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CBO AND DPSO FOR OPTIMUM DESIGN OF REINFORCED CONCRETE CANTILEVER RETAINING WALLS

A. Kaveh[1,2*] and N. Soleimani[1]
[1]Centre of Excellence for Fundamental Studies in Structural Engineering, School of Civil Engineering, Iran University of Science and Technology, Tehran, P.O. 16846-13114, Iran
[2]Guest Professor, Institute for Mechanics of Materials and Structures, TU Wien, Karlsplatz 13, A-1040 Wien, Austria

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ABSTRACT

This paper reports on optimal design of reinforced concrete cantilever retaining walls of a given height under static and earthquake loading conditions utilizing Colliding Bodies Optimization (CBO) and Democratic Particle Swarm Optimization (DPSO). The design is based on ACI 318-05. Two theories known as Coulomb and Rankine have been applied for estimating earth pressures under static loading condition and Mononobe-Okabe method have been applied for estimating earth pressures under dynamic loading condition. The objective function is the cost of materials used in retaining walls. This function is minimized subjected to the considered constraints. A numerical example is optimized to illustrate the performance of the CBO and DPSO algorithms compared with Particle Swarm Optimization (PSO) and Improved Harmony Search (HIS) algorithms.

Keywords: Colliding bodies optimization; democratic particle swarm optimization; reinforced concrete cantilever retaining wall; coulomb and rankin theory; mononobe-okabe method.

1. INTRODUCTION

Earth retaining structures are designed and constructed to provide lateral support to vertical slopes of soils. These structures are constructed for a variety of applications, most commonly in the construction of roads, canals, bridge abutments, transportation systems and other constructed facilities. Some examples of these structures are cantilever and gravity retaining walls. Cantilever retaining walls are constructed of reinforced concrete. These structures consist of a relatively thin stem and a base slab. The base is also divided into two parts, the heel and toe. The heel is the part of the base under the backfill and the toe is the other part of the base.

*E-mail address of the corresponding author: alikaveh@iust.ac.ir (A. Kaveh)

In order to obtain a design with economic cost in minimum time, optimization methods must be used. Some studies have been made in this field by Dembicki and Chi [1], Keskar and Adidam [2], Saribas and Erbatur [3], Rhomberg and Street [4], Basudhar and Lakshman [5], Sivakumar and Munwar [6], and Yepes [7]. In recent years Kaveh and Shakouri [8] employed harmony search algorithm for optimization of cantilever retaining walls, and Kaveh and Behnam [9] used Charged System Search algorithm for optimization of cantilever retaining walls. In both of these works the wall designed under static loading condition utilizing Coulomb theory.

Recently, two new techniques known as Democratic Particle Swarm Optimization and Colliding Bodies Optimization are developed for optimization problems. Democratic Particle Swarm Optimization is proposed by the work of Kaveh and Zolghadr [10] in order to improve the exploration capabilities of the PSO and thus to address the problem of premature convergence. Colliding Bodies Optimization algorithm, proposed in the work of Kaveh and Mahdavi [11-12], is based on collision between two objects in one-dimension, in which one object collide with other object and they moves toward minimum energy level. This algorithm was enhanced by Kaveh and Ilchi Ghazaan [13]. DPSO can alleviate the premature convergence of the PSO by enhancing the performance of the algorithm in two ways: 1) helping the agents to receive information about good regions of the search space other than those experienced by themselves and the best particle of the swarm and 2) letting some bad particles take part in the movement of the swarm and thus improving the exploration capabilities of the algorithm. In this new technique a term is added to velocity vector of PSO which represents the democratic effect of the other particles of the swarm on the movement on the desired particle. In CBO algorithm, each CB is considered as an object with a specified mass and velocity before the collision. After collision occurs, each CB moves to a new position according to the new velocity. This process is repeated until a termination criterion is satisfied and the optimum CB is found.

In this study, the DPSO, CBO, IHS and PSO algorithms are used to determine the optimum design of reinforced concrete cantilever retaining walls. The objective function considered is taken as the cost of structure. This function is minimized subjected to strength and stability constraints. The design is performed under static loading condition utilizing Rankine and Coulomb theories, and under different earthquake loading conditions utilizing Mononobe-Okabe method. A numerical example is presented in order to illustrate the performance of the present algorithms.

2. DPSO ALGORITHM

The method consists of five basic steps. Detailed explanation of these steps can be found in the work of Kaveh and Zolghadr [10] which are summarized in the following:

Step 1. Select the value of the DPSO parameters.

Step 2. Select random values for particles. In this study, each particle has seven variables including: thickness of top stem $(T1)$, the thickness of key and stem $(T2)$, the toe width $(T3)$, the heel width $(T4)$, the height of top stem $(T5)$, the footing thickness $(T6)$, and the key depth $(T7)$ selected randomly between the lower limit and upper limit of each variable.

Step 3. The algorithm calculate required reinforcement for these dimensions and then checks the wall for stability and strength constraints if these dimensions satisfy these constraints the objective function will be calculated otherwise the penalty function will be calculated and multiply to objective function to penalize it.

Step 4. Calculate vector D_i which represents the democratic effect of the other particles of the swarm on the movement of the i th particle to achieve $d_{i,j}$ (the j th variable of the vector D for the i th particle) which is required for next step.

$$E_{ik} = \begin{cases} 1 & \dfrac{obj(k) - obj(i)}{obj_{worst} - obj_{best}} \rangle rand \vee obj(k) \langle obj(i) \\ 0 & else \end{cases} \tag{1}$$

$$Q_{ik} = \frac{E_{ik} \dfrac{obj_{best}}{obj_{(k)}}}{\sum_{k=1}^{n} E_{ik} \dfrac{obj_{best}}{obj_{(k)}}} \tag{2}$$

$$D_i = \sum_{k=1}^{n} Q_{ik} (X_k - X_i) \tag{3}$$

Step 5. Update particles' velocities by the use of $d_{i,j}$ then update the particles' positions for the next iteration of the search.

$$v_{i,j}^{k+1} = \chi[\omega v_{i,j}^{k} + c_1 r_1 (xlbest_{i,j}^{k} - x_{i,j}^{k}) + c_2 r_2 (xgbest_{j}^{k} - x_{i,j}^{k}) + c_3 r_3 d_{i,j}^{k}] \tag{4}$$

$$x_{i,j}^{k+1} = x_{i,j}^{k} + v_{i,j}^{k+1} \tag{5}$$

where, $v_{i,j}^{k}$ is the velocity or the amount of change of the design variable j of particle i, $x_{i,j}^{k}$ is the current value of the j th design variable of the i th particle, $xlbest_{i,j}^{k}$ is the best value of the design variable j ever found by i th particle, $xgbest_{i,j}^{k}$ the best value of the design variable j experienced by the entire swarm so far, r_1 and r_2 are two random numbers uniformly distributed in the range (1,0), c_1 and c_2 are two parameters representing the particle's confidence in itself and in the swarm, respectively. Parameter c_3 is for controlling the weight of democratic vector. Here, ω is the inertia weight for the previous iteration's velocity and it can be set in order to control the exploration of the algorithm. The χ parameter is used to avoid divergence behavior.

Step 6. Repeat Steps 3 to 5 until the termination criterion is satisfied. Therefore we can find the optimum objective function without any penalties.

3. CBO ALGORITHM

The method consists of three levels and each level has some steps. Detailed explanation of these steps can be found in the work of Kaveh and Mahdavi [11-12]. The levels can briefly be outlined as follows:

Level 1: Initialization

Step 1. The initial positions of CBs (thickness of top stem $(T1)$, the thickness of key and stem $(T2)$, the toe width $(T3)$, the heel width $(T4)$, the height of top stem $(T5)$, the footing thickness $(T6)$, and the key depth $(T7)$) are determined with random initialization of a population of individuals in the search space:

$$x_i^0 = x_{\min} + rand(x_{\max} - x_{\min}), \qquad i = 1,2,...,n \tag{6}$$

where, x_i^0 determines the initial value vector of the ith CB. $x_{\min}$ and $x_{\max}$ are the minimum and the maximum allowable values vectors of variables; rand is a random number in the interval [0,1]; and *n* is the number of CBs and in this paper it is set to 30.

Level 2: Search

Step 1. The magnitude of the body mass for each CB is defined as:

$$m_k = \frac{\frac{1}{fit(k)}}{\sum_{i=1}^{n} \frac{1}{fit(i)}}, \qquad k = 1,2,...,n \tag{7}$$

where, $fit(i)$ represents the objective function value of the agent i; n is the population size. Obviously a CB with good values exerts a larger mass than the bad ones. Also, for maximizing the objective function the term $\frac{1}{fit(i)}$ is replaced by $fit(i)$.

Step 2. The arrangement of the CBs objective function values is performed in ascending order. The sorted CBs are equally divided into two groups:

(1) The lower half of CBs (stationary CBs); These CBs are good agents which are stationary and the velocity of these bodies before collision is zero. Thus:

$$v_i = 0, \qquad i = 1,...,\frac{n}{2} \tag{8}$$

(2) The upper half of CBs (moving CBs): These CBs move toward the lower half. Then the better and worse CBs of each group will collide together. The change of the body position represents the velocity of these bodies before collision as:

$$v_i = x_i - x_{i-\frac{n}{2}}, \qquad i = \frac{n}{2}+1,...,n \tag{9}$$

where v_i and x_i are the velocity and position vector of the i th CB in this group, respectively; $x_{i-\frac{n}{2}}$ is the i th CB pair position of x_i in previous group.

Step 3. After the collision, the velocity of bodies in each group is evaluated using Eq. (10), Eq. (11) and the velocities before collision. The velocity of each moving CB after the collision is:

$$v'_i = \frac{(m_i - \varepsilon m_{i-\frac{n}{2}})v_i}{m_i + m_{i-\frac{n}{2}}}, \qquad i = \frac{n}{2}+1,...,n \tag{10}$$

where, v_i and v'_i are the velocity of the i th moving CB before and after the collision, respectively; m_i is the mass of the i th CB; $m_{i-\frac{n}{2}}$ is mass of the i th CB pair. Also, the velocity of each stationary CB after the collision is:

$$v'_i = \frac{(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}})v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}}, \qquad i = 1,...,\frac{n}{2} \tag{11}$$

where, $v_{i+\frac{n}{2}}$ and v'_i are the velocity of the i th moving CB pair before and the i th stationary CB after the collision, respectively; m_i is mass of the i th CB; $m_{i+\frac{n}{2}}$ is mass of the i th moving CB pair. As mentioned previously, ε is the coefficient of restitution (COR) and for most of the real objects, its value is between 0 and 1. It defined as the ratio of the separation velocity of two agents after collision to the approach velocity of two agents before collision. In the present algorithm, this index is used to control of the exploration and exploitation rate. For this goal, the COR is decreases linearly from unit to zero. Thus, ε is defined as:

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \tag{12}$$

where, $iter$ is the actual iteration number and $iter_{\max}$ is the maximum number of iterations.

Step 4. New positions of CBs are obtained using the generated velocities after the collision in position of stationary CBs.

The new positions of each moving CB is:

$$x_i^{new} = x_{i-\frac{n}{2}} + rand \circ v'_i , \qquad i = \frac{n}{2} + 1, ..., n \tag{13}$$

Where, x_i^{new} and v'_i are the new position and the velocity after the collision of the i th moving CB, respectively; $x_{i-\frac{n}{2}}$ is the old position of the i th stationary CB pair. Also, the new positions of stationary CBs are obtained by:

$$x_i^{new} = x_i + rand \circ v'_i , \qquad i = 1, ..., \frac{n}{2} \tag{14}$$

where, x_i^{new} , x_i and v'_i are the new position, old position and the velocity after the collision of the i th stationary CB, respectively. rand is a random vector uniformly distributed in the range (-1,1) and the sign ''∘'' denotes an element-by-element multiplication.

Level 3: Terminating criterion control

The optimization is repeated search level steps until a termination criterion, specified as the maximum number of iteration, is satisfied.

4. DESIGN VARIABLES OF THE PROBLEM

The continuous design variables utilized in this study are illustrated in Fig. 1. These variables are about dimensions of the wall which consist of the thickness of top stem $(T1)$, the thickness of key and stem $(T2)$, the toe width $(T3)$, the heel width $(T4)$, the height of top stem $(T5)$, the footing thickness $(T6)$, and the key depth $(T7)$.

5. OBJECTIVE FUNCTION

By minimizing a suitable cost function, we can reach to an optimum solution for a concrete cantilever retaining wall. In this problem similar to Kaveh and Shakouri Mahmud Abadi [8] and Kaveh and Behnam [9] the objective function is considered as following:

$$Q = V_{conc} \times (C1 + C2) + W_{steel} \times (C3 + C4) \tag{15}$$

By considering $\overline{Q} = Q/(C1 + C2)$, it is converted to:

$$\overline{Q} = V_{conc} + W_{steel} (\frac{C3 + C4}{C1 + C2}) \tag{16}$$

where V_{conc} and W_{steel} are the volume of concrete and the weight of reinforcement steel in the unit of length (ft^3/ft or m^3/m, lb/ft or kg/m), $C1$ and $C2$ are the cost of the concrete and steel ($\$/lb$ or $\$/kg$), $C3$ and $C4$ are the cost of concreting and erecting reinforcement ($\$/lb$ or $\$/kg$).

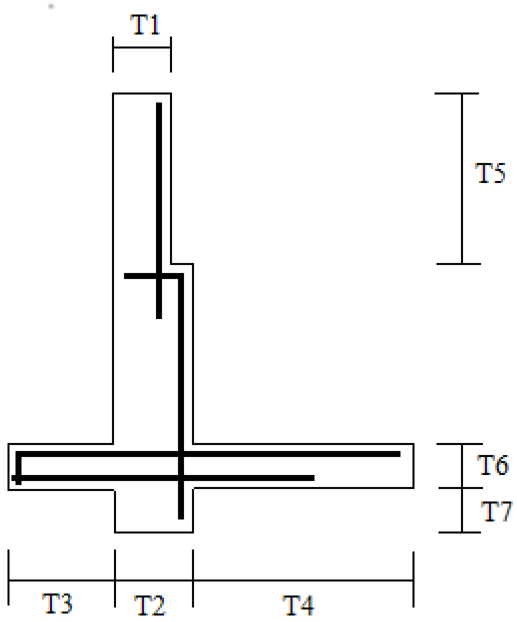


Figure 1. The design variables of cantilever retaining wall

Experience show the value of $\dfrac{C3+C4}{C1+C2}$ is in the range of 0.035 to 0.045. The constraints of this problem are considered as following:

$$FS_{(overturning)} \geq 1.5 \tag{17}$$

$$FS_{(sliding)} \geq 1.5 \tag{18}$$

$$FS_{(\text{bearing capacity})} \geq 2 \tag{19}$$

$$M_u/(\phi_b M_n) \leq 1 \tag{20}$$

$$V_u/(\phi_v V_n) \leq 1 \tag{21}$$

Eq. (17) and Eq. (18) refer to the constraints which are about stability of the cantilever retaining wall, and Eq. (20) and Eq. (21) refer to the constraints which are about shear and flexural strength. AASHTO [14] permits the factors of safety against sliding and overturning failure under dynamic loading condition reduced to 75% of the factors of safety used for the static loading designs.

6. OPTIMUM DESIGN PROCESS

The DPSO and CBO algorithms initiate the design process by selecting random values from design variables upper and lower band for the thickness of top stem $(T1)$, the thickness of key and stem $(T2)$, the toe width $(T3)$, the heel width $(T4)$, the height of top stem $(T5)$, the footing thickness $(T6)$, and the key depth $(T7)$.Then both of the algorithms check the wall for stability and if the dimensions satisfy stability criteria, the algorithms calculate the required reinforcement and check the strength. The design process of DPSO algorithm consists of 9 steps while the CBO algorithm consists of 7 steps. Both of them are explained as follow:

DPSO algorithm:

Step 1. Select the value of the DPSO parameters ($c1$, $c2$, $c3$, χ, number of population)

Step 2. Create particles as number of population.

Step 3. Initialize particles' positions (values for thickness of top stem ($T1$), thickness of key and bottom stem ($T2$), toe width ($T3$), heel width ($T4$), height of top stem ($T5$), footing thickness ($T6$) and key depth ($T7$) are chosen) and velocities.

Step 4. Calculate objective function for each particle.

Step 5. The algorithm checks the wall for stability and if these dimensions satisfy the stability criteria, the algorithm calculates the required reinforcement and checks the strength otherwise the objective function will be penalized.

Step 6. Evaluate objective functions to update local and global best and global worst.

Step 7. Calculate the vector D_i which represents the democratic effect of the other particles of the swarm on the movement of the i th particle.

Step 8. Update particles' velocities and positions.

Step 9. Repeat steps 4 to 8 until the terminating criterion is satisfied.

CBO algorithm:

Step 1. Initialize an array of CBs with random positions (each array of CBs consists of values for thickness of top stem ($T1$), thickness of key and bottom stem ($T2$), toe width ($T3$), heel width ($T4$), height of top stem ($T5$), footing thickness ($T6$) and key depth ($T7$) and calculate their associated values of the objective function. The n the algorithm checks the wall for stability and if these dimensions satisfy the stability criteria, the algorithm calculates the required reinforcement and checks the strength otherwise the objective function will be penalized.

Step 2. Compare the value of the objective function for each CB, and sort them in an increasing order.

Step 3. CBs are divided into two equal groups: (i) stationary group, (ii) moving group. Then, the pairs of CB are defined for collision.

Step 4. The value of mass and velocity of each CB for each group are evaluated before the collision.

Step 5. The value of velocity of each CB in each groups are evaluated after the collision.

Step 6. The new position of each CB is calculated.

Step 7. Repeat step 2 to 6 until a terminating criterion is satisfied.

7. NUMERICAL EXAMPLE

The process of optimization is described in Section 6. For this purpose a computer program is written in Matlab for analysis, design and optimization. The design is performed under static and dynamic loading condition. The analysis and design are in the form of a function which is called by the optimization program.

The features of two backfills are defined in Table 1, and design is based on 1.0m wide strip of the retaining wall. The static design is performed for both F1 and F2 backfills, utilizing Rankine and Coulomb theories. The design under dynamic loading condition is performed for F1 backfill, utilizing Mononobe-Okabe method. This method performs the analysis and design of retaining walls by pseudo-static approach in which the transient earthquake force and static thrust are simultaneously imposed on the retaining wall as an equivalent static force. The value of horizontal and vertical acceleration coefficients, k_h and k_v, are defined in Table 2. Critical sections are illustrated in Fig. 2. Ground water level is assumed to be below the foundation level of the wall and therefore not affecting the soil properties. The total height of stem is constant and equal to 6.1 m. Surcharge load is 10 kN/m^2. The 28 days concrete cylinder strength is 25 MPa, Rebar yield stress is 300 MPa, and the allowable soil pressure is taken as $q_a = 300\ kN/m^2$ ($3\ kg/cm^2$). The h_p is equal to zero. Upper and lower bounds for the design variable are shown in Table 3. A schematic view of a concrete retaining wall is illustrated in Fig. 3. The parameters of PSO [15, 16, 17, 18], IHS [19], DPSO and CBO algorithms are taken as:

$\omega = 0.7$, $c1=c2=2$, $\chi = 0.5$, $c3=5.5$, $HMCR=0.85$, $PAR_{\min} = 0.35$, $PAR_{\max} = 0.99$, $HMS=30$, $n=30$

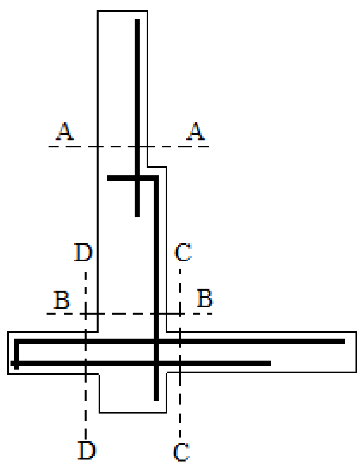


Figure 2. Critical Sections

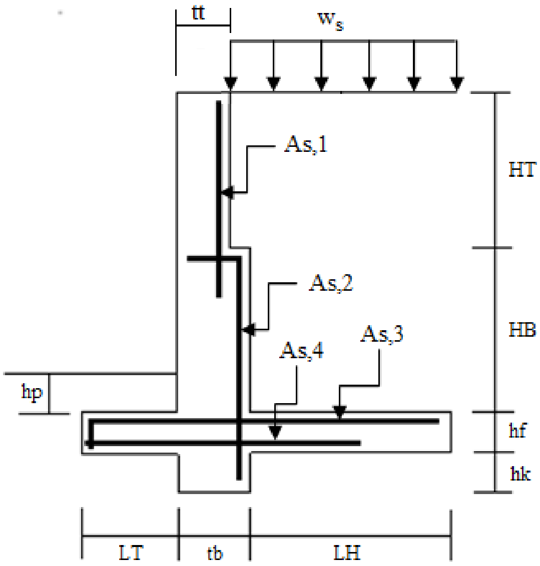


Figure 3. Schematic view of a reinforced concrete cantilever retaining wall

Table 1: Types of the backfills considered in the present work

Type of back fill	**Description**	**Density** kN/m^3	**Internal friction angle** (°)	**Cohesion** kN/m^2
F1	Coarse granular fills (GW,GP)	22	35	0
F2	Granular soils with more than 12% of fines (GW, GS, SM, SL) and fine soils with more than 25% of coarse grains (CL-ML)	20	30	15

Table 2: The value of horizontal and vertical acceleration coefficients

Case number	**Value of vertical and horizontal acceleration coefficients (** k_v, k_h **)**
Case 1	$k_v = 0, k_h = 0.3$
Case 2	$k_v = 0.15, k_h = 0.3$
Case 3	$k_v = 0.3, k_h = 0.3$
Case 4	$k_v = 0, k_h = 0.15$
Case 5	$k_v = 0.075, k_h = 0.15$
Case 6	$k_v = 0.15, k_h = 0.15$

Table 3: Lower and Upper bound for design variables

Design variables	**Thickness of the top stem**	**Thickness of the key and bottom stem**	**Toe width**	**Heel width**	**Height of the top stem**	**Footing thickness**	**Key depth**
	(T_1)	(T_2)	(T_3)	(T_4)	(T_5)	(T_6)	(T_7)
Upper bound	0.3m	0.3m	0.45m	1.8m	1.5m	0.3m	0.2m
Lower bound	0.6m	0.6m	1.2m	3m	6.1m	0.9m	0.9m

Optimum design results of each soil type under static and dynamic loading condition are presented in Tables 4 to 9.

The shear and stability capacity ratios are defined as:

$$Stability\ Capacity\ Ratio = \frac{Allowable\ safy\ factor}{Existing\ safy\ factor} \tag{29}$$

$$Shear\ Capacity\ Ratio = \frac{Existing\ shear\ force}{Allowable\ shear\ force} \tag{30}$$

These ratios have been calculated in four critical sections under different earthquake loading conditions and the results are presented in Table 10. In one cell of this table, the written capacity ratio is greater than 100%. For instance, the greatest numeral is 100.01%. However, this error is negligible, because the corresponding error of this numeral is 0.0001.

Table 4: Optimum results for back fill F1 utilizing Rankine theory

Algorithm	T_1	T_2	T_3	T_4	T_5	T_6	T_7	As_1	As_2	As_3	As_4
DPSO	33 cm	59 cm	120 cm	257 cm	150 cm	32 cm	20 cm	13.39 cm^2	33.29 cm^2	19.76 cm^2	12.94 cm^2
CBO	30 cm	59 cm	120 cm	248 cm	150 cm	31 cm	20 cm	17.43 cm^2	33.04 cm^2	21.33 cm^2	12.53 cm^2

Table 5: Optimum results for back fill F2 utilizing Rankine theory

Algorithm	T_1	T_2	T_3	T_4	T_5	T_6	T_7	As_1	As_2	As_3	As_4
DPSO	31 cm	60 cm	118 cm	284 cm	210 cm	35 cm	20 cm	14.27 cm^2	30.12 cm^2	16.06 cm^2	14.29 cm^2
CBO	30 cm	60 cm	120 cm	251 cm	194 cm	30 cm	23 cm	11.74 cm^2	30.12 cm^2	22.33 cm^2	11.66 cm^2

Table 6: Optimum results for back fill F1 utilizing Coulomb theory

Algorithm	T_1	T_2	T_3	T_4	T_5	T_6	T_7	As_1	As_2	As_3	As_4
DPSO	31 cm	60 cm	118 cm	185 cm	150 cm	33 cm	23 cm	12.18 cm^2	25.66 cm^2	14.68 cm^2	13.44 cm^2
CBO	33 cm	58 cm	119 cm	182 cm	183 cm	32 cm	22 cm	14.07 cm^2	25.16 cm^2	14.70 cm^2	12.51 cm^2

Table 7: Optimum results for back fill F2 utilizing Coulomb theory

Algorithm	T_1	T_2	T_3	T_4	T_5	T_6	T_7	As_1	As_2	As_3	As_4
DPSO	34 cm	59 cm	118 cm	215 cm	150 cm	30 cm	22 cm	13.38 cm^2	32.16 cm^2	42.10 cm^2	11.66 cm^2
CBO	31 cm	60 cm	120 cm	216 cm	150 cm	31 cm	21 cm	14.09 cm^2	31.58 cm^2	38.54 cm^2	12.04 cm^2

Table 8: Optimum results under dynamic loading condition utilizing CBO algorithm

	Optimal dimensions (cm^2) and reinforcement (cm)					
	Case1	**Case2**	**Case3**	**Case4**	**Case5**	**Case6**
T_1	32	31	31	32	31	31
T_2	60	60	59	60	60	60
T_3	49	48	49	46	46	45
T_4	300	300	297	273	269	267
T_5	153	150	150	156	151	151
T_6	53	52	53	53	53	52
T_7	21	20	20	20	20	20
As_1	12.42	12.21	12.22	12.49	12.11	12.11
As_2	44.52	43.9	43.86	39.89	39.10	37.92
As_3	30.91	30.94	30.38	24.68	22.56	22.14
As_4	22.52	22.15	22.35	22.33	22.33	21.84

Table 9: Optimum results under dynamic loading condition utilizing DPSO algorithm

	Optimal dimensions (cm^2) and reinforcement (cm)					
	Case1	**Case2**	**Case3**	**Case4**	**Case5**	**Case6**
T_1	32	33	31	33	33	33
T_2	60	60	59	60	60	60
T_3	50	49	49	46	48	45
T_4	291	300	298	274	262	262

T_5	170	150	150	150	175	150
T_6	56	53	53	54	53	53
T_7	21	20	20	24	20	25
As_1	15.2	13.48	12.42	12.75	14.96	13.06
As_2	44.86	44.14	44.13	40.09	39.33	38.57
As_3	28.34	30.59	29.84	24.31	22.64	22.54
As_4	24.07	22.44	22.85	22.78	22.46	21.39

Table 10: Capacity assessment with capacity ratio

Algorithm	Case	Shear Capacity (%)				Stability Capacity (%)		
		A:A	B:B	C:C	D:D	Sliding	Bearing	Overturning
CBO	1	11.98	66.51	80.64	100	57.98	100.01	47.94
	2	11.78	66.37	84	100	56.53	99.33	47.31
	3	11.67	66.46	81.5	99.70	56.81	99.80	47.77
	4	12.30	66.45	78.82	100	51.23	99.98	48.54
	5	11.90	66.48	79.56	99.84	50.67	99.66	48.38
	6	11.97	66.38	84.76	100	49.85	98.29	47.46
DPSO	1	13.88	66.37	69.72	99.36	59.21	100	50
	2	10.83	67.72	84.38	100	56.81	99.33	47.16
	3	11.59	66.62	79.05	97.86	56.81	100	47.77
	4	11.29	66.56	77.85	98.44	49.78	100	48.38
	5	14.15	66.67	80.78	100.01	50.37	99.33	49.50
	6	11.02	67.16	78.52	99.25	49.24	99.80	48.38

8. RESULTS AND DISCUSSION

The design historic under static loading condition for two types of backfills by applying Rankine and Coulomb methods is shown in Figs. 4-7. Fig. 8 depicts the design historic under earthquake loading condition for F1 backfill by applying Mononobe-Okabe method for Case 2. As these figures show the CBO and DPSO algorithms find better fitness for design and in initial steps their rapid convergence and downfall illustrate the power of these algorithm in evaluating more solutions and exploration. After initial steps the exploitation part of optimization process begins and finally the minimum solution is found. Fig. 9 and Fig. 10 depict the effect of vertical and horizontal acceleration coefficients on the objective function. Based on these figures, by increasing k_h the objective function increases it means that a more vigorous cantilever retaining wall is needed, But by increasing k_v, the inverse of this state happens. Table 10 depicts that the most important controlling factor among the stability capacity ratios is bearing capacity of the soil under the toe region and among the shear capacity ratios is shear capacity in the toe region.

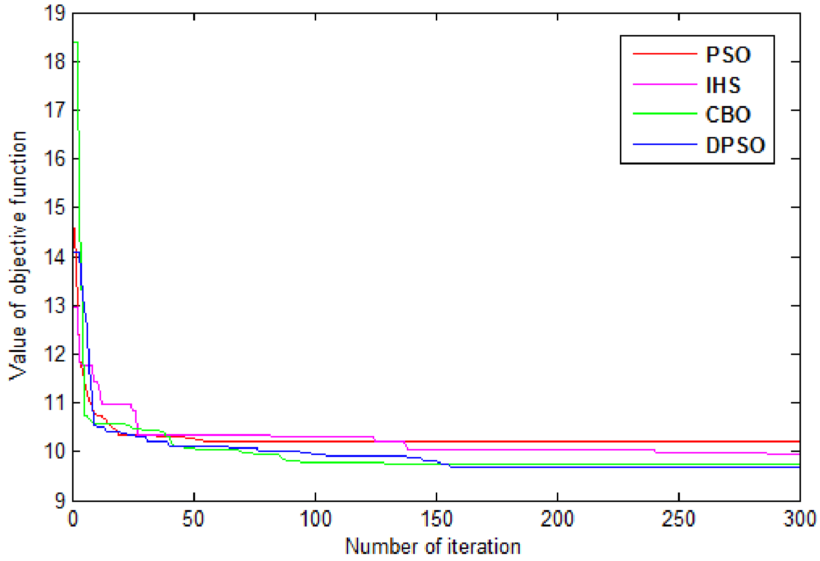


Figure 4. Design history for backfill F1 utilizing Rankine Method

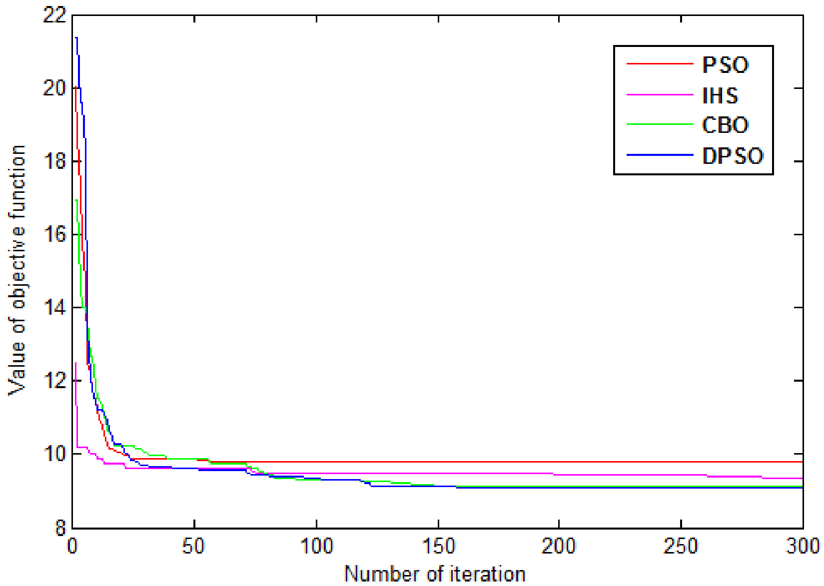


Figure 5. Design history for backfill F2 utilizing Rankine Method

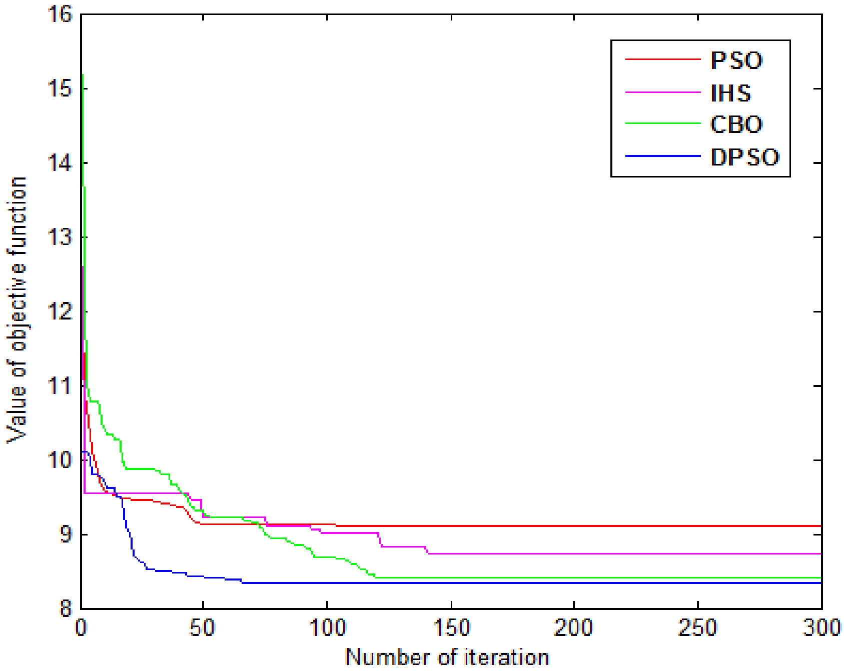


Figure 6. Design history for backfill F1 utilizing Coulomb Method

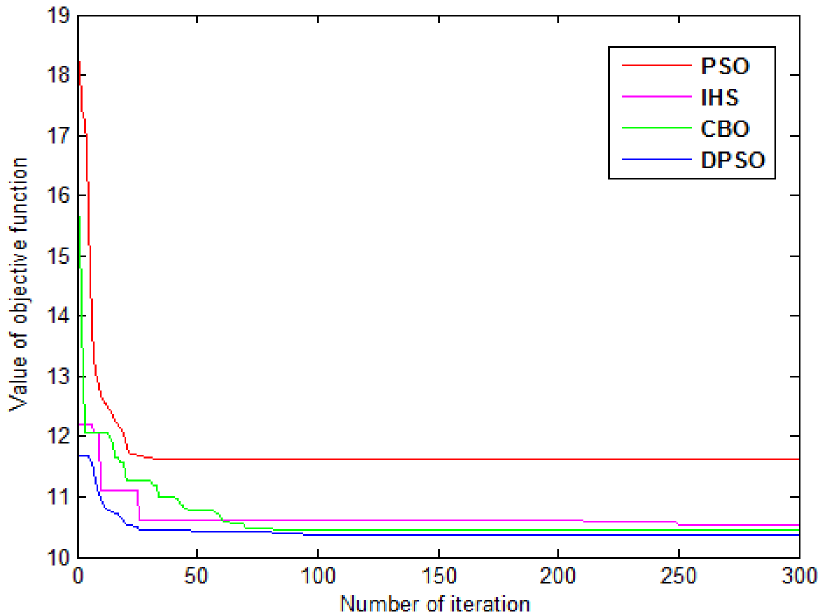


Figure 7. Design history for backfill F2 utilizing Coulomb Method

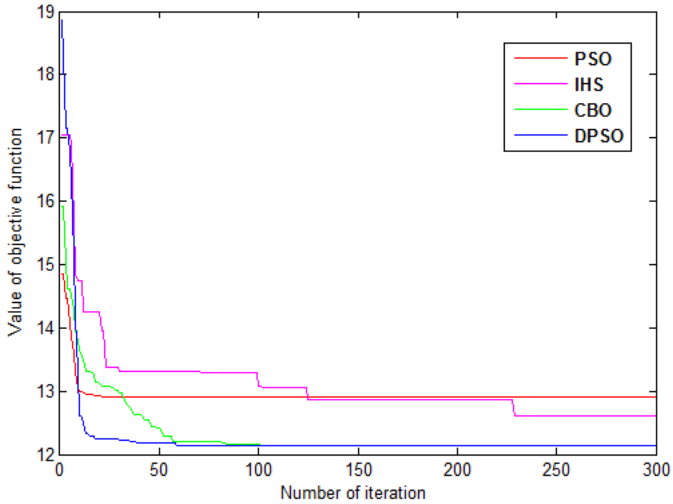


Figure 8. Design history for Case 2 utilizing Mononobe-Okabe method

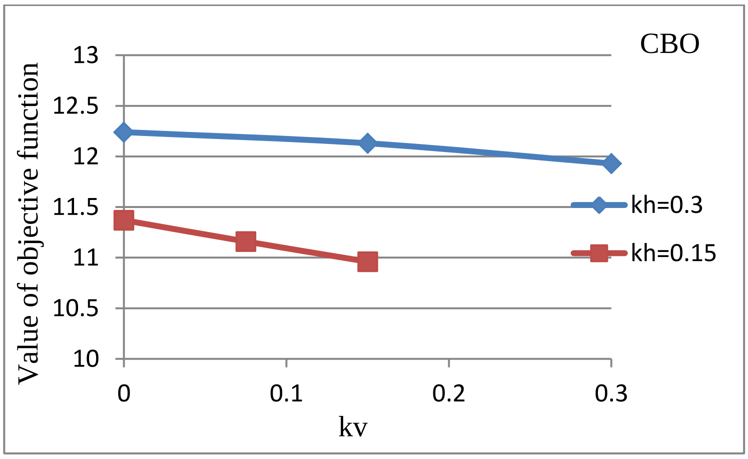


Figure 9. Effect of k_v and k_h on the objective function in CBO algorithm

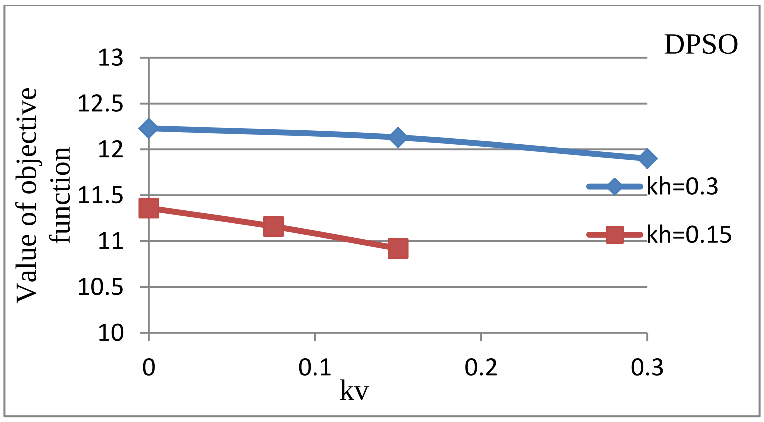


Figure 10. Effect of k_v and k_h on the objective function in DPSO algorithm

9. CONCLUDING REMARKS

In this study the optimization is performed by DPSO, CBO, IHS and PSO algorithms to provide a design of reinforced concrete cantilever retaining walls under static and different earthquake loading conditions which not only satisfies the stability and strength constraints, but also is economical. The design under earthquake loading condition indicates that the bearing capacity of soil under the toe region and the shear strength of critical section in the toe region are the design controlling factors. The effect of horizontal and vertical acceleration coefficients on objective function has also been studied. The results depict that the vertical acceleration coefficient has a reverse effect on the design of the retaining walls while the increase of horizontal acceleration coefficient leads to an increase in the dimensions of the retaining walls.

APPENDIX A: DEFINITIONS OF SYMBOLS

δ	Angle of friction between the wall and soil
i	Slope of the backfill with respect to the horizontal
k_v	Vertical acceleration coefficient
k_h	Horizontal acceleration coefficient
tt	Top stem thickness
tb	Bottom stem and key thickness
HT	Top stem height
HB	Bottom stem height
LT	Toe length
LH	Heel length
L	Total length of the base of the footing
hf	Footing thickness
γ_b	Density of the backfill
φ	Internal friction angle of the backfill
β	Slope of the back of the wall with respect to the vertical
μ	Base friction coefficient
γ_c	Density of the concrete
$W_{w,t}$	Weight of the top stem
$W_{w,b}$	Weight of the bottom stem
W_b	Weight of the fill on the heel
W_s	Surcharge weight
hk	Key depth
kp	Soil over toe
C1	Cost of the concrete
C2	Cost of the steel

C3	Cost of the concreting
C4	Cost of the erecting reinforcement
F1,F2	Considered cases for sensitivity study
T1,…T7	The selected variables

APPENDIX B: ANALYSIS AND DESIGN OF CONCRETE CANTILEVER RETAINING WALL

The content of this section is based on Refs. [8] and [20] to [22].

A1. Active and Passive Earth Pressure

The active and passive earth pressure coefficients under static loading condition are computed using the Rankine and Coulomb earth pressure theories. The details of the Rankine and Coulomb earth pressure are shown in Fig. A1(a),(b) respectively.

$$K_{A,coulomb} = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha\cdot\sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta)\cdot\sin(\phi-i)}{\sin(\alpha-\delta)\cdot\sin(i+\alpha)}}\right]^2} \tag{A-1}$$

$$K_{P,coulomb} = \frac{\sin^2(\alpha-\phi)}{\sin^2\alpha\cdot\sin(\alpha+\delta)\left[1-\sqrt{\dfrac{\sin(\phi+\delta)\cdot\sin(\phi+i)}{\sin(\alpha+\delta)\cdot\sin(i+\alpha)}}\right]^2} \tag{A-2}$$

$$K_{A,Rankine} = \tan^2(45-\frac{\phi}{2}) \tag{A-3}$$

$$K_{P,Rankine} = \tan^2(45+\frac{\phi}{2}) \tag{A-4}$$

The active and passive earth pressure coefficients under dynamic loading condition are computed using Mononobe-Okabe method. The dynamic active and passive earth pressures (P_{AE}, P_{PE}) are shown in Fig. A1(c), (d), respectively.

$$K_{AE} = \frac{\cos^2(\phi-\beta-\theta)}{\cos\theta\cos^2\beta\cdot\cos(\theta+\beta+\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta)\cdot\sin(\phi-\theta-i)}{\cos(\beta+\delta+\theta)\cdot\cos(i-\beta)}}\right]^2} \tag{A-5}$$

$$K_{PE} = \frac{\cos^2(\phi+\beta-\theta)}{\cos\theta\cos^2\beta\cdot\cos(\theta-\beta+\delta)\left[1-\sqrt{\dfrac{\sin(\phi+\delta)\cdot\sin(\phi-\theta+i)}{\cos(\theta-\beta+\delta)\cdot\cos(i-\beta)}}\right]^2} \tag{A-6}$$

$$\theta = \tan^{-1}\left(\frac{k_h}{1-k_v}\right) \tag{A-7}$$

$$k_h = \frac{\text{horizontal earthquake acceleration component}}{\text{acceleration due to gravity, g}} \tag{A-8}$$

$$k_v = \frac{\text{vertical earthquake acceleration component}}{\text{acceleration due to gravity, g}} \tag{A-9}$$

$$P_{AE} = \tfrac{1}{2}\gamma H^2 (1-k_v) K_{AE} \tag{A-10}$$

$$P_{PE} = \tfrac{1}{2}\gamma H^2 (1-k_v) K_{PE} \tag{A-11}$$

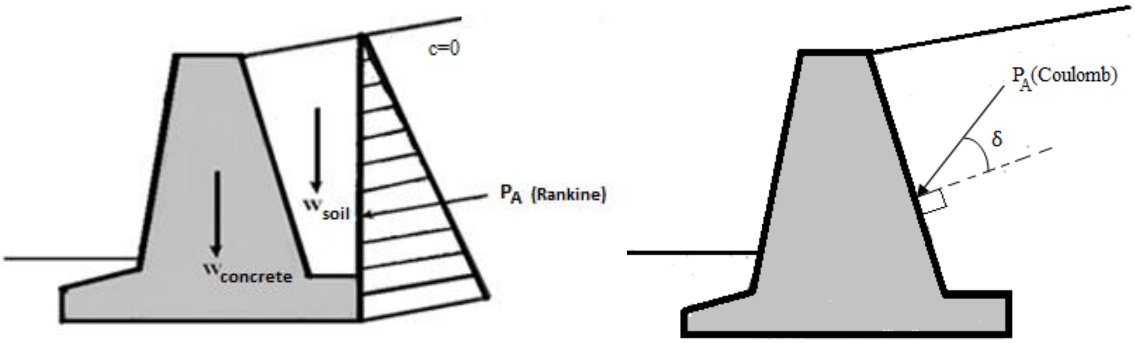
H=Height of the backfill
γ =Density of the backfill

The active force P_{AE} acts at $\overline{H}$ from the bottom of the base slab, given by:

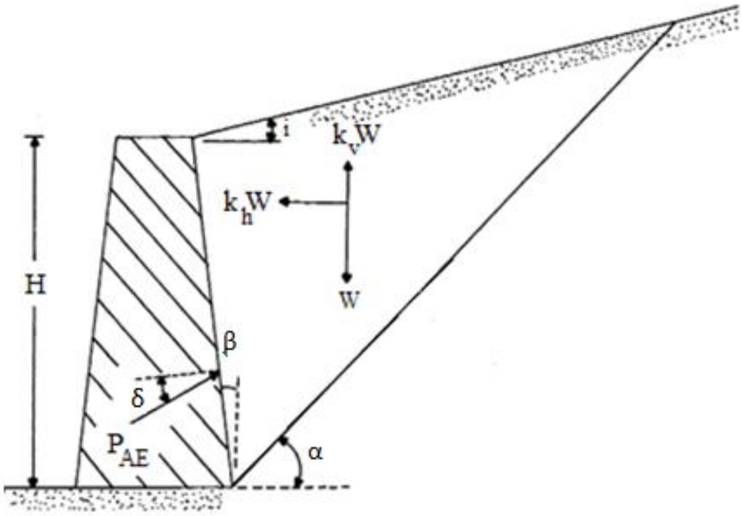
$$\overline{H} = \frac{P_a(\frac{H}{3}) + (\Delta P_{AE})(0.6H)}{P_{AE}} \tag{A-12}$$

Where:

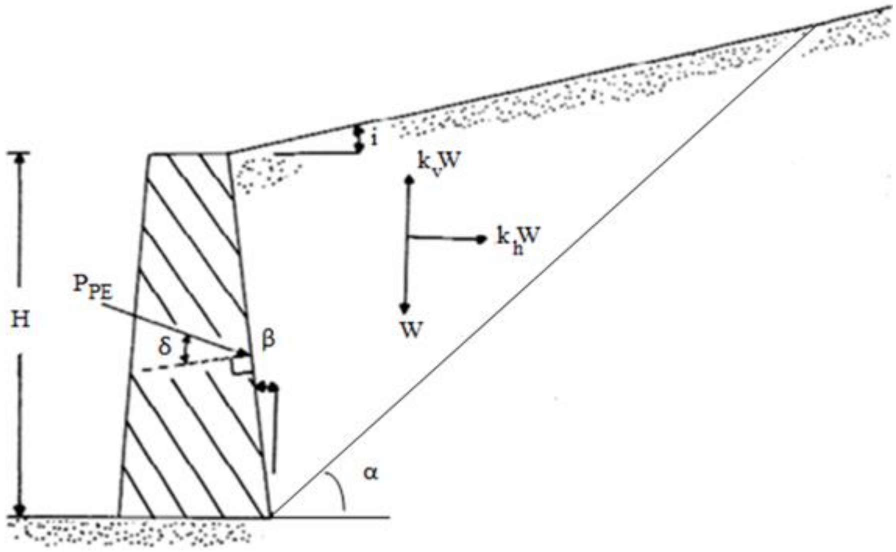
$\Delta P_{AE} = P_{AE} - P_a$
P_a =Coulomb active earth pressure



(a) (b)



(c)



(d)

Figure A1. Details for a) Rankin earth pressure b) Coulomb earth pressure c) dynamic active earth pressure d) dynamic passive earth pressure

A.2 Stability Control

All the loads acting on the reinforced concrete cantilever retaining wall are shown in Fig. A2.

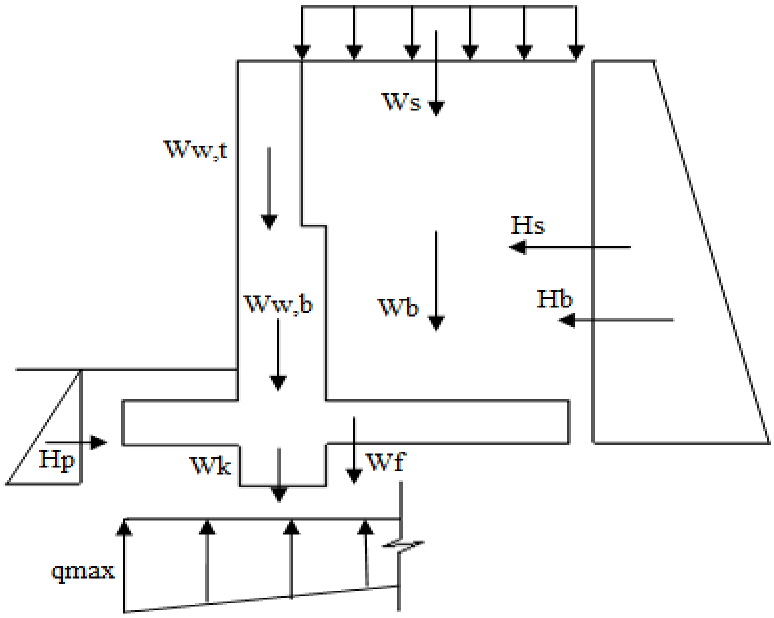


Figure A2. Loads in cantilever retaining wall

Check for Overturning:

$$FS_{(overturning)} = \frac{\sum M_R}{\sum M_O} = \frac{M_{Hp} + \sum Wx}{\sum Hy} \tag{A-13}$$

Where:

$\sum M_O$ = sum of the overturning moments

$\sum M_R$ = sum of the resisting moments

Check for sliding along the base:

$$FS_{(sliding)} = \frac{F_{R'}}{F_d} = \frac{Hb + Hs}{Hp + \mu \sum W} \tag{A-14}$$

Where:

$\sum F_{R'}$ = sum of horizontal resisting forces

$\sum F_d$ = sum of horizontal driving forces

Check for bearing capacity failure:

$$FS_{(\text{bearing capacity})} = \frac{q_u}{q_{\max}} \tag{A-15}$$

Where:

q_u = Ultimate bearing capacity

$q_{\max}$ = Maximum bearing pressure

AASHTO [12] permits the value of q_u used for static loading designs to be increased by 33% for the seismic loading conditions, in the other word:

$$FS_{(\text{bearing capacity})} = \frac{1.33\, q_u}{q_{\max}} \tag{A-16}$$

$$q_{\max} = \begin{cases} \dfrac{\sum W(1+\frac{6e}{L})}{BL} & for \quad e \le \dfrac{L}{6} \\ \dfrac{2\sum W}{3B(0.5L-e)} & for \quad e > \dfrac{L}{6} \end{cases} \tag{A-17}$$

$$e = \frac{L}{2} - \frac{\sum Wx - \sum Hy - M_{Hp}}{\sum W} \tag{A-18}$$

A3. Strength Control

Check the flexure and shear capacities in critical sections:

$$M_u / (\phi_b M_n) \le 1 \tag{A-19}$$

$$V_u / (\phi_v V_n) \le 1 \tag{A-20}$$

Where M_u and V_u are the ultimate moment and shear in critical sections. The required area cross section of rebar is calculated as follow:

$$m = \frac{f_y}{f'_c} \tag{A-21}$$

$$R_n = \frac{M_u}{\phi_b B d^2} \tag{A-22}$$

$$A_s = \frac{Bd}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) \tag{A-23}$$

Where d and B are the effective depth and length of the critical section.

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