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PROBABILITY OF FATIGUE FAILURE OF STEEL FIBROUS CONCRETE CONTAINING MIXED FIBRES

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ABSTRACT

The paper presents a study on the fatigue strength of Steel Fibre Reinforced Concrete (SFRC) containing fibres of mixed aspect ratio. To obtain the fatigue lives of SFRC at various stress levels, beam specimens of size 500 mm x 100 mm x 100 mm were tested under four-point flexural fatigue loading. Static flexural tests were also carried out to determine the static flexural strength of SFRC prior to fatigue testing. The specimens incorporated 1.0% volume fraction of corrugated mixed steel fibres of size 0.6 x 2.0 x 25 mm and 0.6 x 2.0 x 50 mm in different proportions. All the fatigue tests were conducted on MTS servo-controlled test system where as all the static tests were conducted on a similar INSTRON machine. Fatigue life data obtained has been analysed in an attempt to determine the relationship among stress level S, number of cycles to failure N and probability of failure P_f for SFRC. It was found that this relationship can be represented reasonably well graphically by a family of S-N-P_f curves. The experimental coefficients of the fatigue equation have been obtained from the fatigue test data to represent the S-N-P_f relationships analytically.

Keywords: fatigue-life; failure probability; flexural strength; sfrc; stress level; survival probability

1. INTRODUCTION

Concrete is considered a brittle material, primarily because of its low tensile strength and low tensile strain capacity. Based on the experimental as well as theoretical evidence available, it can be concluded that the low tensile strength of concrete is due to internal flaws and micro-cracks present in the concrete. The use of randomly distributed, discrete fibres to improve the physical properties of the matrix is an age-old concept. Concrete can be modified to perform in a more ductile form, by the addition of randomly distributed discrete fibres in the concrete matrix. This results in a composite or two-phase system

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wherein the presence of one phase improves the basic properties of the other phase. The composite or two-phase concept of materials led to the use of new materials in which the weak matrix is reinforced by strong stiff fibres to produce a composite material with superior properties and performance.

The increased application of steel fibre reinforced concrete as an engineering material demands an additional knowledge of its behaviour under several types of loading to which it is subjected. Such knowledge is necessary not only to provide safe, efficient, and economical designs for the present, but also to serve as a rational basis for extended future applications.

Since Feret's pioneer tests [1] many researchers have carried out laboratory fatigue experiments to investigate the fatigue behaviour of plain as well as steel fibre reinforced concrete. However, specimen size, loading conditions and fatigue failure criteria have varied over a wide range. Most of the studies on steel fibre reinforced concrete were mainly confined to the determination of its flexural fatigue endurance limit for different type/volume fraction/aspect ratio of fibres [3, 5–9]. Some studies focussed attention on studying other aspects of fibre reinforced concrete in respect of fatigue. Yin et al. [10] studied the fatigue behaviour of steel fibre reinforced concrete under uniaxial and biaxial compression and observed that the S-N curves can be approximated by two straight lines connected by a curved knee instead of a single straight line. Wei et al. [11] studied the effect of fibre volume fraction, amount of silica fume and their composite action on fatigue performance of SFRC. Ong et al. [12] investigated the behaviour of steel fibre mortar overlayed concrete beams under cyclic loading whereas the behaviour of composite concrete sections reinforced with conventional steel bars and steel fibres, and subjected to flexural cyclic loading was analysed by Spadea and Bencardino [13] by means of a mechanical model. Jun and Stang [14] reported that the accumulated damage level in fibre reinforced concrete in fatigue loading was 1-2 order of magnitude higher than the level recorded in static testing of the same material. Effect of fly ash on fatigue properties of hardened cement mortar was studied by Taylor and Tait [15] whereas, Daniel and Loukili [16] investigated the behaviour of high strength fibre reinforced concrete under cyclic loading. SFRC with two types of hooked end steel fibres was tested by Cachim et al. [17] in an experimental study to evaluate the performance of plain and fibre reinforced concrete under compressive fatigue loading. In a review paper, Lee and Barr [18] provided a general overview of recent developments in the study of the fatigue behaviour of plain and fibre reinforced concrete.

2. RESEARCH SIGNIFICANCE

Literature review indicates that no research has been reported which addresses the behaviour of steel fibre reinforced concrete containing fibres of mixed aspect ratio under flexural fatigue loading. Steel fibre reinforced concrete containing mixed fibres can be used in many applications by combining fibres of varying length i.e. mixed aspect ratio in a matrix. Therefore, the present investigation was planned to investigate the flexural fatigue strength of steel fibre reinforced concrete containing mixed steel fibres. Although, the fatigue test data of SFRC shows considerable variability, even at a given stress level, due to the random orientation of fibres in concrete, a little effort has been directed to develop relationships among stress level, fatigue life and probability of failure or survival probability. The research work of McCall [2] provides good opportunity to generate a family of S-N-P_f curves for SFRC containing fibres of mixed aspect ratio. It is also proposed to develop a mathematical model for SFRC from the fatigue test data obtained in this investigation to represent the S-N-P_f curves analytically.

3. EXPERIMENTAL PROGRAM

The concrete mix used for casting the test specimens, its 28 days compressive strength and static flexural strength is shown in Table 1. Ordinary Portland Cement, crushed stone coarse aggregates with maximum size 12.5 mm and river sand were used. The materials used conformed to relevant Indian Standard specifications. The specimen incorporated two different aspect ratios of xerox type steel fibres namely 20 (fibre size 2.0 x 0.6 x 25 mm) and 40 (fibre size 2.0 x 0.6 x 50 mm) by weight of the longer and shorter fibres in the mix proportions by weight of 65%-35% and 35%-65% at fibre volume fractions of 1.0%. The detail of the mixes along with 28 day compressive strength and static flexural strength are presented in Table 2. The specimen used for flexural fatigue tests as well as static flexure tests were fibre concrete beams of size 100 x 100 x 500 mm. The mixing of the concrete was done in a rotary mixture and the fibres were gradually sprinkled into the drum by hand. Concrete was placed in the moulds in two layers, each layer vibrated properly such that no preferred orientation of the fibres in the matrix was obtained. The cube specimens of size 150 x 150 x 150 mm were used to determine the 28 days compressive strength of concrete.

The static flexural tests on a particular batch were conducted just before the fatigue testing of the same. All the static flexural tests were conducted on a 100 kN INSTRON closed loop electrohydraulic Universal Testing Machine. The beams were simply supported on a span of 450 mm and loaded at third points. Flexural fatigue testing of steel fibre reinforced concrete was the main thrust of the investigation. All the fatigue tests were conducted on a 100 kN MTS closed loop electrohydraulic Universal Testing Machine. The span/points of loading in the fatigue tests were kept the same as those for the static flexural tests. Fatigue tests were conducted at different stress levels *'S'* ($S = f_{max}/f_r$, f_{max} = maximum fatigue stress, f_r = static flexural stress), ranging from 0.90 to 0.80. Constant amplitude, sinusoidal, non-reversed loads were applied at a frequency of 12 Hz. The load applied to the specimen was sensed by the load cell attached to the cross head of the machine, and the load cell output was used as a feedback signal to control the load applied by the actuator. The number of cycles to failure of the specimen at a particular stress level were noted from the cycle counter of the machine and recorded as its fatigue-life.

4. ANALYSIS OF FATIGUE TEST DATA

The fatigue life data at different stress levels, arranged in ascending order, obtained for SFRC containing fibres of mixed aspect ratio is summarized in Tables 3 and 4.

Table 1. Concrete mix proportions, compressive and static flexural strength of plain concrete.

Water/Cement Ratio	Sand/Cement Ratio	Coarse Aggregate/Cement Ratio	28 Days Compressive Strength (MPa)	Static Flexural Strength (MPa)
0.35	1.35	2.12	57.82*	5.35**

*** Average of 10 tests; ** Average of 12 tests.**

Table 2. Steel fibrous concrete mixes, compressive and static flexural strength.

Fibre Volume Fraction (%)	Fibre Mix Proportion by Weight (%)		28 Days Average Compressive Strength (MPa)	Average Static Flexural Strength (MPa)
	50 mm Long Fibres+	25 mm Long Fibres++		
1.0	65	35	62.40	7.61
1.0	35	65	64.69	7.49

+ 50 mm long, 2 mm wide and 0.6 mm thick; ++ 25 mm long, 2 mm wide and 0.6 mm thick.

Table 3. Fatigue life data for SFRC at different stress levels, $V_f = 1.0\%$, (65% 50 mm + 35% 25 mm Long Fibres)

S.No.	Stress Level 'S'			Probability of Failure $P_f = m/(n+1)$
	0.80	0.85	0.90	
1	70161	6555	848	0.1000
2	103431	8439	1483	0.2000
3	111396	9616	1761	0.3000
4	139749	9690	1915	0.4000
5	162420	10627	2526	0.5000
6	226486	14048	2872	0.6000
7	303981	22628	2958	0.7000
8	330739	26036	3036	0.8000
9	450736	30853	5262	0.9000

5. FAMILY OF S–N–P_F CURVES

In this analysis, the graphical method similar to that employed by McCall [2] is used with a slight modification. As might be expected, the different specimens tested at a given stress level failed at different number of cycles. The data is analyzed by ranking the specimens in the order of the number of cycles to failure and the probability of failure P_f is calculated by dividing the rank of each specimen 'm' by (n + 1), where 'n' equals the total number of specimens tested at a particular stress level. The calculated values of probability of failure P_f are shown in Tables- 3 and 4 for 65% 50 mm + 35% 25 mm and 35% 50 mm + 65% 25 mm long fibres respectively. The reason for dividing by (n + 1), rather than 'n' is to avoid obtaining a probability of failure equal to 1.0 for the specimen having greatest fatigue life [4].

Table 4. Fatigue life data for SFRC at different stress levels, V_f = 1.0%, (35% 50 mm + 65% 25 mm Long Fibres)

S.No.	**Stress Level 'S'**			**Probability of Failure $P_f = m/(n+1)$**
	0.80	**0.85**	**0.90**	
1	94812	12428	1116	0.1000
2	99194	19177	1716	0.2000
3	120991	20918	2053	0.3000
4	199551	23165	2299	0.4000
5	209142	26665	3222	0.5000
6	284033	38227	3728	0.6000
7	325261	48842	3790	0.7000
8	462541	62119	3879	0.8000
9	583010	70436	7330	0.9000

Figure 1 presents the S-N-P_f diagram for fatigue life data for SFRC with 65% 50 mm + 35% 25 mm long fibres. The probability of failure P_f is plotted against the fatigue life N for each stress level as a first step. This plot is shown in the lower left part of the Figure 1. This may be termed as a family of N-P_f curves. The next step consists of generating a family of S-N curves. Using the N-P_f curves obtained in the previous step, a family of S-N curves are drawn. This plot is shown in the upper right part of Figure 1. From the S-N curves, S-P_f curves are plotted and these are shown in the upper left part of Figure 1. Figures 2 presents the family of S-N-P_f curves obtained in the same manner as described above for fatigue life data of SFRC with 35% 50 mm + 65% 25 mm long fibres.

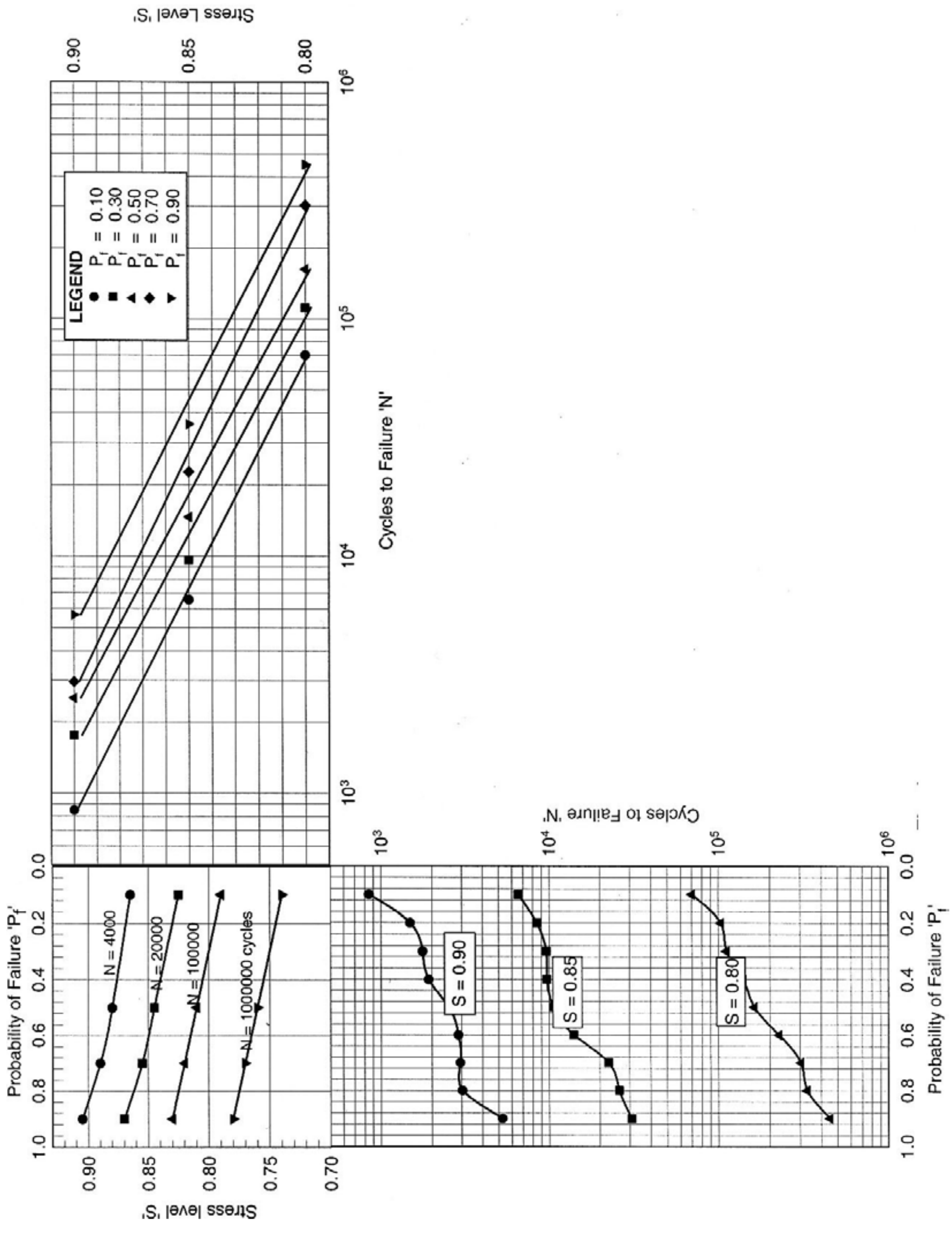

Figure 1. S-N-P_f diagram for SFRC under flexural fatigue loading, $V_f = 1.0\%$, (65% 50mm+35%25mm Long Fibres)

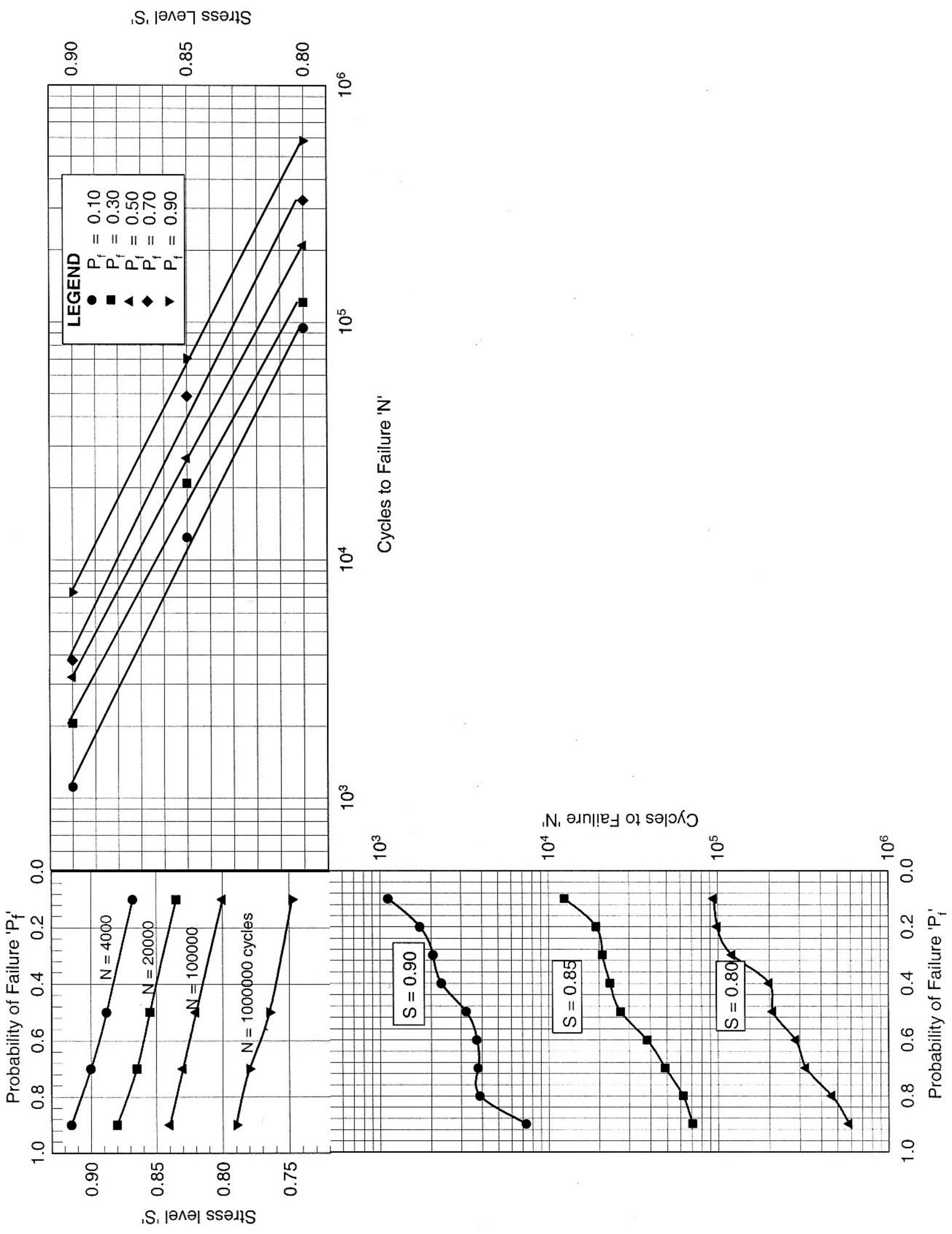

Figure 2. S-N-P_f diagram for SFRC under flexural fatigue loading, V_f = 1.0%, (35% 50mm+65%25mm Long Fibres)

6. THEORETICAL ANALYSIS

Another way to describe the S-N-P_f relationship is by using a mathematical relation. Following function used by McCall [2] to describe the S-N-P_f relationship for plain concrete is adopted here to represent S-N-P_f relationships for steel fibre reinforced concrete:

$$L = (10)^{-a(S)^b (\log N)^c} \tag{1}$$

where a, b and c are the experimental constants, S is stress level and L is the survival probability which equal to 1 – P_f, where P_f is probability of failure. It is simpler to use this equation in terms of survival probability L rather than probability of failure P_f.

To develop the equations for the proposed S-N-P_f relationships, it is necessary to evaluate the experimental constants a, b and c using fatigue life data for SFRC containing fibres in different proportions. In this way, two different relationships i.e. one each for SFRC with 65% 50 mm + 35% 25 mm and 35% 50 mm + 65% 25 mm long fibres representing the S-N-P_f curves will be obtained. These relationships have been developed as explained below:

Taking logarithms twice of the both sides of Eq.(1), one gets

$$\log(-\log L) = \log(a) + b\log(S) + c\log(\log N)$$

The following equation can be obtained after changing variables:

$$Y = A + bX + cZ \tag{2}$$

where Y = log(-logL), A = log(a), X = log(S) and Z = log(logN)

Since it is required to determine Z from X and Y i.e. to determine fatigue life for a given stress level and certain survival probability, the Eq.(2) can be modified as follows:

$$Z = A' + B'X + C'Y \tag{3}$$

in which $A' = -\frac{A}{c}$, $B' = -\frac{b}{c}$ and $C' = \frac{1}{c}$

As it is convenient to work with the variables measured from their sample means than the variables themselves [2], the following relationship can be derived:

$$\sum Z = \sum A' + B'\sum X + C'\sum Y$$

$$\frac{1}{n}\sum Z = A' + B'\frac{\sum X}{n} + C'\frac{\sum Y}{n}$$

$$\overline{Z} = A' + B'\overline{X} + C'\overline{Y} \tag{4}$$

Subtracting Eq.(4) from Eq.(3), the following expression is obtained:

$$Z - \overline{Z} = B'\left(X - \overline{X}\right) + C'\left(Y - \overline{Y}\right)$$

or

$$z = b'x + c'y \tag{5}$$

in which $z = Z - \overline{Z}$, $x = X - \overline{X}$ and $y = Y - \overline{Y}$

From the fatigue life data as obtained in this investigation for SFRC containing fibres of mixed aspect ratio, the following statistics are calculated:

For SFRC containing 65% 50 mm + 35% 25 mm long fibres

$$\sum x^2 = 0.0118 \qquad \sum y^2 = 4.3933 \qquad \sum z^2 = 0.1836$$
$$\sum xy = 0.0000 \qquad \sum yz = 0.2544 \qquad \sum xz = -0.0450$$
$$\overline{X} = -0.0710 \qquad \overline{Y} = -0.5750 \qquad \overline{Z} = 0.6190$$

For SFRC containing 35% 50 mm + 65% 25 mm long fibres

$$\sum x^2 = 0.0118 \qquad \sum y^2 = 4.3933 \qquad \sum z^2 = 0.1812$$
$$\sum xy = 0.0000 \qquad \sum yz = 0.2609 \qquad \sum xz = -0.0436$$
$$\overline{X} = -0.0710 \qquad \overline{Y} = -0.5750 \qquad \overline{Z} = 0.6390$$

By means of least square normal equations [2], the following expressions are obtained:

$$b'\sum x^2 + c'\sum xy = \sum xz \tag{6}$$

$$b'\sum xy + c'\sum y^2 = \sum yz \tag{7}$$

Using the statistics calculated above, the constants b' and c' in Eqs. (6) and (7) are determined for fatigue life of SFRC for each combination of fibres and the following equations are obtained by substituting these in Eq. (5):

$z = -3.8266x + 0.0579y$ for SFRC with 65% 50 mm + 35% 25 mm long fibres

$z = -3.7265x + 0.0594y$ for SFRC with 35% 50 mm + 65% 25 mm long fibres

or in the other form

$Z = 0.3807 - 3.8266X + 0.0579Y$ for SFRC with 65% 50 mm + 35% 25 mm long fibres

$Z = 0.4086 - 3.7265X + 0.0594Y$ for SFRC with 35% 50 mm + 65% 25 mm long fibres

and the final equations can be written in the following form:
for SFRC with 65% 50 mm + 35% 25 mm long fibres

$$L = (10)^{-2.67 x 10^{-7} (S)^{66.08} (\log N)^{17.27}} \quad (8)$$

and for SFRC with 35% 50 mm + 65% 25 mm long fibres

$$L = (10)^{-1.33 x 10^{-7} (S)^{62.72} (\log N)^{16.83}} \quad (9)$$

Equations (8) and (9) can be used by the design engineers to predict the flexural fatigue strength of steel fibre reinforced concrete for the desired level of survival probability.

7. CONCLUSIONS

Flexural fatigue life data obtained in this investigation for SFRC with 1.0% fibre content with mixed aspect ratio of fibres i.e. 65% 50 mm + 35% 25 mm and 35% 50 mm + 65% 25 mm long fibres has been analyzed in an attempt to determine the relationship among stress level S, fatigue life N and probability of failure P_f or survival probability L. A family of S-N-P_f curves have been generated for SFRC containing fibres in different proportions. The following equations
for SFRC with 65% 50 mm + 35% 25 mm long fibres

$$L = (10)^{-2.67 x 10^{-7} (S)^{66.08} (\log N)^{17.27}}$$

and for SFRC with 35% 50 mm + 65% 25 mm long fibres

$$L = (10)^{-1.33 x 10^{-7} (S)^{62.72} (\log N)^{16.83}}$$

have been developed to represent the S-N-P_f relationships mathematically and can be useful for predicting the flexural fatigue strength of steel fibre reinforced concrete for desired level of survival probability and for a given combination of fibres.

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NOTATIONS

S = stress level = f_{max}/f_r
f_r = static flexural stress
f_{max} = maximum fatigue stress
f_{min} = minimum fatigue stress
L = survival probability
P_f = probability of Failure
R = stress ratio = f_{min}/f_{max}
N = fatigue-life or number of cycles to failure

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