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DAMAGE IDENTIFICATION OF TRUSS STRUCTURES USING CBO AND ECBO ALGORITHMS

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ABSTRACT

In the method of identification of structural damage utilized in the structural health monitoring problems, a search is carried out to find the location and damage severity of structural members by checking every possible value of these damage severities. Recently, researchers have been solved this problem using optimization algorithms. This paper examines the application of recently developed optimization algorithms, so-called Colliding Bodies Optimization (CBO) and Enhanced Colliding Bodies Optimization (ECBO), in conjunction with structural modal information for damage detection of steel trusses. The performance of the presented technique has been is verified through three numerical examples. Comparative studies illustrate the superiority the ECBO algorithm compared to the standard CBO algorithm.

Keywords: Damage detection; natural frequency; optimization algorithms; colliding bodies optimization.

1. INTRODUCTION

The problem of damage detection of structures, for structural health monitoring, consists of obtaining information about the existence, location, and extent of damage in the structure using non-destructive methods. One approach is to monitor and interpret changes in structural dynamic measurements based on experimental modal analyses and signal-processing techniques. The extraction of the natural frequency and mode shape of a vibrating structure can be accomplished using modern vibration testing equipment and instrumentation [1].

Several strategies have been reported in the damage detection field based on changes in the measured natural frequencies [2-11]. This problem may be treated as a bounded nonlinear optimization problem. The basic idea is to change the properties of the numerical

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model to fit the values provided by the experimental data, identifying damaged regions and the extent of damage on the structure. In other words, the optimization algorithm seeks the optimal parameter values, which are the reduction factors of element stiffness, to achieve a pre-defined performance in terms of the modal parameters defined by the experimental data. This procedure leads to a target performance optimization problem, which is usually very complex to solve because it generally leads to nonconvex and multimodal objective functions [12, 13]. Under these conditions, deterministic optimization algorithms such as gradient methods, Newton methods or sequential simplex methods may not converge to the global minimum of the problem due to their dependence on the quality of the starting point of the search. That is, if a given starting point is not on a basin of attraction of the global optimum, these methods will not converge to the global solution; the use of a global optimization algorithm is then required. For instance, GAs have been applied by several researchers for vibration-based damage detection including Mares and Surace [13], Chou and Ghaboussi [14], Hao and Xia [15], Rao et al. [16], Vakil-Baghmisheh et al. [17], Gomes and Silva [18], Srinivas et al. [19], Perera et al. [20], Na et al. [21] and Srinivas et al. [22]. Recently, the problem was also solved using other more recent metaheuristics such as the bee algorithm (BA) [23], the PSO algorithm [24], the HS algorithm [25], the improved CSS algorithm [10].

Colliding bodies optimization (CBO) belongs to a family of meta-heuristic algorithms which recently developed by the authors [26, 27]. This algorithm can be considered as a multi-agent method, where each agent is a Colliding Body (CB). Simple formulation and no internal parameter tuning are advantages of this algorithm. The enhanced colliding bodies optimization (ECBO) was introduced by [28, 29] and it used memory to save some historically best solution to improve the CBO performance without increasing the computational cost and some components of agents was also changed to jump out from local minimum. The present study presents damage detection of truss structures based on changes in the natural frequencies. The damage detection is first formulated in the form of optimization problem as mentioned before. Then, both mentioned optimization algorithms are applies for the optimization of this problem.

The present paper is organized as follows: In next section, the problem formulation of damage detection problem is briefly introduced. The optimization algorithms are then presented, followed by a section consisting of the study of three truss structural damage detection problems. Conclusions are derived in the final section.

2. PROBLEM FORMULATION

In this section, the approach for structural damage detection based on changes in natural frequency is briefly described. A simplest technique to represent the damage state of structure is the degradation in stiffness properties of structure. Thus, it is useful to introduce the damage in the structure through the consideration of an elemental stiffness reduction factor (α_i). In this approach, the global stiffness matrix of damaged structure ($[k^d]$) can be formulated as the summation of damaged and undamaged element stiffness matrices ($[k_e]$), where the damaged local element stiffness ($[k_e^d]$) is multiplied by the reduction factor (α_i), such as:

$$\left[K^d\right]=\sum_{i=1}^{N}\left[K_e^d\right]_i=\sum_{i=1}^{N}\alpha_i\left[K_e\right]_i \tag{1}$$

where N is number of structural elements. The reduction factor (α_i) indicates the damage severity at the ith element in the finite element model whose values are between 1 for an element without damage and 0 for a ruptured element. It should be noted that the damage severity of the ith element is defined as $(1-\alpha_i)$. Moreover, it is assumed that no change will occur after damage in the mass matrix $[M]$, which seems to be reasonable in most real problems [7].

The jth eigenvalue equation of the damaged structure will be derived by substitution of the structure's stiffness matrix by that of the damaged one:

$$\left[K^d\right]\left\{\varphi_{jd}\right\}-\omega_{jd}^2\left[M\right]\left\{\varphi_{jd}\right\}=\left\{0\right\} \tag{2}$$

in which, ω_{jd} and φ_{jd} are the jth natural frequency and the jth shape mode of the damaged structure, respectively.

In this study, the objective function is considered the fractional changes in natural frequencies and mode shapes before and after damage as:

$$F(\alpha)=\sum_{i=1}^{NM}\left(\left(\frac{\delta\omega_i(\alpha)}{\omega_i}\right)^D-\left(\frac{\delta\omega_i}{\omega_i}\right)^E\right)^2+\sum_{i=1}^{NM}\sum_{j=1}^{NP}\left(\left(\delta\varphi_{ij}(\alpha)\right)^D-\left(\delta\varphi_{ij}\right)^E\right)^2 \tag{3}$$

Where, NM is the number of modes analyzed, NP is the number of nodal displacement that is measured, the superscripts D and E represent numerical and experimental quantities, respectively, ω_i is the natural frequency for the ith mode of the undamaged state, $\delta\omega_i$ and $\delta\varphi_{ij}$ are fractional change of the experimental and analytical natural frequencies and displacement nodal for the ith mode of the structure, respectively. The stiffness reduction factor (α) of the finite element model should be updated until the differences of the numerical frequencies in the healthy and damaged states converge to the observed experimental frequencies in the pre- and post-damaged states [8].

3. OPTIMIZATION ALGORITHMS

As mentioned before, the CBO and ECBO algorithms have been employed for solving Eq. (3). In this part, we briefly present these algorithms.

3.1 Colliding bodies optimization algorithm

The colliding bodies optimization is based on momentum and energy conservation law for 1-dimensional collision [26]. This algorithm contains a number of Colliding Body (CB) where each one is treated as an object with specified mass and velocity which collide to others. After collision, each CB moves to a new position with new velocity with respect to

old velocities, masses and coefficient of restitution. CBO starts with a set of agents determined with random initialization of a population of individuals in the search space. Then, CBs are sorted in an ascending order based on the value of cost function (see Fig. 1). The sorted CBs are divided equally into two groups. The first group is stationary and consists of good agents. This set of CBs is stationary and their velocity before collision is zero. The second group consists of moving agents which move toward the first group. Then, the better and worse CBs, i.e. agents with upper fitness value, of each group collide together to improve the positions of moving CBs and to push stationary CBs towards better positions (see Fig. 1). The change of the body position represents the velocity of the CBs before collision as:

$$v_i = \begin{cases} 0, & i = 1,\ldots,n \\ x_i - x_{i-n}, & i = n+1,\ldots,2n \end{cases} \tag{4}$$

where, and are the velocity vector and position vector of the *i*th CB, respectively. 2*n* is the number of population size.

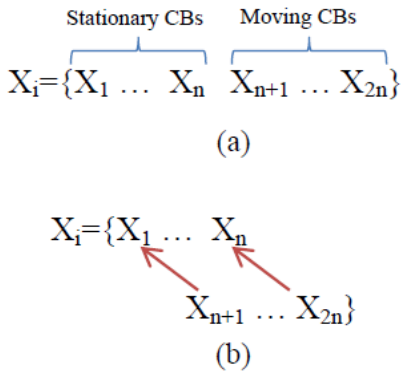


Figure 1. (a) The sorted CBs in an increasing order, (b) The mating process for the collision.

After the collision, the velocity of bodies in each group is evaluated using momentum and energy conservation law and the velocities before collision. The velocity of CBs after the collision is:

$$v_i^{'} = \begin{cases} \dfrac{(m_{i+n} + \varepsilon m_{i+n})v_{i+n}}{m_i + m_{i+n}}, & i = 1,\ldots,n \\ \dfrac{(m_i - \varepsilon m_{i-n})v_i}{m_i + m_{i-n}}, & i = n+1,\ldots,2n \end{cases} \tag{5}$$

where, and are the velocities of the *i*th CB before and after the collision, respectively; is the mass of the *i*th CB defined as:

$$m_k = \frac{\dfrac{1}{fit(k)}}{\sum_{i=1}^{n} \dfrac{1}{fit(i)}}, \quad k = 1,2,...,2n \tag{6}$$

where *fit(i)* represents the objective function value of the *i*th agent. Obviously a CB with good values exerts a larger mass and fewer moves than the bad ones. Also, for maximizing the objective function, the term is replaced by $fit(i)$. ε is the coefficient of restitution (COR) and is defined as the ratio of the separation velocity of the two agents after collision to approach velocity of two agents before collision. In this algorithm, this index is defined to control of the exploration and exploitation rates. For this purpose, the COR decreases linearly from unit value to zero. Here, ε is defined as:

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \tag{7}$$

where *iter* is the actual iteration number, and $iter_{max}$ is the maximum number of iterations. Here, COR is equal to unity and zero representing the global and local search, respectively. In this way a good balance between the global and local search is achieved by increasing the iteration.

The new positions of CBs are evaluated using the generated velocities after the collision in the position of stationary CBs:

$$x_i^{new} = \begin{cases} x_i + rand \circ v_i^{'}, & i = 1,...,n \\ x_{i-n} + rand \circ v_i^{'}, & i = n+1,...,2n \end{cases} \tag{8}$$

where, x_i^{new} and are the new position and the velocity after the collision of the i th CB, respectively.

3.2 Enhanced colliding bodies optimization algorithm

In order to improve the CBO to obtain faster and more reliable solutions, Enhanced Colliding Bodies Optimization (ECBO) is developed which uses memory to save a number of historically best CBs and also utilizes a mechanism to escape from local optima (Kaveh and Ilchi [28]). The steps of this technique are given as follows:

Level 1: *Initialization*

Step 1: The initial positions of all the CBs are determined randomly in the search space.

Level 2: *Search*

Step 1: The value of mass for each CB is evaluated according to Equation (6).

Step 2: Colliding memory (CM) is utilized to save a number of historically best CB vectors and their related mass and objective function values. Solution vectors which are saved in CM are added to the population and the same number of current worst CBs are removed. Finally, CBs are sorted according to their masses in a decreasing order.

Step 3: CBs are divided into two equal groups: (i) stationary group, (ii) moving group (Fig. 1).

Step 4: The velocities of stationary and moving bodies before collision are evaluated by Equation (4).

Step 5: The velocities of stationary and moving bodies after the collision are evaluated using Equation. (5).

Step 6: The new position of each CB is calculated by Equation (8).

Step 7: A parameter like **Pro** within (0, 1) is introduced and it is specified whether a component of each CB must be changed or not. For each colliding body **Pro** is compared with rn_i (i=1,2,…,n) which is a random number uniformly distributed within (0, 1). If rn < **Pro**, one dimension of the ith CB is selected randomly and its value is regenerated as follows:

$$x_{ij} = x_{j,\min} + random.(x_{j,\max} - x_{j,\min}) \tag{9}$$

where x_{ij} is the jth variable of the ith CB, and $x_{j,min}$ and $x_{j,max}$ are the lower and upper bounds of the jth variable, respectively. In order to protect the structures of CBs, only one dimension is changed.

Level 3: *Terminal condition check.*

Step 1: After a predefined maximum evaluation number, the optimization process is terminated.

4. NUMERICAL EXAMPLES

In this section, damage identification of three truss structures are studied utilizing the proposed method. The final results of both algorithms are compared demonstrate the efficiency of the ECBO algorithm. For all examples a number of 40 and 30 CBs are utilized for CBO and ECBO algorithms, respectively. For first, second and third examples, the maximum number of iterations are considered as 500, 2000 and 2000, respectively. In order to assess the effect of the initial population on the final result, 20 independent runs are carried out using both algorithms with different initial populations. The algorithms are coded in Matlab.

For all examples, two damage scenarios are used as elements with reduction in elastic modulus. The first and second damage scenarios are considered as the single damage case and multiple damage case, respectively.

4.1 Example 1: A 10-bar planar truss

A planar steel truss, as illustrated in Fig. 2, with a ten-element model consisting of ten elements and four free nodes is considered as the first example. This example has been considered as benchmark in the field of optimal design by many researchers (e.g., Kaveh and Zolghadr [30]). Non-structural masses are attached to all free nodes as 454.0kg. The material density is taken as $2770 kg/m^3$ and the modulus of elasticity is $69{,}800 MPa$. The cross section of members is considered as $A=0.0025\ m^2$.

For both damage scenarios of this example, the damage severity in each element is given by the reduction factor (as listed in Table 1): (1) 5% damage in element 1, (2) 10% damage in element 2 and 5% damage in element 4. The objective function employed in this example is described by Equation (3). In this equation, the first 8 natural frequencies and 8 nodal displacements were adopted in the mode shapes.

Table 1 shows the obtained best and mean results of 20 independent runs in this example with both algorithms. As seen in Table 1, the damage locations and the severity damage are accurately identified in the best result of first scenario using CBO and ECBO algorithms. For multiple damage scenario, the CBO accurately identified the location of damaged elements in the best result, although, it cannot predict the exact value of the severity damage. The ECBO is also accurately identified the location and severity damage of damaged elements for multiple damage scenario in the best result. Fig. 3 shows the mean predicted damage severities of 20 runs in elements using both algorithms. It can be seen, the mean predicted severity damage using ECBO are closer to exact one compared with the outcome of CBO. The evolution processes of mean fitness value obtained of 20 runs by both algorithms are shown in Fig. 4.

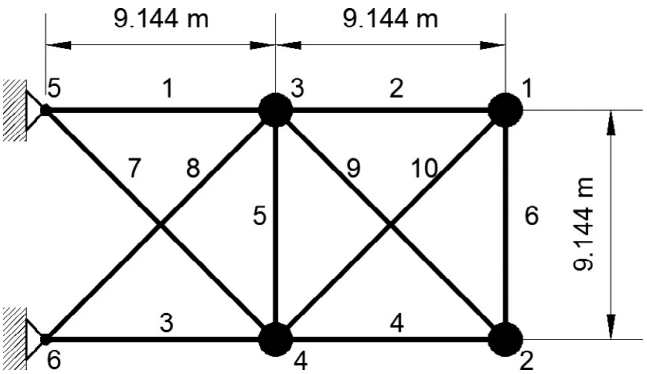
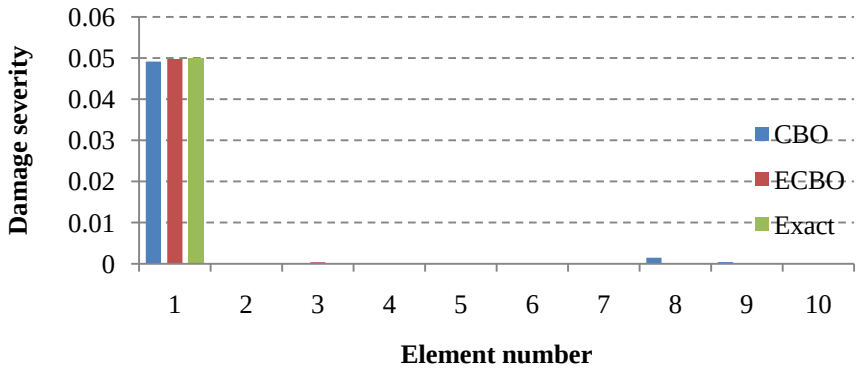


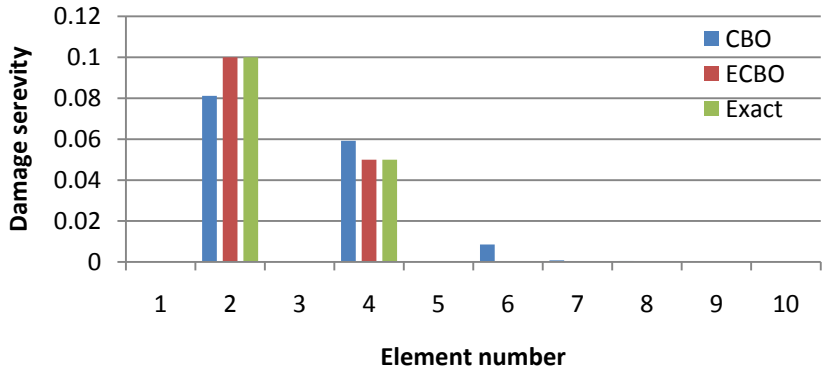
Figure 2. Schematic of a 10 bar truss.

Table 1: Results obtained by different optimization algorithms for both damage scenarios of the 10-bar planar truss.

	Damage location	**Exact severity damage**	**Best predicted severity damage**		**Mean predicted severity damage**	
			CBO	ECBO	CBO	ECBO
Scenario 1	1	%5	%5	%5	%4.916	%4.977
Scenario 2	2	%10	%10.00	%10.00	%8.11	%10.00
	4	%5	%5.10	%5.00	%5.98	%4.99



(a)



(b)

Figure 3. The mean damage severities of both algorithms for the 10-bar planar truss: a) scenario 1, b) scenario 2.

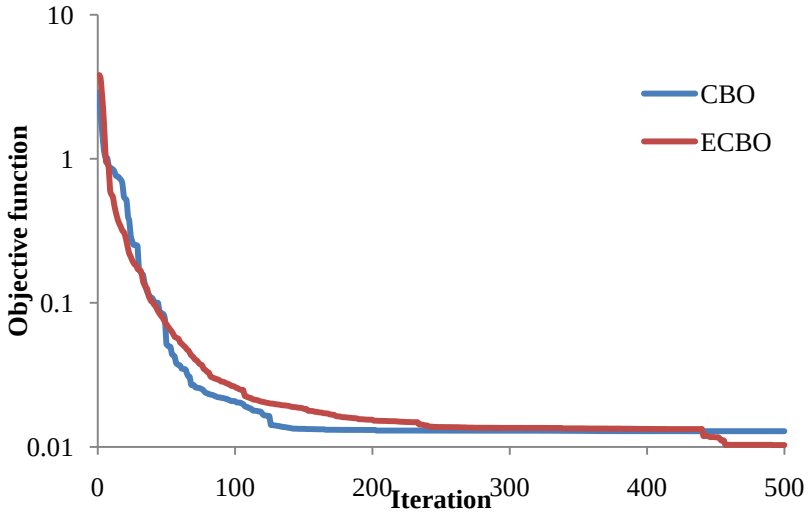


Figure 4. The mean convergence curves of both algorithms for the multiple scenario of 10-bar planar truss.

4.2 Example 2: A 37-bar planar truss

A simply supported 37-bar Pratt type truss, as depicted in Figure 5, is examined as the second example. This example has been considered as a benchmark in the field of optimal design by many researchers (e.g., see Kaveh and Zolghadr [30]). The material density is 7800 kg/m^3 and the modulus of elasticity is 210,000*MPa*. On the lower chord, a non-structural mass of 10 kg is attached to all the free nodes. The cross section of members is considered as $A=0.002\ m^2$.

For both damage scenarios of this example, the damage severity in each element is given by the reduction factor (as listed in Table 2): (1) 10% damage in element 6, (2) 15% damage in element 6 and 10% damage in element 17. In this example, the first 10 natural frequencies and 20 nodal displacements are adopted in the mode shapes.

Table 2 proposes both damage states and the results obtained using CBO and ECBO algorithms. Similar to the first example, the locations and extent of damage are accurately identified in the best result of the single damage scenario using CBO and ECBO algorithms. For multiple damage scenario, while CBO accurately identified the location of damaged elements in the best result, it could not predict the exact value of the severity of damage. The ECBO also accurately identified the location and severity of damaged elements for multiple damage scenario in the best result. Fig. 6 shows the mean predicted damage severities of 20 runs in elements using both algorithms. It can be seen, the mean predicted severity damage using ECBO is closer to exact result compared with the outcome of the CBO. Fig. 7 shows the mean convergence curves of 20 individual run using the CBO and ECBO algorithms for the multiple scenario.

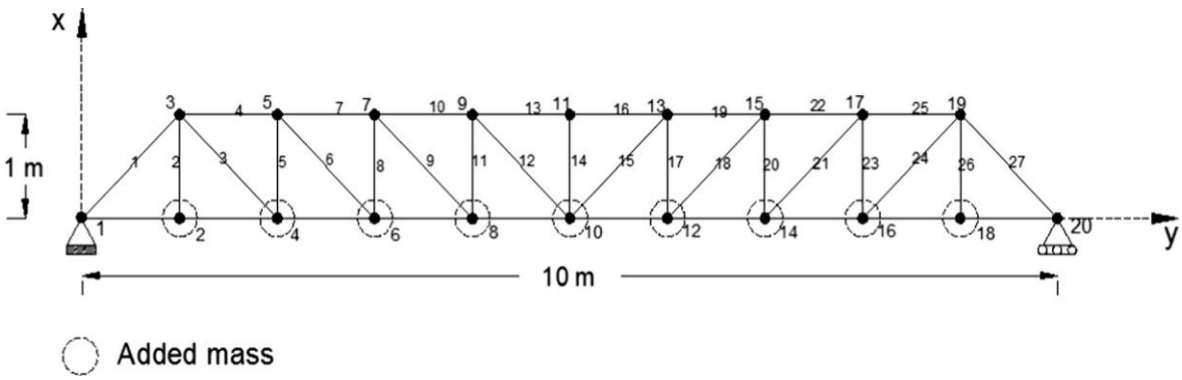
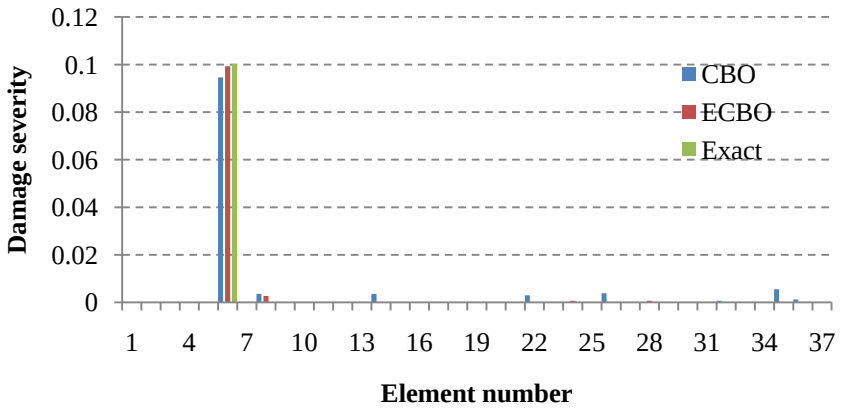


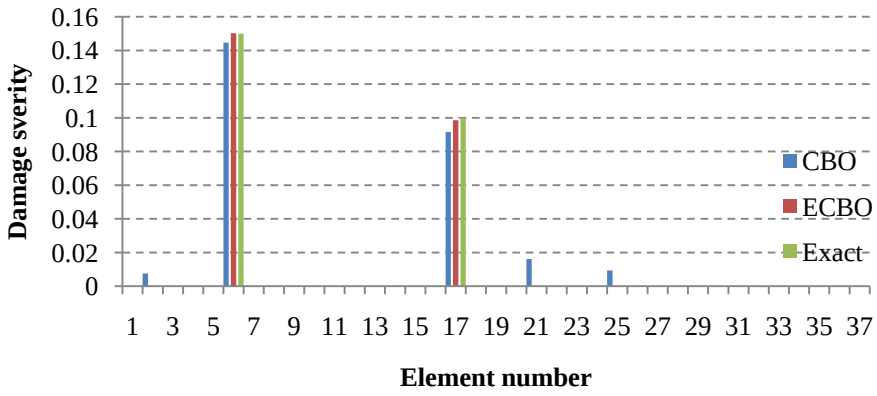
Figure 5. A simply-supported planar 37-bar truss.

Table 2: Results obtained by different optimization algorithms for both damage scenarios of the 37-bar planar truss.

	Damage location	**Exact severity damage**	**Best predicted severity damage**		**Mean predicted severity damage**	
			CBO	ECBO	CBO	ECBO
Scenario 1	6	%10	%10	%10	%9.46	%9.93
Scenario 2	6	%15	%14.92	%15.00	%14.46	%15.07
	17	%10	%8.33	%10.00	%9.16	%9.57



(a)



(b)

Figure 6. The obtained mean damage severities of both algorithms for the 37-bar planar truss: a) scenario 1, b) scenario 2

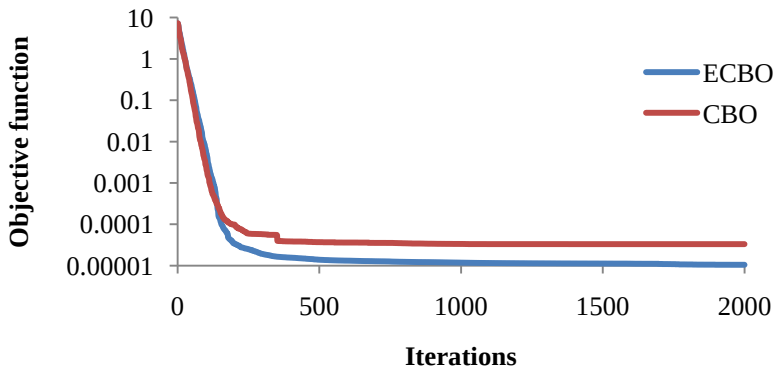


Figure 7. The mean convergence curves of both algorithms for the multiple scenario of 37-bar planar truss

4.3 Example 3: A 72-bar space truss

Fig. 8 shows the topology and element numbering of a 72-bar space truss for this example. This example has been also considered as benchmark in the field of optimal design by many researchers (for example, see Kaveh and Zolghadr [30]). For this example, the material density is 2770 kg/m^3 and the modulus of elasticity is 69,800*MPa*. Four non-structural masses of 2270 kg are attached to the nodes 1-4. The cross section of members is considered as A=0.0025 m^2.

For both damage scenarios of this example, the damage severity in each element is given by the reduction factor as listed in Table 3: (1) 15% damage in element 55, (2) 10% damage in element 4 and 15% damage in element 58. In this example, the first 16 natural frequencies and 20 nodal displacements were adopted in the mode shapes.

Table 3 compares the results obtained for both scenarios using CBO and ECBO algorithms. The identified damage locations and severities after optimizing using CBO and ECBO algorithms for both scenarios are shown in Fig. 9. According to Table 3 and Fig. 9, similar to two first examples, the mean result of 20 runs obtained by the ECBO is meaningfully closer to actual values compared to the standard CBO algorithm. Fig. 10 shows the mean convergence curves of the CBO and ECBO algorithms for the multiple damage scenario of this method.

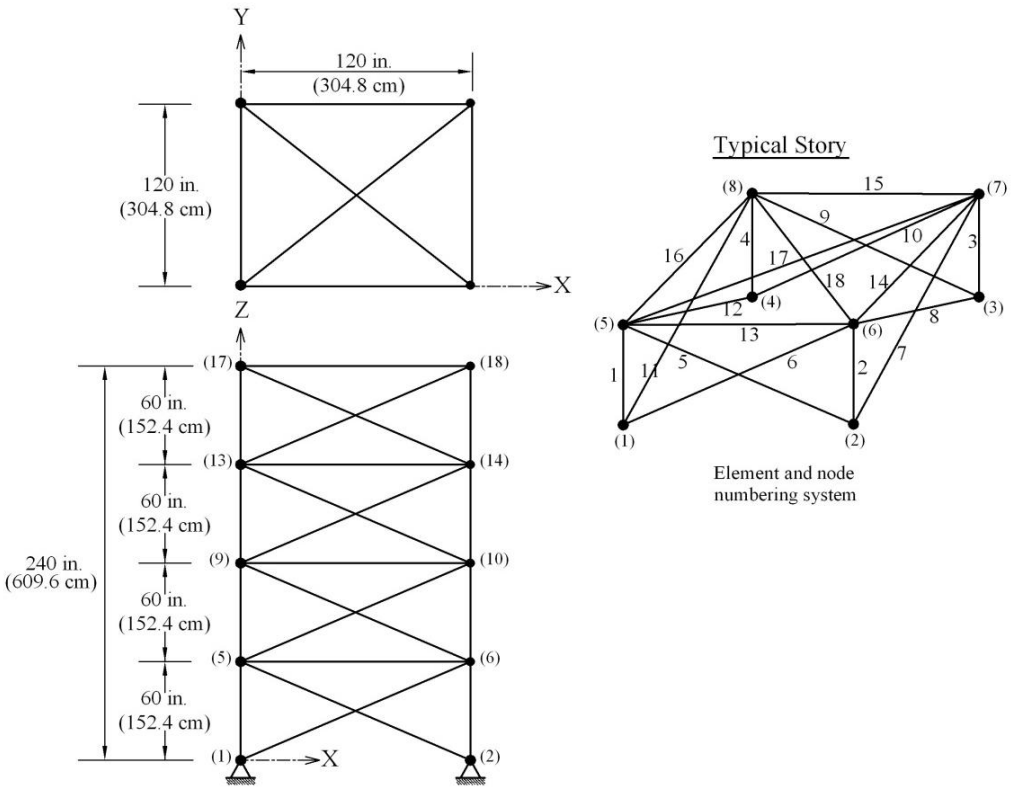
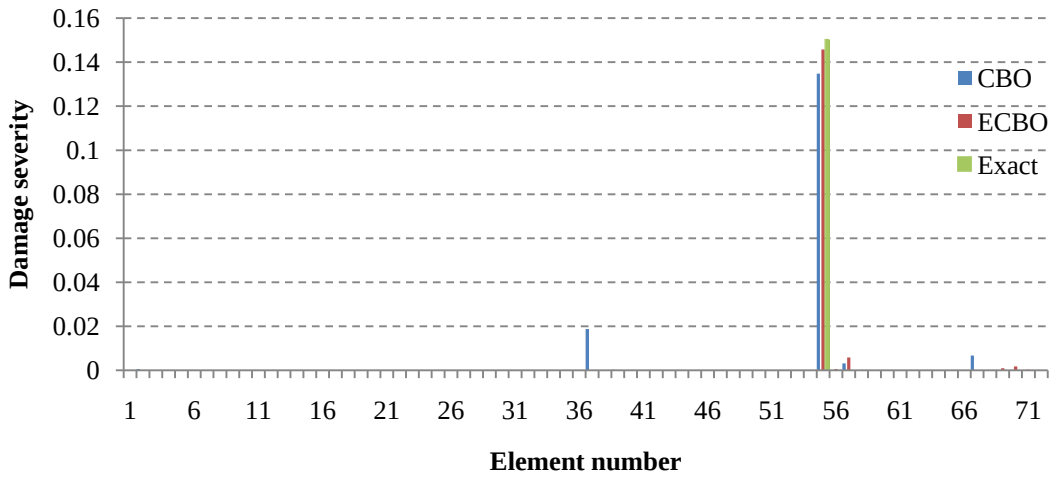


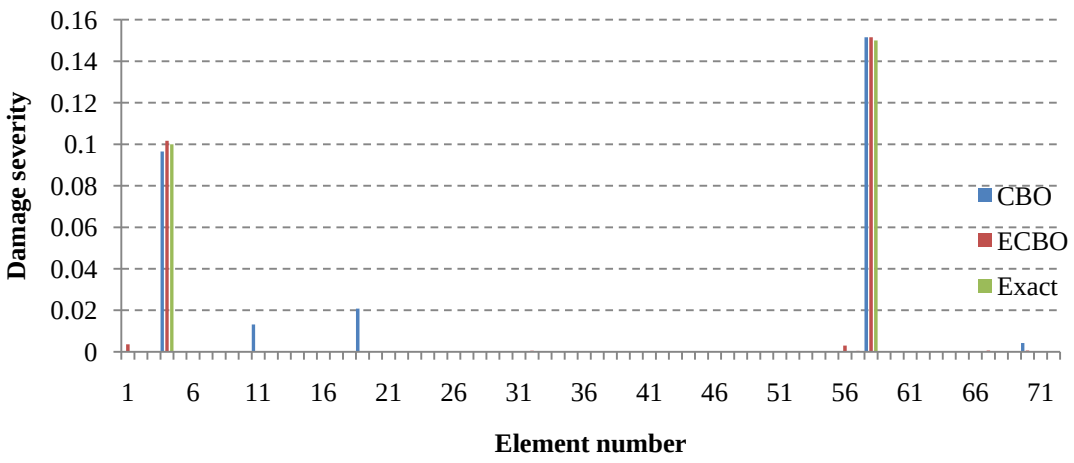
Figure 8. A 72-bar space truss

Table 3: Results obtained by different optimization algorithms for both damage scenarios of the 72-bar space truss

	Damage location	Exact severity damage	Best predicted severity damage		Mean predicted severity damage	
			CBO	ECBO	CBO	ECBO
Scenario 1	55	%15	%15	%15	%13.48	%14.57
Scenario 2	4	%10	%9.89	%10.00	%9.16	%10.17
	58	%15	%15.00	%15.00	%15.15	%15.15



(a)



(b)

Figure 9. The mean damage severities obtained using both algorithms for the single scenario of 72-bar spatial truss: a) Scenario 1, b) Scenario 2

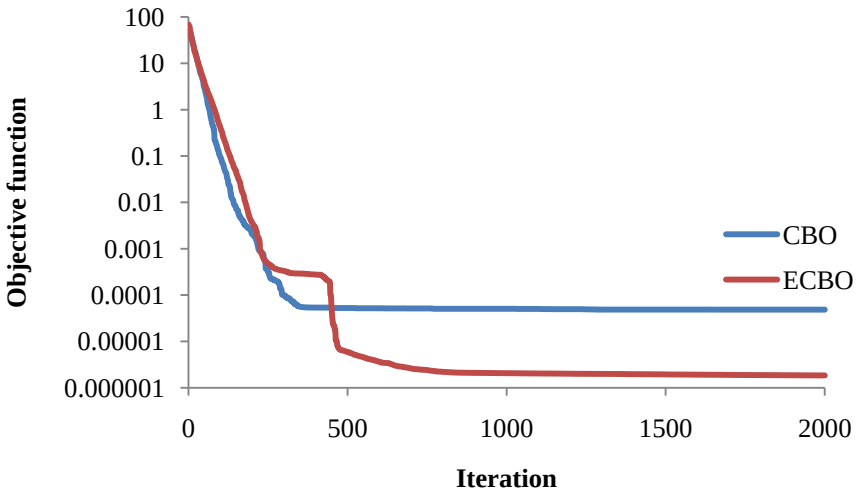


Figure 10. The mean convergence curves of both algorithms for the multiple scenario of 72-bar spatial truss

5. CONCLUDING REMARKS

In the present study, two meta-heuristic algorithms, so-called the Colliding Bodies Optimization (CBO) and Enhanced Colliding Bodies Optimization (ECBO), are utilized for damage detection of truss structures using changes in natural frequencies. First, an objective function is defined for finding the location and qualification of damage severity of structural elements based on modal information of damaged structures. Then, the optimization algorithms are used to determine the damage in structures by optimizing a cost function.

The capabilities of the proposed algorithms are assessed using three truss structural examples. The performances are measured using noise-free modal data through simulated damage scenarios. Two damage scenarios, namely single and multiple damage scenarios, are considered for comparison study in each example. To show the effect of initial damage severity in final results, the optimization algorithms were conducted through 20 independent runs with different initial populations. The obtained results from the numerical studies indicated that both optimization algorithms are viable to the problem of damage detection in truss structures. In both scenarios of examples, both algorithms were successful to predict the location and damage severity of damaged structural elements in some optimization algorithm runs. The mean results of 20 runs provided by both algorithms showed superior of the ECBO algorithm for finding the exact location of damaged structural elements compared to the CBO algorithm. The mean convergence curves obtained using both algorithms shows the capability of ECBO algorithm for finding the global optimal solution in the last iterations due to implementation of strategies in its formulation to escape from local optimal solution. For multiple damage scenarios, the optimization algorithms were less accurate than single damage scenarios. Thus, improving or proposing new optimization scheme may be necessary for the future problem of damage detection of large scale or frame structures and damage scenarios with higher multiplicity.

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