point has an outward vertical relocation. The nodal points with fixed positions are indicated by little circles and the original form of the elliptic body is indicated by dotted line. Another two similar examples of novation of the elliptic body are shown in Figs 10b and 10c.

![Figure 10. Some novated forms obtained using the supple conformity rule](image)
Of course, there is no end to the forms that can be generated using the concept of novation and the examples provided so far are just representative samples.

Figs 8, 9 and 10 use 2-dimensional forms to represent 3-dimensional bodies. This is done, so that it would be easy to visualise the effects of the relocations. But, in these examples, the kinds of forms in mind are three dimensional bodies an example of which is shown in Fig 11a. This is an ellipsoidal geodesic form consisting of 180 triangular panel elements with corner nodal points. The novated form of the body under the effects of three relocations is as shown in Fig 11b, where the relocations are indicated by vectors. The supple conformity rule has been used in this example.

From what has been discussed in the paper so far, the process of novation may be summarised as follows:

To create a desired form using the concept of novation, a suitable initial form is selected. This initial shape is then deformed into the required shape using the novation function of Formian with appropriate mode and suitable relocations. It is like the process of creating a sculpture when, with the ultimate objective in mind, the sculptor begins by choosing a block of material of suitable initial form and then uses various tools to turn the block into the intended shape. However, in the case of novation, the tools are conceptual. The successful application of the novation process, usually, involves some try and error. Also, a User needs some experience, as well as imaginative power and creativity.

Earlier, in describing the behaviour of the soft novation (in relation to Fig 1c), a simple model of behaviour was suggested which was based on the assumption that the configuration was made from such materials as ropes and membrane. This model works well in many cases. However, to symbolise the mode of action of the soft novation, a generally applicable model is the one shown in Fig 12b.

Fig 12a shows the elliptic body of Fig 8 under 8 relocations that are applied at 8 regularly
EXPLORING THE CONCEPT OF NOVATION

spaced nodal points of the body. The directions of these relocations pass through the centre of the ellipse. Also, the magnitudes of the relocations are proportional to the distances of their nodal points from the centre of the ellipse.

The novated form of the elliptic body, shown in Fig 12b, is a good model for symbolising the effects of the soft conformity rule under inwards relocations. The corresponding situation, when the relocations are outwards, is shown in Fig 13, where Fig 13a shows the body with the relocations and Fig 13b shows the novated body with the body itself in dotted line.

Figs 12b and 13b represent what is called the ‘balloon model, symbolising the effects of the soft conformity rule. The name reflects the fact that the forms resemble inflated rubber balloons which are pushed or pulled at a number of points.

![Figure 13. (a) Elliptic body under outward relocations (b) Soft novated form of the body (balloon model)](image)

The balloon model symbolises the behaviour of the soft novation. However, as a matter interest, the elliptic body has been also novated using the relocations of Figs 12a and 13a with other novation modes and the results are shown in Fig 14.

![Figure 14. Effects of inward and outward relocations with sharp, linear and supple novations](image)

Fig 14a is the sharp novated form of the elliptic body for the inward relocations of Fig 12a and Fig 14d is for the outward relocations of Fig 13a. Also, Figs 14b and 14e are the corresponding ones for the linear novation and Figs 14c and 14f are for the supple novation. It is interesting to note that for the supple novation, with the regular relocations of Figs 12a and 13a, the novated forms are almost scaled versions of the elliptic body itself.

6. NOVATION OF A DOME USING GROUP RELOCATIONS

The starting configuration for novational processes need not necessarily be a simple
configuration. Any configuration may be the starting point of the novational processes, irrespective of whether it is the outcome of an elaborate long sequence of configuration processing operations or it is a simply generated primitive form. To illustrate the point made, in this section, a dome configuration is chosen as a starting form for novational operations. A view of the chosen dome is shown in Fig 15 and a Formian scheme (program) that is used to generate the dome is shown in the Editory display of Fig 16 (see [4], Fig 1.9.14).

![Figure 15. A lamella dome](image)

The scheme in the Editory display of Fig 16 uses a spherical reference system (spherical normat). The use of the spherical normat makes the formulation of the dome very convenient (see FCP, Section 1.9). The dome is formulated parametrically with the parameters being listed at the beginning of the scheme (program) of Fig 16:

- The span of the dome is represented by S, with an initial value of 50 m.
- The rise of the dome is represented by H, with an initial value of 8 m.
- The number of nodes in each circumferential ring is denoted by m, with an initial value of 18.
- The number of rings is denoted by n, with an initial value of 9. The value of n should be ‘odd’.

The dome of Fig 15 may be formulated either as a lattice dome (with two-ended bar elements) or as a dome with three-noded triangular panel elements. The formulation with bar elements has been chosen for the present example.

However, it is to be noted that, since the novational processes only affect the nodal coordinates, the coordinates of the nodal points of the configuration in this example will be the same, irrespective of whether the formulation is with bar elements or panel elements.

The specification of the relocations for ‘novational work’ can be done in different ways. In exemplifying the novation process, as described in the paper so far, the relocations were always considered individually. However, in some cases it is more convenient to specify the relocations in terms of ‘groups of nodes’. To illustrate this approach, in this section, a number of novations of the dome of Fig 15 are obtained in terms of three rings of nodes, referred to as ‘hoops’, as shown in Fig 17.
EXPLORING THE CONCEPT OF NOVATION

(*) Scheme for generation of lamella dome of Fig 15 (*)

(*) Defining parameters (*)
S=50; (*) Span of dome, in metres (*)
H=8; (*) Rise of dome, in metres (*)
m=18; (*) Number of nodes in every circumferential ring (*)
n=9; (*) Number of rings, must be odd (*)
A=2*atan([2*H/S]); (*) Sweep angle of dome, see FCP, Fig 1.9.2 (*)
R=S/(2*π|A|); (*) Circumradius of dome, see FCP, Fig 1.9.2 (*)

(*) Formulation of dome in spherical coordinates (spherical norm) (*)
θ=rin(m,[n-1]/2,2,2)||lamit(1,3)||1,1,2; 1,0,3|#
λb(i=0,n-1)||trani(i)||rin(2,m,2)||1,1,2, 1,3,2;.

(*) Transferring to Cartesian coordinates (*)
Dome=hs(R,180/m,A/(n+1))/R;

(*) Place for insertion of statements relating to the novational process (*)

(*) Setting viewing particulars (*)
use &.vm(2).vt(2),vh(R,R,3^R, 0,0,0, 0,0,1);
clear; draw Dome;
clear;

Figure 16. A scheme for the lamella dome of Fig 15

Figure 17. Hoops of nodes for the control of the process of novation

The hoops of nodes are indicated by dots in Fig 17a. These hoops are also shown in Fig 17b, with the ‘top hoop’ represented by T1, the ‘middle hoop’ represented by M1 and the ‘base hoop’ represented by B1. The scheme of Fig 16, together with the insertion of Fig 18, is used to novate the dome of Fig 15 into the form of Fig 19a. The statement

T2=dilid(0.5,1)|T1;

appearing under Fig 19a is a Formian instruction for scaling of the top hoop for obtaining the elliptic form of its relocated position.
In fact, the scheme of Fig 16, together with the insertion of Fig 18, has also been used to
generate the other eight novated forms of Fig 19. The only necessary changes to the scheme
for the other eight cases are the instructions appearing under the figures that should be
incorporated into the insertion. Brief descriptions of the relocations for novation of the forms
of Figs 19b to 19i are as follows:

- The form of Fig 19b is obtained by fixing the positions of the top and middle hoops and
  relocating the base hoop by 18 m downwards (B2=tran (3,-18)|B1).
- The dome of Fig 19c is obtained by fixing the top hoop, moving the middle hoop
downwards by 9 m as well as enlarging it by 50% and by moving the base hoop downwards by
25 m.
- The form of Fig 19d is obtained by fixing the position of the base hoop and by moving the
top and middle hoops downwards by 20 m and 6 m, respectively.
- The form of Fig 19e is generated by fixing the position of the base hoop and moving the
top and middle hoops upwards by 16 m and 4 m, respectively.
- The form of Fig 19f is created by carrying out the relocations in the same way as for Fig
  19e, with the additional enlargement of the middle hoop by 100%.
- The form of Fig 19g is obtained by fixing the top hoop and moving the middle and base
  hoops downwards by 10 m and 25 m, respectively, and by enlarging the middle and base hoops
50% and 100%, respectively.
- The freeform of Fig 19h is generated by enlarging the top hoop in one direction by 100%
  (thus making the ring elliptic) and by moving the middle hoop downwards by 4 m and sideways
  by 10 m and by moving the base hoop downwards by 16 m.
- Finally, the freeform of Fig 19i is created by enlarging the top and middle hoops by 100%
in one direction and by moving the base hoop downwards by 8 m.
There is no end to the forms that can be created through the above process. Also, each one of the forms that is created can be considered to represent a family that has many other similar members. For instance, consider Fig 19d. This is a ‘funnel-shaped’ configuration but, while preserving its funnel shape, its particulars can be varied in many different ways. For example, a few funnel-shaped configurations are shown in Fig 20. These are generated in the same way as explained for the forms in Fig 19. However, Formian instructions appearing under Figs 20a to 20f include an additional parameter ‘n’. This parameter refers to the number of rings, as given in the scheme of Fig 16 (the number of rings for all the forms of Fig 19 is 9).

The supple conformity rule has been used for all the novated forms of Figs 19 and 20. However, in the case of these forms, the other conformity rules would have given rise to similar results. This is due to the fact that a large number (54) of relocations are involved. This implies that the resulting novated forms are mainly governed by relocations rather than conformity rules. This matter will receive more attention in the next section.

An interesting question may be raised at this point, namely, all the nodes of the dome of Fig 15 lie in a spherical surface. Thus, the surface of this dome has an equation. But, what about the configurations in Figs 19 and 20 which are obtained by novation from the dome of
Fig 15? Each of the fifteen configurations in Figs 19 and 20 has a surface that contains all the nodes of the configuration, but does this surface have an equation? The answer is that the surface may not have a traditional mathematical equation, but if the problem is formulated in a 'mathematical form language' like Formian, then the formulation serves as an equation. This is so because the formulation is capable of determining the precise values of all the coordinates of the nodes. However, it would be more appropriate to say that the surface has a 'formulation' rather than an 'equation'.

7. NOVATING BARREL VAULTS

Barrel vaults constitute a major family of spatial structures. Also, a barrel vault as the starting configuration for novation can give rise to many interesting forms. To exemplify this, consider the barrel vault of Fig 21a. This is a cylindrical lattice barrel vault with a lamella pattern. The span is 30 m, the central rise is 8 m and the length is 45 m. Fig 21a, also shows a side view of the barrel vault under its perspective view.

Fig 21b shows a novated form of the barrel vault of Fig 21a. This novated form is obtained by fixing the positions of the corner nodes of the barrel vault (indicated by little circles) and relocating the central point of the barrel vault by 4 m vertically upwards. The novated barrel vault of Fig 21c is obtained in a similar manner, except for the central
relocation being downwards rather than upwards. The novated forms in Figs 21d, 21e and 21f are obtained by fixing the nodes indicated by little circles and applying the relocations indicated by vectors. In these figures, the dotted lines show the original form of the barrel vault.

Another example of a barrel vault to be novated is shown in Fig 22. In this example, novations will be applied in terms of group relocations. The barrel vault of Fig 22a has all the particulars of the barrel vault of Fig 21a, except that it is longer. Specifically, it is 60 m long instead of 45 m. In Fig 22a, a number of nodes are indicated by ‘dots’. These nodes are divided into seven groups, referred to as ‘Arcs’, as shown in Fig 22b. The Arcs are used for six supple novations of the barrel vault of Fig 22a and the results are shown in Fig 23.
The ‘double humped’ barrel vault of Fig 23a is obtained by fixing the positions of Arcs 2, 3, 5 and 6 and scaling Arcs 1, 4 and 7 by 0.5, 0.9 and 0.9, respectively. The dotted outline in Fig 23a shows the original position of the barrel vault before novation.

The ‘undulated’ barrel vault of Fig 23b is obtained by fixing the positions of Arcs 2, 4 and 6, scaling Arc1, by 0.6 and scaling Arcs 3, 5 and 7 by 0.9.

The form of Fig 23c is obtained by using only Arcs 1, 4, and 7, scaling them by 0.5, 1.15 and 0.9, respectively.

The form of Fig 23d is obtained by fixing the position of Arc7, scaling the span and rise of Arc1 and Arc5 by 0.6 and 1.1, respectively, and scaling the span and rise of Arc3 by 1.1 and 0.6, respectively.

The freeform of Fig 23e is obtained by scaling the span and rise of Arc1 by 0.7 and 1.4, respectively, scaling of span and rise of Arc4 by 0.6 and 1.2, respectively, and by translating Arc7 by 7 m to the left.

Finally, the freeform of Fig 23f is obtained by scaling the span and rise of Arc1 by 0.2 and 1.5, respectively, enlarging Arc4 by 1.2 and by translating Arc7 by 5m to the right.

In formex configuration processing, usually there is a choice of formulating a problem in a Cartesian environment (based on Cartesian coordinate system) or using another environment (based on another coordinate system) like cylindrical, polar, spherical, … etc. The reason for wanting to use a coordinate environment other than Cartesian is convenience of formulation. For instance, when formulating a dome, it is normally convenient to use a
spherical coordinate system (as it was done for the lamella dome of Fig 15 in the scheme of Fig 16). Also, for a barrel vault, it is, normally, convenient to formulate it in a cylindrical or annular (toroidal) coordinate system. However, when a non-Cartesian coordinate system is used for the formulation, subsequently, the results are transferred into Cartesian coordinates. For instance the formulation of the barrel vault of Fig 23a is initially carried out in a cylindrical coordinate system, and the information is then transferred into Cartesian coordinates. This formulation is given in the Editory display of Fig 24.

\[
\text{(*) Scheme for generation of novated barrel vault of Fig 23a (*)}
\]

\[
\text{(*) Defining parameters (*)}
\]
\[S=30; \text{(*) Span (*)} \ H=8; \text{(*) Rise (*)} \ L=60; \text{(*) Length (*)}
\]
\[m=7; \text{(*) Frequency in the circumferential direction (*)}
\]
\[n=12; \text{(*) Frequency in the longitudinal direction (*)}
\]
\[A=27\text{atan}[(2^*H/S)]; \text{(*) Sweep angle (*)}
\]
\[R=S/(2^*\sin[A]; \text{(*) Circumradius (*)} \ m1=m+1;
\]

\[
\text{(*) Formulation of barrel vault in cylindrical coordinates (*)}
\]
\[E=\text{rinit}(m,n,2,2)[\text{lambda}(1,1))[1,0,0; 1,1,1]
\]
\[=\text{rinit}(m,2,2^*n)[1,0,0; 1,2,0]
\]
\[=\text{rinit}(2,n,2^*m,2)[1,0,0; 1,0,2]
\]

\[
\text{(*) Formulation of the arcs in cylindrical coordinates (*)}
\]
\[\text{Arc1=rin}(2,m,1,2)[1,0,0]; \text{Arc2=rin}(2,m,1,2)[1,0,4];
\]
\[\text{Arc3=rin}(2,m,1,2)[1,0,8]; \text{Arc4=rin}(2,m,1,2)[1,0,12];
\]
\[\text{Arc5=rin}(2,m,1,2)[1,0,16]; \text{Arc6=rin}(2,m,1,2)[1,0,20];
\]
\[\text{Arc7=rin}(2,m,1,2)[1,0,24];
\]

\[
\text{(*) Formulation of relocations and application of novation (*)}
\]
\[\text{fix=Arc2#Arc3#Arc5#Arc6; ListA=fix#Arc1#Arc4#Arc7;}
\]
\[\text{ListB=fix#dil(1,0,5)[Arc1#dil(1,0,5)[Arc4#dil(1,0,5)[Arc7;}
\]
\[F=\text{nov}(3,1,ListA,ListB)];
\]

\[
\text{(*) Transferring to Cartesian coordinates (*)}
\]
\[\text{Vault=verad([0.0,0.90-A])bc(RA/m,L/[2^*n])};
\]

\[
\text{(*) Setting viewing particulars (*)}
\]
\[\text{use \&vm(2),vt(2),v(0.3^\*R,1.5^\*R,0,0,0,1,0,8);}
\]
\[\text{clear; draw Vault;}
\]

Figure 24. A scheme for the lamella barrel vault of Fig 23a

In the context of the present paper, an important question is: Should the specification of relocations and the application of the novation function be carried out before or after the transfer of information into Cartesian coordinates? The answer is that it can be done either way.

For example, the ‘novation work’ for the creation of Fig 19a, as formulated in the insertion of Fig 18, was carried out in a Cartesian environment after the transfer of information. But the novation work for the creation of Fig 23a, as given in the Editory display of Fig 24, is carried out in a cylindrical environment, before the transfer of
information.

The merits of doing the novation work in the pre-transfer or post-transfer stage depend on the type of problem, ease of formulation of relocations and the personal preferences. However, as far as the final results are concerned, there may be minor differences, but the overall effects are similar.

The reason for differences is that the effects of conformity rules are influenced by the coordinate system in which they are used. For instance, consider Fig 25a. This figure shows different versions of Fig 23a for the soft, linear and supple conformity rules superimposed on top of each other. Fig 25b shows a similar arrangement, except that Fig 25a corresponds to pre-transfer and Fig 25b corresponds to post-transfer novation work. It is seen that the differences between all the six versions are rather small.

Actually, the smallness of the differences raises an important question. Namely, why should the differences be so small and why different conformity rules do not show their characteristic features? The answer is that in the case of the barrel vault of Fig 23a, there are a large number of relocations (56 relocations related to 7 Arcs) and these relocations are evenly distributed throughout the configuration.

This implies that the novated shapes are mainly determined by the specification of the relocations and the conformity rules do not have enough space to influence the shapes. In general, for a conformity rule to have a major influence, the configuration should have regions whose forms are not mainly dictated by relocations.

8. NOVATIONAL FORM FINDING OF TENSION STRUCTURES

There is a wide variety of structural forms that come under the general description of ‘tension structures’. Examples include membrane structures and pre-stressed cable nets. The design of tension structures often involves a preliminary stage for evolving shapes that are likely to be suitable as tension forms. This is referred to as ‘form finding’.

The concept of novation is an effective tool for form finding of tension structures. Novation provides a convenient intuitive approach for exploring various possibilities that are
suitable as tension forms. To illustrate this approach, a high-point membrane form is chosen as an example. A view of this form is shown in Fig 26a. It consists of 12 fabric segments that are joined together, with the 12 perimeter corners pulled out and the central ring pulled up, as shown.

To produce the configuration of Fig 26a, the initial form of the fabric segments is chosen to be represented by a 16 m diameter flat circular arrangement of line elements, as shown in Fig 26b. Each of the 12 radial parts of the initial form consists of 14 equal line elements. Also, each of the outer and inner circular parts between the radial lines consists of 5 equal line elements.

The initial arrangement representing the fabric segments is subjected to three sets of novational relocations, as shown in Fig 27.

In Fig 27a, the initial form is seen with all the 12 corner nodes of the fabric segments on the perimeter fixed in position (zero relocation, indicated by little circles). The initial form in Fig 27a is also subjected to vertical relocations of 6 m at all the 60 nodes of the inner ring. With soft novation, and the control parameter $C=4$, the resulting configuration is shown in Fig 27d.

In Fig 27b, the nodes on the perimeter at the corners of the fabric segments have out-radiating relocations. Also, the nodes on the inner ring have vertical relocations similar to
those in Fig 27a. With the soft novation and the control parameter C=5, the resulting configuration is as shown in Fig 27c (which is the same as Fig 26a). The formulation of the high-point membrane form of Fig 27e is shown in the Editor display of Fig 28.

In Fig 27c, only one of the nodes of the inner ring has been vertically relocated. This eccentric application of the vertical relocation gives rise to the interesting ‘pointed shape’ of the top ring in Fig 27f. Also, in Fig 27c, every other corner points of the fabric segments on the perimeter have been relocated. This causes the perimeter of the resulting high-point membrane to have 6 curved edges, rather than 12.

It should be noted that, when applying the soft novation for form finding of tension structures, the value of the control parameter C, has a significant effect on the resulting forms. To illustrate this, the high-point membrane form of Fig 27d with C=4 is also generated with another two values of C, namely, C=1 and C=8. The results are shown in Fig 29.

It is a characteristic of the ‘soft conformity rule’ that can give rise to forms suggestive of tension structures. This is a valuable characteristic and provides a noble convenient technique for form finding of tension structures. An important feature of the novational form finding is that it works through imposition of movements (relocation) and, therefore, the approach is intuitive.
In the process of novational form finding of tension structures, the initial shape for the application of novation may be ‘sketchy’ or ‘more detailed’. For example, suppose that instead of the sketchy initial form of Fig 26b, a finite element mesh like the one in Fig 30a is employed. The application of novation to the initial form of Fig 30a, with the particulars of Fig 27b, will result in the high-point membrane form of Fig 30b. Comparison shows that all the nodal points of the form of Fig 27e coincide with the corresponding nodal points of Fig 30b, although the latter form contains information about the positions of many more points.

Actually, in relation to any conformity rule, one can choose the form for the application of novation to be ‘very sketchy’ or ‘fully represented’, or anywhere in between, to suit a particular case. However, all these forms will give compatible results as long as:

- The overall dimensions of the forms are the same,
- The control parameter C is the same and
- The same relocations are applied.

Incidentally, to generate the high-point membrane of Fig 30b, all that is required is to use the scheme of Fig 28 with the formex variable ‘form’ replaced by:

\[
\text{form} = \text{pex}\{\text{bp}(1,360/(\text{seg}\times\text{m}))\}
\text{rinid}(n/k,\text{seg}\times\text{m},k,1)\}
\{(1,0,0; 1+k,0,0; 1+k,1,0)\#(1,0,0; 1,1,0; 1+k,1,0)\};
\]

9. CONCLUSIONS

It has been shown that the concept of novation is an effective tool in ‘shaping’ of geometric configurations. The concept is particularly powerful in creation of freeform configurations.
The examples used in the paper relate to the field of structural configurations. However, the idea is general and could, for example, be used in the geometric design of the body of a car, an aircraft or a ship. The concept can also be used in so many other applications.

An attractive feature of the process of novation is that it is ‘intuitive’. To elaborate, the shaping of a geometric body, obviously, involves deforming the body towards a desired target. This, on the other hand, is exactly what the novation does. So, the required deformations of the body, as guided by the desired target, can be effected through relocations.

The ideal environment for the use of concepts such as novation is a ‘configuration processing environment’ such as formex algebra and its programming language Formian. The mathematical nature of formex algebra and Formian allows configuration processing to become a serious discipline, a discipline that may be taught and learnt at various levels from elementary schools to the universities.

REFERENCES


APPENDIX: CONFORMITY RULES

This Appendix contains a brief explanation of the mathematical formulae relating to the soft, linear and supple conformity rules used in the novation process.

Suppose that there is a single specified relocation applied to a configuration. Then, with the soft conformity rule, the $k^{th}$ coordinate of a typical node $j$ of the configuration will be modified by adding to it the quantity $T_k = T_{sk} (10^{-Cd_j})$ where

- $k$ is 1, 2 or 3 representing the $1^{st}$, $2^{nd}$ or $3^{rd}$ coordinate direction, respectively.
- $T_{sk}$ is the $k^{th}$ component of the specified relocation.
• $d_j$ is the ‘relative distance’ of node $j$ from the position of the specified relocation. The relative measure of $d_j$ is with respect to the overall dimension of the configuration.

• $C$ is a ‘control parameter’ whose range is from zero to infinity and the larger it is the more ‘local’ the effects of the specified relocation will be.

Fig A1 represents the variations of $T_k$ with respect to $T_{sk}$ and $d_j$ for different values of $C$ as determined by the soft conformity rule. The soft conformity rule is also referred to as the ‘exponential decay’ conformity rule or ‘ED’ conformity rule.

The variations shown in Fig A1 relate to a situation when there is only one single relocation. However, when there are two or more relocations (including zero relocations that imply fixed positions), then it is necessary to examine the interaction between the relocations in order that all the relocations are satisfied simultaneously. For more details see Nooshin et al. (1997).

The details presented in Fig A1 relate to the soft conformity rule. The corresponding details for the linear and supple conformity rules are presented in Figs A2 and A3, respectively.

The linear conformity rule is also referred to as the ‘linear decay’ conformity rule or ‘LD’ conformity rule. The supple conformity rule is also referred to as the ‘cubic decay’ conformity rule or ‘CD’ conformity rule.

The three conformity modes described above and exemplified in the paper are available in Formian. However, theoretically there are many other possibilities. For instance, it may be noticed that the formulae governing the linear and supple conformity rules are similar. In fact, the difference between these only concerns the exponent at the end, which is 1 for the linear case and 3 for the supple one. The general form of these formulae is $T_k = T_{sk} (1+(10^{-C}-1)d_j^3)$. This formula works for any positive value of $P$ giving rise to various conformity rules. Any conformity rule based on this formula is referred to as a ‘power conformity rule or ‘power decay conformity rule’ or PD conformity rule.
The diagrams showing the behaviour of two of these PD conformity rules are given in Figs A4 and A5. Fig A4 shows the diagrams for $P = 0.5$ and Fig A5 shows the diagrams for $P = 5$.

However, in most cases the effects of various PD novations can be approximately obtained from the soft and supple novations by appropriate choices of the control parameter $C$. To elaborate, the behaviour of the PD novation with $P = 0.5$ is approximately similar to the behaviour of the soft novation with the control parameter $C$ in the range $0$ to $2$. Also, the behaviour of the PD novation with $P = 5$ is approximately similar to that of the supple novation with smaller values of $C$.

Regarding the influence of the magnitude of parameter $C$, it can be seen from the above diagrams that the behaviour of the soft conformity rule is sensitive to the magnitude of $C$ throughout its range of zero to infinity. However, the behaviour of the other conformity rules, represented by Figs A2 to A5, is only sensitive to the values of $C$ from zero to two.
EXPLORING THE CONCEPT OF NOVATION

Finally, it should be mentioned that not every set of specified relocations can be achieved within a conformity rule. A set of specified relocations should be satisfied simultaneously within the constraints of a conformity rule. When the set of specified relocations cannot be achieved within the conformity rule, then the corresponding system of simultaneous equations becomes ‘singular’. Also, when the system is near singularity then the results become inaccurate.