

SIMULATION OF SEQUENTIAL CONSTRUCTION OF EMBANKMENT BY FINITE ELEMENT METHOD

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ABSTRACT

In this paper, finite element method is used for the analysis of sequential construction of embankment. The idealized study gives information on the behavior of a multi-stage construction of an embankment over a soft cohesive deposit. If the response is determined after the construction of entire structure, it is likely to be different from the results which are obtained by considering sequential construction procedure. The displacement and stresses have been calculated during each stage of sequential construction of embankment. The developed computer program shows bearing capacity failure at final lift where the algorithm fails to converge leading to failure.

Keywords: Earthen embankment; bearing capacity failure; material non-linearity; Mohr-Coulomb and Von-Mises failure criteria; visco-plastic framework; finite element method.

1. INTRODUCTION

The main purpose of construction of earthen embankment is to hold back water in order to prevent flooding from seas, lakes, or rivers onto adjacent land and to support transportation. These typically strong barriers provide protection to lower-lying grounds by acting as a levee. Recent social and economic development around the world has brought about an increase in the construction of embankment used in highway and railway systems, flood and irrigation projects and harbor and airport installations. Staged construction can be used to construct embankments with relatively steep side slopes [1, 2]. This method relies on the increase in the undrained shear strength of the subsoil during consolidation and is therefore most beneficial when used with vertical drains. The study by He and Zhang [3] is based on Mohr-Coulomb yield and Drucker-Prager yield criteria considering equivalent area circle as yield criterion. The results showed that equivalent area circle D - P yield criterion was

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suitable for slope stability analysis and that strength reduction technology could be applied in simple homogeneous side slope stability analysis. Chen et al. [4] modeled the sequential construction of a high embankment dam over 300 m and its effect of settlement has been discussed. The simulation results have demonstrated that at least 25 layers are required to accurately model the stage construction of a high embankment dam over 300 m. Huzjak et al. [5] presented the field and laboratory data, design and construction phase analyses, construction monitoring data and conclusions about the design predictions and embankment performance during construction.

In reality, the soil/rock foundation has nonlinear, inelastic stress-strain behavior. Therefore, in order to truly asses the behavior of the soil-structure system during interaction analysis, the nonlinear material properties of the foundation should be considered. Various types of constitutive models are available to characterize the nonlinear behavior of geologic materials such as nonlinear elastic models, elasto-plastic models and elastoviscoplastic models. The hyperbolic nonlinear, elastic model proposed by Duncan and Chang [6] is very popular among the researchers throughout. The Von-Mises, Tresca, Mohr-Coulomb and Drucker-Prager models form the major part of the elasto-plastic models available for geomaterials. Ozcoban et al. [7] conducted field and laboratory tests to determine the geotechnical characteristics of the foundation soils for the construction of Alibey dam. They used elastoplastic soil model that yielded realistic predictions of field behavior in response to the complex construction history. Swan and Seo [8] developed slope stability analysis framework, wherein the soil mass is treated as a continuum and in-situ soil stresses and strengths are computed accurately using inelastic finite element methods with general constitutive models. Lin and Wei [9] analyzed one side widened embankment. They used ABAQUS software and Mohr-Coulomb constitutive model was used to set up the finite element model of the stability analysis of one side widened-embankments. Yu and Fei [10] found that the magnitude and rate of settlement are the key elements subjected to design analysis of embankments on soft ground.

The analysis of non-linear problems is far from straight forward and traditional techniques are not always sufficiently accurate to be reliable methods for design. Numerical analysis using the finite element method overcomes several of the disadvantages of traditional methods and produces predictions for displacements and stresses. Kelln et al. [11] used elastic-viscoplastic (EVP) soil model to simulate the measured deformation response of a soft estuarine soil loaded by a stage-constructed embankment. The simulation incorporates prefabricated vertical drains installed in the foundation soils and reinforcement installed at the base of the embankment. Ding et al. [12] studied a highway embankment constructed over wash pond sediments. To improve the engineering properties of the soft soils for the proposed embankment, prefabricated vertical (PV) wick drains in combination with staged construction of the embankment were implemented to accommodate the anticipated settlement and to meet the required minimum factor of safety for the embankment slope stability. A case history of road embankment over very weak clay was studied by Sura and Othman [13]. Stability analyses indicated that construction of the embankment using typical methods would lead to embankment failure. However, the analyses showed that staged construction will result in acceptable factors of safety for stability. Furthermore, the consolidation data suggested the soft clay will consolidate relatively quickly, making staged construction feasible.

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Totsev and Jellev [14] presented the comparison between two different directions in slope stability analysis for a particular example and the way these results can affect various parameters. Clough and Woodward [15] applied finite element method to evaluate embankment stresses and deformations. Construction sequence plays an important part in the deformations developed in earth embankment. Analysis demonstrates the influence of basic parameters such as material properties, size, and geometry on the stresses and deformations developed in embankments, and the effect of foundation flexibility on the embankment. Sakai et al. [16] proposed an elasto-plastic constitutive model of soil in which the tensile failure is considered. They performed couple of non-linear dynamic finite element analysis of embankment using proposed constitutive model. One is the simulation of static inclination experiment of embankment models and the other is analysis of the railway embankment structure damaged during the 1968 Tokachi-oki earthquake. Hammah et al. [17] uses the application of finite element analysis to determine the factor of safety of rock slopes, for which strength is modeled by the Generalized Hoek-Brown failure criterion.

In this particular work an attempt has been made to simulate the sequential construction procedure of earthen embankment. For that a finite element method program has been developed using constitutive non-linear model such as Von-Mises and Mohr-Coulomb. Because the stress and strain behavior of soil material is essentially non-linear in nature therefore it is important to consider a proper non-linear model with either visco-plastic or elasto-plastic framework. In this particular work, visco-plastic method has been used for simulation purpose. The program used for simulation of sequential construction of embankment leading up to bearing capacity failure is an extension of that given by Smith and Griffiths [18]. Here, results have been established for displacement by varying several soil parameters like cohesion, unit weight, tolerance limit, Poisson's ratio. Result also shows the variation of iteration limit and number of load increment. In this present work, contour plot of major and minor principal stress is presented. Also, effect of stress (compressive and tensile) has been shown in each lift. This helps to understand that in which lift the embankment suffers bearing capacity failure. The use of finite element method has been found to be very useful since it describes stress, strain and displacement behavior at every sequential loading stage. The algorithm presented here is robust in terms of runtime as well as memory allocation considerations.

2. THEORETICAL FORMULATION

2.1 Failure criteria

The embankment has been analyzed following a plane strain idealization. Several failure criteria have been proposed to represent the strength of soils as engineering materials. For soils with both frictional and cohesive components of shear strength, conical failure criteria are appropriate, which is best known to be the Mohr-Coulomb criterion. Also, Von-Mises failure criteria has also been used in this work and the results from both criteria are compared. The details of both of these failure criteria may be referred to Smith and Griffiths [18].

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2.2 Visco-plastic model for soil material

The present work uses visco-plastic framework to model the material nonlinear behaviour of the embankment soil. The detail of visco-plastic algorithm used in this work may be found in the book by Smith and Griffiths [18].

2.3 Generation of body load

According to Smith and Griffith [18], constant stiffness methods use repeated elastic solutions to achieve convergence by iteratively varying the loads on the system. Within each load increment, the system of equations

$$\begin{bmatrix} K_m \end{bmatrix} U_i^{i} = \{F\}^i \tag{1}$$

Must be solved for the global displacement increments $\{U\}^i$, where *i* represents the iteration number, $[K_m]$ the global stiffness matrix, and $\{F\}^i$ the global external and internal (body) loads. The element displacement increments $\{u\}^i$ are extracted from $\{U\}^i$, and these lead to strain increments via the element strain-displacement relationships:

$$\{\Delta \in\}^i = [B]\{u\}^i \tag{2}$$

Assuming the material is yielding, the strains will contain both elastic and (visco) plastic component, thus

$$\{\Delta \in\}^{i} = \{\Delta \in e^{i}\}^{i} + \{\Delta \in vp\}^{i}$$
(3)

It is only the elastic strain increments $\{\Delta \in e^{i}\}^{i}$ that generates stress through elastic stress-strain matrix, hence

$$\{\Delta\sigma\}^{i} = [De] \{\Delta \in e^{i}\}^{i}$$

$$\tag{4}$$

These stresses increment are added to stresses already existing from the previous load step and updated stresses substituted into the failure criterion Eq. (6). If stress redistribution is necessary (F > 0), this is done by altering the load increment vector $\{F\}^{i}$ in Eq. (16). In general, this vector holds two types of load, as given by

$$\{F\}^{i} = \{F_{a}\} + \{F_{b}\}^{i} \tag{5}$$

Where $\{F_a\}$ the actual is applied external load increment and $\{F_b\}^i$ is the body loads vector that varies from one iteration to next. The $\{F_b\}^i$ vector must be self-equilibrating so that the net loading on the system is not affected by it.

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2.4 Visco-plasticity

Accolrding to Zienkiewicz and Cormeau [19], the material is allowed to sustain stresses outside the failure criterion for finite "periods". Overshoot of the failure criterion as signified by a positive value of F, is an integral part of the method and is actually used to drive the algorithm. Instead of plastic strains, we now refer to viscoplastic strains and these are generated at a rate that is related to the amount by which yield has been violated through the expression

$$\begin{cases} \bullet^{vp} \\ \in \end{cases} = F \left\{ \frac{\partial Q}{\partial \sigma} \right\} \tag{6}$$

Where F is the yield function and Q is the plastic potential function.

It should be noted that a pseudo-viscosity property equal to unity is implied on the right hand side of Eq. (6) from dimensional considerations. Multiplication of the viscoplastic strain rate by a pseudo-time step gives an increment of viscoplastic strain which is accumulated from one "time-step" or iteration to the next;

Thus
$$\left\{ \delta \in \mathcal{V}p \right\}^i = \Delta t \left\{ \in \mathbb{R} \right\}^i$$
 (7)

and
$$\{\Delta \in {}^{\nu p}\}^{i} = \{\Delta \in {}^{\nu p}\}^{i-1} + \{\delta \in {}^{\nu p}\}^{i}$$
 (8)

The body loads $\{F_b\}^i$ are accumulated at each "time-step" within each load step by summing the following integrals for all elements containing a yielding (F > 0) Gauss point:

$$\{F_b\}^i = \{F_b\}^{i-1} + \sum_{elements}^{all} \iint [B]^T [D^e] \{\Delta \in {}^{vp}\}^i dxdy$$
(9)

This process is repeated at each "time-step" iteration until no integrating point stresses violate the failure criterion within the tolerance. The convergence criterion is based on a dimensionless measure of the amount by which the displacement increment vector $\{U\}^i$ changes from one iteration to the next.

2.5 Gravity loading

The forces generated by the self-weight of the soil are computed by using a procedure involving integrals over each element of the form:

$$p(e) = \gamma \int_{V^e} [N]^T dV^e \tag{10}$$

Where [N] is the shape of the functions of the element and the superscript *e* refers to the element number. This integral evaluates the volume of each element, multiplies by the total

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unit weight of the soil and distributes the net vertical force consistently to all the nodes. These element forces are assembled into a global gravity force vector that is applied to the finite element mesh in order to generate the initial stress state of the problem.

The present work applies gravity in a single increment to an initially stress-free slope. Others have shown that under elastic conditions, sequential loading in the form of incremental gravity application or embanking, affects deformations but not stresses [15]. In nonlinear analyses it is recognized that the stress paths followed due to sequential excavation may be quite different to those followed under a gravity "turn-on" procedure, however the factor of safety appears unaffected when using simple elasto-plastic models.

2.6 Computation of stresses

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In finite element analysis, once the nodal displacements of an element are obtained, the stresses $\{\sigma\}$ can be calculated as

$$\{\sigma_e\} = [D][B]\{q_e\} \tag{11}$$

where [D] is the constitutive matrix, [B] is the strain displacement matrix, $\{\sigma_e\}$ is the elemental stress vector and $\{q_e\}$ is the elemental displacement vector. The stresses $\{\sigma_e\}$ at any point inside an element are estimated by interpolating the stresses obtained at sampling points. Unfortunately, while determining the stresses at the nodal points, it has been found that they are discontinuous in nature. Efforts to calculate stresses directly at the nodal points have proved them to be very bad sampling points [20], though the nodes are most useful points for output and interpretation of stresses. For two dimensional isoparametric elements, the 2×2 Gauss points have been found out to be the optimal sampling points [21]. The nodal point stresses are calculated as per the method suggested by Hinton and Campbell [22]. Major and minor principal stresses are calculated as follows:

$$\boldsymbol{\sigma}_{1} = \frac{\boldsymbol{\sigma}_{x}^{+} \boldsymbol{\sigma}_{y}}{2} + \sqrt{\left(\frac{\boldsymbol{\sigma}_{x}^{-} \boldsymbol{\sigma}_{y}}{2}\right)^{2} + \tau^{2}}$$
(12)

$$\sigma_{2} = \frac{\sigma_{x} + \sigma_{y}}{2} - \sqrt{\left(\frac{\sigma_{x} - \sigma_{y}}{2}\right)^{2} + \tau^{2}_{xy}}$$
(13)

2.7 Degeneration of quadrilateral elements into triangular elements

At the left portion of the embankment, where shape of the ground is inclined, it is necessary that finite element chosen should also represent in proper diagram. So 8-noded elements are converted into triangular element. In whole domain, 8-noded elements are used. But at the inclination 8-noded element has been squeezed in type shown in Fig. 1. In Fig. 1, the quadrilateral element is numbered from 1 to 8 in clockwise manner. These are being squeezed to a triangular element. So the node numbers 2, 3 and 4 will assemble on the hypotenuse.



Figure 1. Nodal positions for elements on the sloping side

2.8 Staged construction of embankment

Engineering structures are usually constructed in a definite sequence of operations. A conventional linear analysis of such structures is performed by assuming that the entire construction takes place in single operation. In other words stresses and deformation are computed by considering loads on completed structures. However, for the non-linear problems typical in soil and foundation engineering, the behavior of soil at a particular stage of loading is dependent upon the state of stress and stress history. Thus the stresses in final configuration are dependent upon the sequence of intermediate configurations and loadings.

Gravity is one of the main agencies causing deformation and it is common to employ gravity turn-on as the loading mechanism. In "Numerical Results" part (section 3.8), the contour plots of major and minor principal stresses for the embankment are shown which clearly depicts sequential construction procedure of the embankment. The embankment is assumed to be raised in a series of lifts, the first of which merely stresses the foundation block gravitationally under at rest condition.

3. NUMERICAL RESULTS

3.1 Selection of an optimum mesh size

The stress-strain behaviour of the earthen embankment is non-linear in nature which is dealt with visco-plastic method. The dimensions of the earthen embankment are shown in Fig. 2. The material properties of earthen embankment are: unit weight of soil = 20 kN/m^3 , dilation angle = 0^0 , angle of friction = 0^0 , cohesion = 14 kN/m^2 , Poisson's ratio = 0.49, modulus of elasticity = 10^5 kN/m^2 . The parameters are initially same as those considered by Smith and Griffiths [18]. The bottom boundary nodes are attached to hinges as shown in Fig. 2. Also all the right and left side nodes are provided with rollers support.

In order to investigate the response of different parameters like (displacement, stress and loading), an analysis of staged construction of an earthen embankment has been done. Therefore, a solution with minimum error and discretization and with maximum accuracy is sought by selecting an economic number of divisions. Satisfactory solution may be arrived at either by (i) selecting the required number of subdivisions of the continuum from past experience or (ii) if an analysis is attempted for the first time, the convergence can be tested based on varied mesh grading and thereafter making a suitable choice. In this analysis body loads are being increased by increasing the number of lifts.



Figure 2. Geometry and Finite Element Discretization of Earthen Embankment

3.2 Effect of number of load increments

The maximum displacement is occurring at the final lift when the embankment has failed to achieve equilibrium within a specified number of iterations. Five lifts are applied one after each other to reach the final height of the embankment before it fails. For each of the lift, there are provisions of applying several load increments in the program designated by the parameter INCS. The value of displacement is plotted at point 'A' as shown in Fig. 2 has been shown in Table 1 when the embankment has failed to achieve equilibrium. It is observed that the values of displacements for both the yield criterion (i.e. Mohr-Coulomb and Von-Mises) at failure are nearly same used for representing the inclined slope of the embankment.

The displacement obtained before failure is achieved at node designated by point 'A' (as shown in Fig. 2). The displacements at failure are nearly same for both Von-Mises and Mohr-Coulomb criteria. However, it has to be understood that when the algorithm has failed to converge within specified iteration limits, it only indicates that the failure has occurred. As the nonlinear algorithm is basically incremental in nature, the displacements, stresses and strains will increase manifold if the algorithm has not converged, as observed in lift 5, the displacements, stresses and strains obtained will not be correct. Only thing that can be said for sure is the structure has failed to reach the state of equilibrium indicating failure. However, the displacements and stresses and strains obtained in lift 4, where algorithm has converged, indicates correct values.

Table 1: Effect of change in load increment				
Number of		Maximum	Maximum	
Load	Failure criteria	displacement (m)	displacement (m)	
increment		at failure at lift 5	before failure at lift 4	
2	Mohr-Coulomb	1.1377×10^{-1}	3.3105×10^{-4}	
3	Von-Mises	1.1379×10^{-1}	3.3113×10^{-4}	

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4	Mohr-Coulomb Von-Mises	1.1328×10^{-1} 1.1329×10^{-1}	$\begin{array}{c} 2.5031 \times 10^{-4} \\ 2.5035 \times 10^{-4} \end{array}$
5	Mohr-Coulomb Von-Mises	1.1259×10^{-1} 1.1261×10^{-1}	$\begin{array}{c} 2.0194 \times 10^{-4} \\ 2.0203 \times 10^{-4} \end{array}$

3.3 Effect of variation of iteration limit on embankment

In this section, iteration limit is changed and change in displacement is observed. Table 2 shows the numerical values of maximum displacement at failure and at the previous lift when the algorithm has converged for different values of iteration. From Table 3, it is clear that before failure the values are converging for both the yield criteria i.e. Von-Mises and Mohr-Coulomb yield criteria. But at lift 5, the algorithm is failing to converge and the value of displacement is increasing rapidly with great magnitude indicating failure of the embankment. If the iteration limit is taken equal to 5000, the maximum displacement increases to 1.0704 m for Mohr-coulomb yield criteria and approximately same for Von-Mises yield criteria. As the algorithm fails to converge for iteration limit 500, it is considered to be sufficient to work with iteration limit equal 500.

Table 2: Effect of displacement and iteration limit

Iteration limit	Failure criteria	Maximum displacement (m) before failure at lift 4	Maximum displacement (m) at failure at lift 5
mmt	Mohr-Coulomb	2.0194×10^{-4}	1.1259×10^{-1}
500	Von-Mises	2.0203×10^{-4}	1.1261×10^{-1}
1000	Mohr-Coulomb	2.0194×10^{-4}	2.2105×10^{-1}
1000	Von-Mises	2.0232×10^{-4}	2.2107×10^{-1}
5000	Mohr-Coulomb	2.0194×10^{-4}	1.0704
5000	Von-Mises	2.0203×10^{-4}	1.0705

3.4 Effect of variation of tolerance limit on embankment

In this section tolerance limit has been changed and change in displacement has been analyzed. Table 3 shows the numerical values of maximum displacement at failure and at the previous lift when the algorithm has converged for changing values of tolerance limit. It is seen that if the tolerance limit is reduced below 0.0001, the obtained displacements are converging. Therefore, 0.0001 has been safely considered as convergence limit for the present work.

Table 3: Effect of displacement and tolerance limit

Tolerance	Failure criteria	Maximum displacement	Maximum displacement
Limit	i anure enteria	before failure at lift 4 (m)	at failure at lift 5 (m)
0.001	Mohr-Coulomb	2.0195×10^{-4}	1.1263×10^{-1}
0.001	Von-Mises	2.0203×10^{-4}	1.1264×10^{-1}
0.0001	Mohr-Coulomb	2.0194×10^{-4}	1.1259×10^{-1}
	Von-Mises	2.0203×10^{-4}	1.1261×10^{-1}
0.00001	Mohr-Coulomb	2.0194×10^{-4}	1.1259×10^{-1}
	Von-Mises	2.0203×10^{-4}	1.1260×10^{-4}

3.5 Effect of variation of cohesion

Table 4 shows the numerical values of maximum displacement at failure which occurs at lift 5 at point 'A' as shown in Fig. 2. Also, the values of maximum displacement at the previous lift when the algorithm has converged for different values of cohesion. Here, the values of cohesion considered are 14 kN/m², 20 kN/m², 25 kN/m²and 50 kN/m² respectively. If material properties are improving, then we find that displacement is decreasing. Also in some cases i.e. c = 20 kN/m² to 25 kN/m², failure might not occur as evident from the result because the algorithm converges before the specified iteration limit. The value of displacement obtained at lift 5 is not correct as the algorithm fails to converge when the analysis is carried out with cohesion value of 14 kN/m². But, for higher value of cohesion, the algorithm might converge before the specified iteration limit of 500. Also, difference between both the yield criteria i.e. Mohr-Coulomb and Von-Mises yield criteria is practically negligible. It is observed that the maximum displacements at failure are occurring at point 'A' in the Fig. 2. The major and minor principal stresses are also tabulated at the moment of failure at point 'A' as shown in Table 5.

Table 4:	Effect of	cohesion	on disj	placement	behavior

Cohesion (kN/m ²)	Failure criteria	Maximum displacement (m) before failure at lift 4	Maximum displacement (m) at failure at lift 5	Major principal stress at failure (kN/m ²) (lift=5)	Minor principal stress at failure (kN/m ²) (lift=5)
14	Mohr-Coulomb	2.0194×10^{-4}	1.1259×10^{-1}	24.9813	-4.9919
14	Von-Mises	2.0203×10^{-4}	1.1261×10^{-1}	24.9864	-4.9873
20	Mohr-Coulomb	1.7158×10^{-4}	2.0297×10^{-4}	7.7958	0.0165
20	Von-Mises	1.7158×10^{-4}	2.0299×10^{-4}	7.7962	0.0165
25	Mohr-Coulomb	1.7158×10^{-4}	2.0274×10^{-4}	7.6687	0.0181
	Von-Mises	1.7158×10^{-4}	2.0274×10^{-4}	7.6563	0.0181
50	Mohr-Coulomb	1.7158×10^{-4}	2.0274×10^{-4}	7.6687	0.0181
	Von-Mises	1.7158×10^{-4}	2.0274×10^{-4}	7.6687	0.0181

It is being observed that for cohesion value equal to 20 kN/m^2 , the algorithm converges at lift 5 before specified iteration count (INCS). For a value of cohesion equal to 14 kN/m^2 , the maximum displacement obtained at lift 5 is 0.11259 m, which is too high for the soil material to sustain without cracking. This large value of displacement is also indicative of the failure of the soil mass. As the cohesion values increases, it is observed that failure does not occur even at lift 5. The corresponding values of displacements, major and minor principal stresses at point 'A' are comparatively very less to that obtained for cohesion value of 14 kN/m^2 .

The variation of maximum displacement with cohesion obtained at the end of last lift (lift=5) for both failure criteria Mohr-Coulomb and Von-Mises are shown in graphical form, represented by Fig. 3. It is shown that in final lift the displacement has decreased by very large extent with the increase of cohesion. From Fig. 3, it is clear that for cohesion = 14 kN/m^2 , failure has occurred at iteration limit 500 (iters = 2, 2 designate two iterations are required to achieve convergence for Mohr-Coulomb and Von-Mises criteria). But for other cohesion values, convergence has been achieved, as they are failing before iteration limit 500. For Fig.

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3 to Fig. 6, the points are plotted along with the numbers of iterations required for Mohr-Coulomb and Von-Mises material to achieve convergence for the algorithm. The displacement obtained at the final lift for the cohesion = 14 kN/m^2 is very large compared to those obtained for other values of cohesion. That is why, the plot seems to align with x axis even though the displacements obtained at cohesion values other than 14 kN/m^2 are non-zero.



Figure 3. Displacement vs. cohesion at failure

The variation of major and minor principal stresses with cohesion obtained at the end of last lift (at lift 5) for both failure criteria Mohr-Coulomb and Von-Mises are shown in graphical form, represented by Fig. 4 and Fig. 5 respectively. From all these figures, it is observed that in final lift the major principal stress has increased by very large extent. The major principal stresses occurring up to lift 4 are several times lower in magnitude from that occurring at lift 5. However, if the cohesion increases, the major and minor principal stresses also decreases. From Fig. 4 and Fig. 5, it is also observed that for higher values of cohesion other than 14 kN/m², the algorithm converges in two iterations as indicated by the term iters = 2, 2 for both Mohr-Coulomb and Von-Mises criteria. This also indicates that the failure does not take place when the cohesion increases. Therefore, the values obtained for displacements, major and minor principal stresses are converged and correct values for higher values of cohesion.



The variation of maximum displacement with cohesion obtained at lift 4 (i.e. just before failure) for failure criteria Mohr-Coulomb and Von-Mises are shown in graphical form, represented by Fig. 6. It is being shown that in lift 4 the displacement has decreased. The displacement occurring at lift 4 is decreasing as cohesion is being increased.

3.6 Effect of variation of unit weight on embankment

In this section, the displacements with respect to changing values of unit weight are investigated keeping all the other material parameters same as mentioned in section 3.1. Table 5 shows the numerical values of maximum displacement at failure and at the previous lift when the algorithm has converged for changing values of unit weight of soil. For lower values of unit weight of soil, it is seen that the algorithm reaches equilibrium indicating no failure. The displacements observed at the final lift are also small. But, when the unit weight of the soil is considered to be 20 kN/m^2 , the displacement suddenly increases to very high value indicating failure has occurred.

Table 5: Effect of displacement and unit weight				
Unit weight	Failure criteria	Maximum displacement (m)	Maximum displacement	
(KN/m^2)		before failure at lift 4	(m) at failure at lift 5	
16	Mohr-Coulomb	1.3726×10^{-4}	2.9950×10^{-4}	
10	Von-Mises	1.3726×10^{-4}	3.0121×10^{-4}	
17	Mohr-Coulomb	1.4720×10^{-4}	5.7744×10^{-4}	
	Von-Mises	1.4721×10^{-4}	5.7958×10^{-4}	
19	Mohr-Coulomb	1.7934×10^{-4}	4.9059×10^{-3}	
	Von-Mises	1.7943×10^{-4}	4.9164×10^{-3}	
20	Mohr-Coulomb	2.0194×10^{-4}	1.1259×10^{-1}	
	Von-Mises	2.0203×10^{-4}	1.1261×10^{-1}	

Table 5: Effect of displacement and unit weight

3.7 Effect of variation of poisson's ratio on embankment

Table 6 shows the numerical values of maximum displacement at failure and at the previous lift (lift 4) when the algorithm has converged for different values of Poisson's ratio of soil for both the yield criterions. It is being shown that in lift 4 (i.e. before failure) the displacement is decreasing. The displacement occurring at lift 4 is decreasing as Poisson's ratio is being increased. It is being observed that the displacements are increasing with the reduction of Poisson's ratio values. As the Poisson's ratio is the ratio of lateral strain and longitudinal strain, these results indicate that the lateral strain is increasing with the decrease of Poisson's ratio values of the material.

Table 6: Effect of displacement and poisson's ratio

Poisson's Ratio	Failure criteria	Maximum displacement(m) at just before failure at lift 4	Maximum displacement (m) at failure. at lift 5	
0.25	Mohr-Coulomb	1.3421×10^{-3}	1.2387×10^{-2}	
	Von-Mises	1.3284×10^{-3}	1.3326×10^{-2}	
0.30	Mohr-Coulomb	1.1335×10^{-3}	1.1501×10^{-2}	
	Von-Mises	1.1460×10^{-3}	1.2180×10^{-2}	

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0.25	Mohr-Coulomb	9.0542 10-4	1.0282 10-2		
0.55	Von-Mises	9.4865×10^{-4}	1.0806×10^{-2}		
0.40	Mohr-Coulomb	6.3572×10^{-4}	8.6544×10^{-3}		
0.40	Von-Mises	7.2175×10^{-4}	9.1173×10^{-3}		
0.45	Mohr-Coulomb	3.1943×10^{-4}	1.1280×10^{-1}		
0.45	Von-Mises	3.4577×10^{-4}	1.1293×10^{-1}		
0.40	Mohr-Coulomb	2.0194×10^{-4}	1.1259×10^{-1}		
0.49	Von-Mises	2.0203×10^{-4}	1.1261×10^{-1}		

The variation of maximum displacement with Poisson's ratio obtained at the end of lift 4 (before failure) for both failure criteria Mohr-Coulomb and Von-Mises are shown in graphical form, represented by Fig. 7. It is observed that in lift 4 (i.e. before failure) the displacement is decreasing. The displacement occurring at lift 4 is decreasing as Poisson's ratio is being increased.



Figure 7. Displacement vs. unit weight at lift 4 (before failure at lift 4)

3.8 Effect of variation of young's modulus

To investigate the effect of Young's modulus on the behaviour of the embankment, the value of the Young's modulus is varied and the corresponding displacements at point 'A' (as shown in Fig. 2) at the final lift as well as at the previous lifts are calculated. The values are provided in Table 7. The data reflect that displacement values reduce as the Young's modulus of soil increases. The displacements at lift 5 are found to be much higher than that obtained at lift 4. It is because the algorithm has failed to converge at the final lift indicating bearing capacity failure of the embankment has taken place. As the algorithm has failed to converged within a specified limit of iterations, the displacements have increased manifold compared to that obtained at previous lift. However, these displacements are not correct values because convergence of algorithm has not been achieved. Only inference that may be drawn is that the embankments has failed.

Young's Modulus (kN/m ²)	Failure criteria	Maximum displacement (m) before failure at lift 4	Maximum displacement (m) at failure at lift 5
1×10^{5}	Mohr-Coulomb	2.0194×10^{-4}	1.1259×10^{-1}
$1 \times 10^{\circ}$	Von-Mises	2.0203×10^{-4}	1.1261×10^{-1}
2×10^{5}	Mohr-Coulomb	1.0097×10^{-4}	5.6299×10^{-2}
	Von-Mises	1.0101×10^{-4}	5.6305×10^{-2}
3×10^{5}	Mohr-Coulomb	6.7314×10^{-5}	3.7533×10^{-2}
	Von-Mises	6.7394×10^{-5}	3.7537×10^{-2}
4×10^{5}	Mohr-Coulomb	5.0486×10^{-5}	2.8149×10^{-2}
	Von-Mises	5.0508×10^{-5}	2.8152×10^{-2}

Table 7: Effect of displacement and young's modulus

3.9 Effect of variation of lifts on embankment

In this section the contour plots of major and minor principal stresses (kN/m^2) are plotted for the embankment cross section. For yield criteria Mohr-Coulomb and Von-Mises yield criteria, it is noticed that the nature and the magnitude of principal stresses are very close to each other. To avoid repetition of similar contour plots, the plots are drawn only for Mohr-Coulomb criteria. Fig. 8 and Fig. 9 show the contour plots for major and minor principal stress in lift 1. When no external loading is present as in the case of lift 1, the entire soil mass is loaded with at rest earth pressure (K_0) , which is evident from Fig. 8 and Fig. 9. It is seen that the value of compressive stress increases with depth. Since the bottom portion of embankment experiences maximum compressive stresses. Fig. 10 and Fig. 11 shows the contour plots for major and minor principal stress in lift 2. This two figures shows the nature of the principal stresses when the second lift is applied. From Fig. 10, it is observed that the magnitude of compressive stress reduces in the left portion. However, the bottom of the embankment is still under the influence of compressive stress. Fig. 12 and Fig. 13 show the contour plots for major and minor principal stress in lift 3. Here also the nature of stress is similar as before. Fig. 14 and Fig. 15 show that the nature of the principal stresses is compressive throughout the embankment in lift 4. Fig. 16 and Fig. 17 shows the contour plots for major and minor principal stress in lift 5. Fig 16 shows that the top right most portion of the embankment is experiencing tensile stress. Since soil is unable to withstand tensile stress, this portion is likely to crack leading to failure. This observation tallies well with the results of the program which shows that the embankment fails in lift 5. In order to validate the results, slope stability analysis of the embankment is also carried out at all the lifts. The slope stability analysis at all the lifts are carried out using the finite element procedure as stated in Smith and Griffiths (2004). Also, Bishop's simplified method (limit equilibrium method) is used to calculate the factor of safety of the embankment at each lift against failure with the help of SLOPEW software. The results are provided in Table 8. The values of factor of safeties obtained from both the types of analysis indicate that the embankment fails at lift 5. This serves as another vindication of the bearing capacity failure analysis carried out in this work.











Figure 10. Contour for major principal stress (kN/m^2) at lift 2







Figure 12. Contour for major principal stress (kN/m²) at lift 3



Figure 13. Contour for minor principal stress (kN/m²) at lift 3







Figure 15. Contour for minor principal stress (kN/m^2) at lift 4







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Figure 17. Contour for minor principal stress (kN/m²) at lift 5

Lift no	Factor of safety against failure using	Factor of safety against failure using Bishop's
LIII IIO.	finite element method	simplified method using SLOPW software
1	Very high (>10)	Very high (>10)
2	3.70	3.92
3	1.89	2.22
4	1.27	1.32
5	0.95	0.99

Table 8: Factor of safeties of the embankment against failure at each lift

4. CONCLUSION

The paper presents a methodology for the analysis of sequential construction of earthen embankment. The proposed method is validated from the literature which shows the accuracy of the developed algorithm. The analysis is carried out in a visco-plastic framework which effectively simulates the failure mechanism during sequential construction of embankments. In the present work two failure criteria namely Mohr-Coulomb and Von-Mises have been used to the study sequential construction of earthen embankment. The results obtained for both the failure criteria are found to be very close. Several results have been obtained by changing the material properties. If values of cohesion are increased, it is found that at failure, the values of displacement have decreased. If the cohesion is too high, failure might not occur because the algorithm converged before the specified iteration limit. So for a higher value of cohesion, the algorithm may converge before specified iteration limit indicating a state of equilibrium has been reached for the soil mass. When the value of unit weight is increased, displacement also increases. The contour plots of major and minor principal stresses drawn at each lift clearly indicates tension is developed at the time of failure at the final lift. This fact is also validated by the slope stability analysis performed at each lift which clearly shows that the slope stability failure occurs at the final lift. The idealized study gives information on the behavior of a multi-stage construction of an embankment over a soft cohesive deposit.

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