ASIAN JOURNAL OF CIVIL ENGINEERING (BUILDING AND HOUSING) VOL. 7, NO. 2 (2006)
PAGES 195-204

Technical Note

EFFECT OF MATERIAL PROPERTIES ON BEHAVIOR OF OVER-REINFORCED CONCRETE BEAMS

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ABSTRACT

Reinforced concrete structures are made up of two materials with different characteristics, namely, concrete and steel. The typical stage in the load-deformation behavior of a reinforced concrete simply supported beam is highly nonlinear and the response can be roughly divided into three ranges of behavior: the uncracked elastic stage, the crack propagation stage and the plastic (yielding of steel or crushing of concrete) stage. The stress-strain relation of concrete is not only nonlinear, but is different in tension than in compression. A nonlinear numerical model has been developed considering material and geometric nonlinearities. Material nonlinearity is simulated by considering parabolic stress-strain relationship of concrete and bilinear relationship of reinforcing steel which can model both elastic-perfectly plastic and plastic strain hardening. The modified Newton-Raphson technique has been used for the solution of the nonlinear equations. The load-deformation behavior of the over-reinforced high strength concrete beams has been carried out with the model. From the analyses it is observed that for the high strength concrete the increase in steel content increases the strength and stiffness but decreases the ductility.

Keywords: over-reinforced concrete beams, nonlinear material behavior, reinforcement ratio, strength of concrete, ductility, numerical analysis

1. INTRODUCTION

Reinforced concrete (RC) is one of the most important building materials and is widely used in many types of engineering structures. The economy, the efficiency, the strength and the stiffness of reinforced concrete make it an attractive material for a wide range of structural applications. The ultimate objective of the designer is to create a structure that is safe and

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economical. The safety and serviceability assessment of the structures necessitate the development of accurate and reliable methods and models for their analysis. The rise in cost of materials used in structures and labor costs encourage engineers to seek more economical alternative designs often resorting to innovative construction methods but without lowering the safety of the structure. In addition, the extent and impact of disaster in terms of human and economical loss in the event of structural failure promote designers to check the design thoroughly.

The development of numerical models for the analysis of the response of RC structures is complicated due to a) Reinforced concrete is a composite material made up of concrete and steel, two materials with very different physical and mechanical behavior; b) Concrete exhibits nonlinear behavior even under low level loading due to nonlinear material behavior, environmental effects, cracking, biaxial stiffening and strain softening; c) Reinforcing steel and concrete interact in a complex way through bond-slip and aggregate interlock. These complex phenomena have led engineers in the past to rely heavily on empirical formulas for the design of concrete structures, which were derived from numerous experiments. With the advent of digital computers and powerful methods of analysis, much effort to develop analytical solutions which would obviate the need for experiments have been undertaken by investigators.

2. NUMERICAL MODEL FOR RC BEAMS

A numerical model [1] has been developed for the analysis of reinforced concrete members. The stiffness of concrete and reinforcing steel is formulated separately. The results are then superimposed to obtain the element stiffness; the reinforcing steel is assumed to carry stress along its axis only and the effect of dowel action of reinforcement is neglected. The end displacements of the steel element are assumed to be compatible with boundary displacements of concrete element so that perfect bond is implied. In stiffness method, at first stiffness matrix of structural element of RC member is developed. Then for a particular load, resulting deflection and forces of an element at node are obtained by iterative method. To develop a nonlinear numerical model it is required to know the stress-strain relationship of constituent materials of RC. In the design of RC structural members, uniaxial compressive strength of concrete obtained from cylinder test is one of the most important design parameter. Typical stress-strain curves presented in [2] and [3] for normal density and lightweight concrete and presented in [4] for prestressed concrete had shown that concrete had a similar nonlinear character even at a normal level of stress in compression. The curves are linear up to about half of the compressive strength. The peak of the curve for high strength concrete is relatively sharp, but for low strength concrete the curve has a flat top. Simply, all the curves consist of an initial relatively straight line portion which then begins to curve to the horizontal, reaching the maximum stress for normal density concrete followed by a falling branch. In describing the uniaxial stress-strain behavior of concrete various idealizations are available. The linearly elastic-perfectly plastic model was used by [5] in a study of reinforced concrete slabs and walls. The inelastic-perfectly plastic model proposed by the European Concrete Committee [6] made up of a parabola and a horizontal line. The model proposed by Hognestad [7] is parabolic shape for ascending branch and linear for descending branch is widely used for idealization of concrete uniaxial compressive stress. In the present model parabolic stress-strain relationship for concrete for both the ascending and descending branch is used. The stress-strain relationship of concrete used in the model is presented in Figure 1.

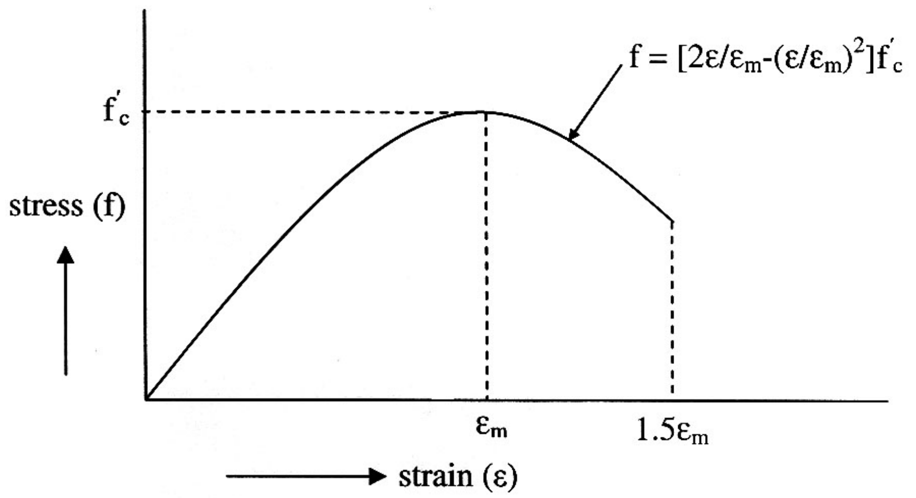

Figure 1. Stress-strain relationship of concrete used in the model

The steel stress-strain relationship exhibits an initial linear elastic portion, a yield plateau, a strain hardening range in which stress again increases with strain and, finally, a range in which the stress drops off until fracture occurs. The extent of the yield plateau is a function of the tensile strength of steel. High-strength, high-carbon steels, generally, has a much shorter yield plateau than relatively low-strength, low-carbon steels.

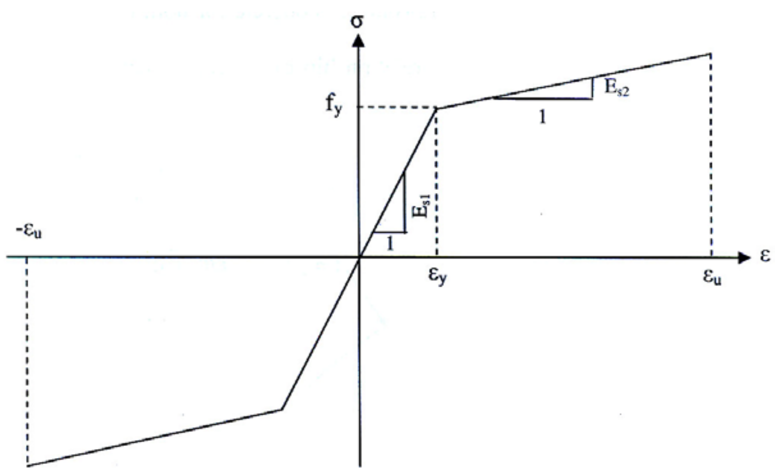

Figure 2. Stress-strain relationship of steel used in the model

For the analysis and design of RC structures it is required to define the shape of stress-strain behaviour of steel reinforcement. There are a number of stress-strain relationships available to idealize the steel as presented in [3]. The elastic perfectly plastic model is widely used for idealization of reinforcing steel for simplicity but it neglects the strength increase due to strain hardening. In the present model for reinforcing steel both the bilinear (elastic strain hardening) and elastic-perfectly plastic stress-strain relationship for steel are used. Figure 2 presents the stress-strain relationship of reinforcing steel used in the present model.

The numerical model has been developed using these nonlinear material properties. Since behavior is nonlinear the stiffness of the RC sections become a function of the state of strain and hence stress. This is due to the fact that strain distribution varies over the length of a member. However, using a large number of small elements the variations of stiffness properties over the length of member can be minimized and stiffness properties corresponding to the middle section of the element can be considered to be that of the element. This simplification makes it possible to derive the stiffness properties analytically. This approach has been utilized in developing a numerical model for limit state analysis of brickwork structures [8]. The complete derivation of stiffness of the beam-column element is presented in [9] for cracked and uncracked concrete sections respectively. Since the stiffness of RC members is the sum of stiffness of concrete and reinforcing steel, stiffness properties for steel derived by [10] has also been utilized for the development of the numerical model.

The nonlinear analysis algorithm consists of four basic steps: the formation of the current stiffness matrix, the solution of the equilibrium equations for the displacement increments, the determination of status of strains and internal forces in all the elements of the model and the convergence check. These steps are presented in some detail in the flow diagram in Figure 3.

3. RESPONSE OF OVER-REINFORCED CONCRETE BEAMS

Many researchers have carried out a number of experimental and analytical investigations and important advances have been made for RC beams. To illustrate the effectiveness of the present model test results by the researchers [11] and [12] are compared with the present model. In both cases the RC beams are over-reinforced. For such a beam, both ultimate strength and ductility can be enhanced by increasing concrete strength. The stress-strain curves for concrete in compression obtained from flexural tests are remarkably similar to those generated from uni-axially loaded specimen [12]. The analysis is based on flexural theory, but the use of stress-strain curves for axially loaded specimen gives close predictions of the experiment data on moment-curvature relationship and ultimate moment capacity of the beams. The stress-strain curve used for normal strength concrete is also used here to describe the behavior of high strength plain concrete.

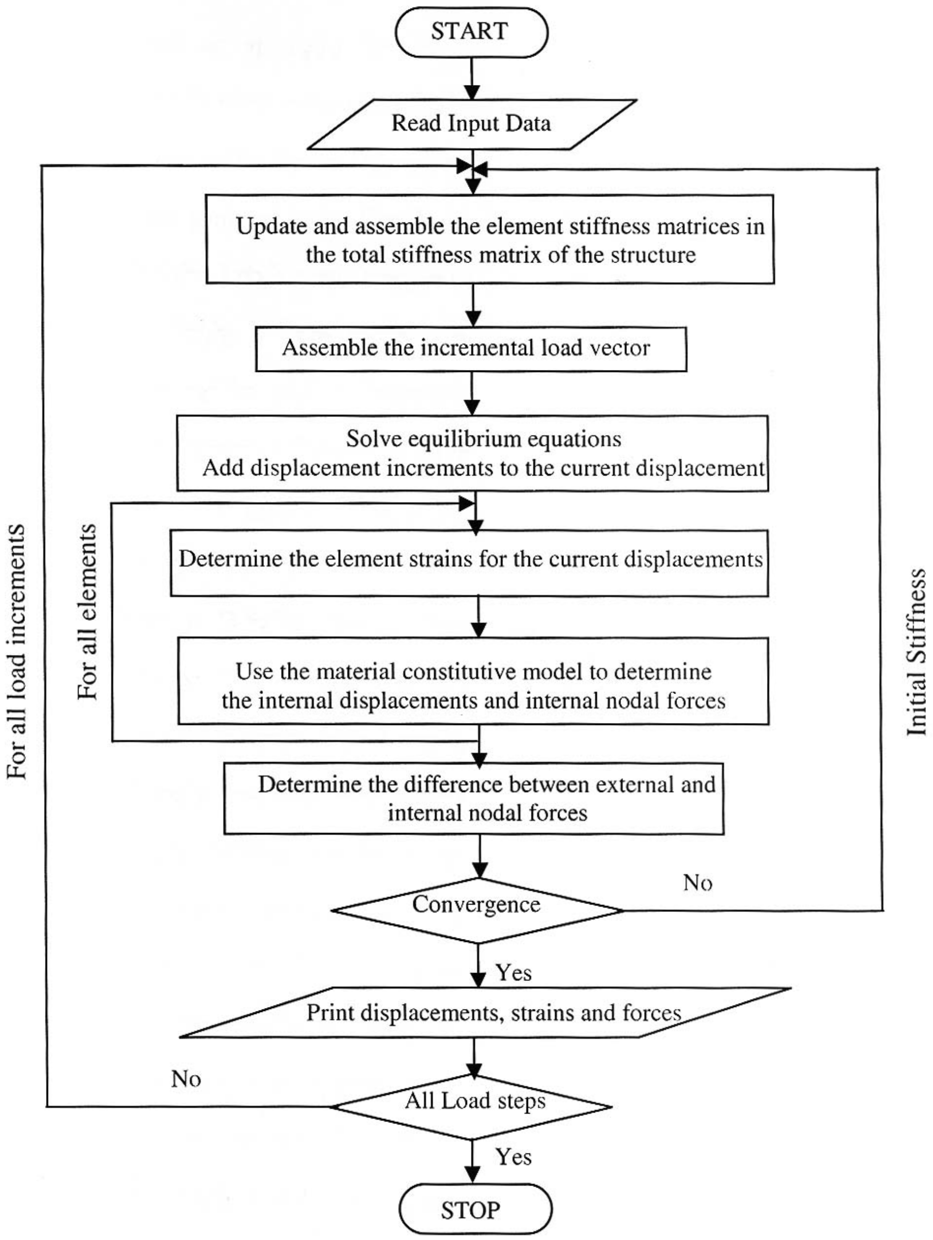

Figure 3. Outline of solution algorithm

To investigate whether the present numerical model is capable of predicting compression failure of concrete structures, two beams of different sizes and made of two different concrete types normal strength concrete (NSC) and high strength concrete (HSC) are studied numerically and compared with the numerical results obtained by [11]. Tables 1 and 2 present the peak load and peak displacement for small RC beams and two different sizes of beams respectively.

Table 1. Summary of peak load and peak displacement for small beam [11]

Concrete type	Test results		Predicted				Test/Predicted			
			Ozbolt et al.		Present analysis		Ozbolt et al.		Present analysis	
	P_u (kN)	δ_u (mm)	P_u (kN)	δ_u (mm)	P_u (kN)	δ_u (mm)	P_{test}/P_u	δ_{test}/δ_u	P_{test}/P_u	δ_{test}/δ_u
NSC	16.4	21.0	15.5	19.3	15.9	16.2	1.06	1.09	1.03	1.30
HSC	56.4	55.7	50.2	44.8	63.2	47.2	1.12	1.24	0.89	1.18

The numerical results obtained by the model observed failure due to concrete crushing in the compressive zone of the beam as was obtained in the tests for all over-reinforced beams. The numerical results obtained using the present model is able to predict the compressive failure of NSC beams.

In the second comparison [12] the test program had been carried out to observe the flexural behaviour of over-reinforced HSC beams with and without confinement in the compression zone. Concrete strengths ranging from 60 to 110 MPa were used to observe the flexural response of the beams. The study included the effect of concrete compressive strength on the beam behavior. The beams were designated as A4-0.0C, B4-0.0C, C4-0.0C, D4-0.0C. It was observed that failure of the beams was characterized by sudden crushing of concrete prior to yielding of the steel. Since the failure of an over-reinforced beam is typically by the spalling of concrete, measurement of curvature at or beyond the ultimate load was very difficult and sometimes impossible. However, the results that were obtained from test reasonably fits well with the model predictions indicating that stress-strain curves obtained from uni-axially loaded specimens may be applicable to predict the flexural response of a HSC beam. The ultimate moments and loads predicted by the model for beams are presented and compared with the experimental results in Tables 3 and 4, respectively.

Table 2. Summary of peak load and peak displacement for NSC beams of two sizes, Small (h = 200mm) and Large (h = 400 mm) [11]

Beam Size	Test results		Predicted				Test/Predicted			
			Ozbolt et al.		Present analysis		Ozbolt et al.		Present analysis	
	P_u (kN)	δ_u (mm)	P_u (kN)	δ_u (mm)	P_u (kN)	δ_u (mm)	P_{test}/P_u	δ_{test}/δ_u	P_{test}/P_u	δ_{test}/δ_u
Small	16.4	21.0	15.5	19.3	15.9	16.2	1.06	1.09	1.03	1.30
Large	64.1	47.8	56.0	41.1	56.5	50.4	1.14	1.16	1.13	0.95

Table 3. Comparison between test and predicted values of ultimate moment [12]

Beam	Test ultimate moment, $M_{u,test}$, kN-m	Calculated or Predicted ultimate moment, M_u, (kN-m)			$M_{u,test}/M_{u,\ predicted}$		
		ACI method	Mansur et al.(1997)	Present analysis	ACI method	Mansur et al.	Present analysis
A4-0.0C	106.6	98.5	112.7	108.9	1.08	0.95	0.98
B4 -0.0C	122.6	115.8	128.2	124.0	1.06	0.96	0.99
C4 -0.0C	130.1	131.0	140.4	139.9	0.99	0.93	0.94
D4-0.0C	137.6	145.1	150.9	152.9	0.95	0.91	0.90

Table 4. Comparison between test and predicted values of ultimate load and deflection [12]

Beam	Test ultimate load and deflection at ultimate load		Predicted ultimate load and deflection at ultimate load		Test/Present analysis	
	$P_{ult,test}$ (kN)	δ_{test} (mm)	$P_{u,pred.}$ (kN)	$\delta_{pred.}$ (mm)	$P_{ult,test}/P_{u,pred.}$	$\delta_{test}/\delta_{pred..}$
A4-0.0C	215	25	217	25.3	0.99	0.99
B4-0.0C	245	26	247	25.5	0.99	1.02
C4-0.0C	255	29	279	29.2	0.91	0.99
D4-0.0C	280	32	305	31.5	0.92	1.02

It may be observed that the ratio of test to predicted ultimate strengths varies from 0.90 to 0.98, with an average of 0.95. The reasonably good agreement between the two again shows that stress-strain curve obtained from the uni-axially loaded specimens can be applied to predict the response of HSC flexural members. From analytical solution it is observed that for HSC beams the uniaxial behaviour may be reasonably simulated by the model.

4. EFFECT OF CONCRETE STRENGTH AND REINFORCEMENT

To observe the effect of concrete strength on the moment-curvature response of the beams without any confinement, but with different concrete strengths an analytical studies have been carried out. Figure 4 presents the moment vs. curvature curves of the beams without any confinement, but with different concrete strengths [12]. It may be observed that an increase in concrete strength increases the ultimate load carrying capacity of the beam.

In order to see the effect of percentage of steel on the response of HSC beam the beam A4-0.0C has been considered for analysis by the model. Three percentages of steel is considered for the analysis. Figure 5 presents the effect of steel percentages on the moment-curvature response of the beam A4-0.0C at the centre of the span.

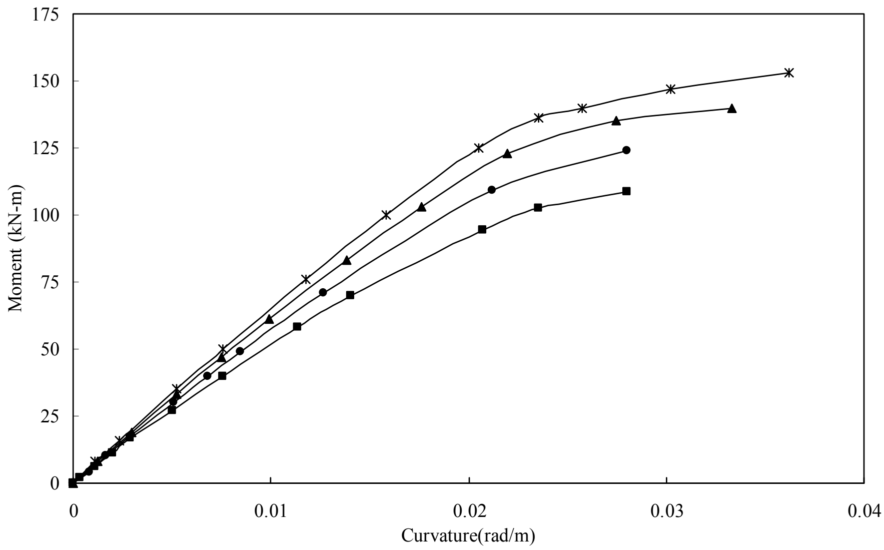

Figure 4. Effect of concrete compressive strength on the moment-curvature response of the beams predicted by the model

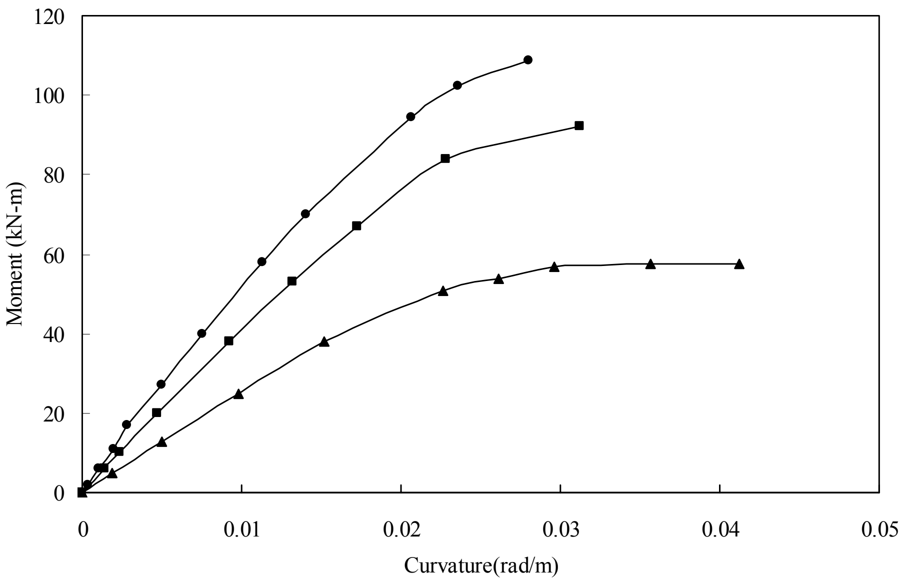

Figure 5. Effect of percentage of steel on the moment-curvature response of the A4-00.C beam predicted by the model

A numerical investigation has also been carried out by the present model for the beam A4-0.0C to observe the effect of percentage of steel on the response of various parameters such as load, deformation, stiffness (force required per unit deflection of the member at centre of the span) and curvature. Figures 6, 7, 8 and 9 present the variations of these parameters due to percentage of steel.

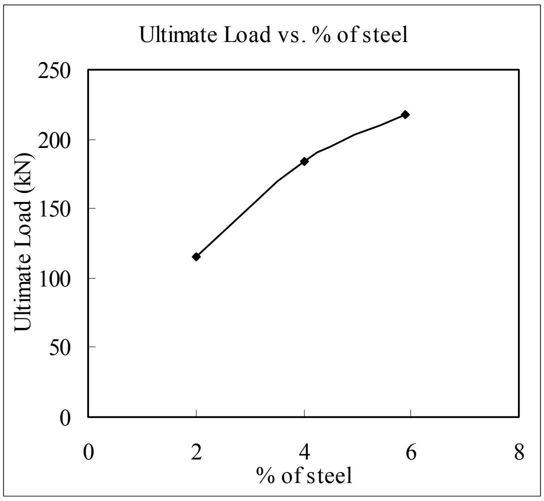

Figure 6. Effect of percentage of steel on load

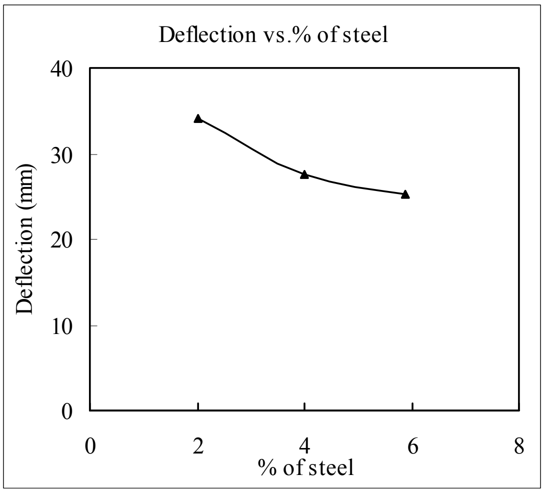

Figure 7. Effect of percentage of steel on deflection at ultimate load

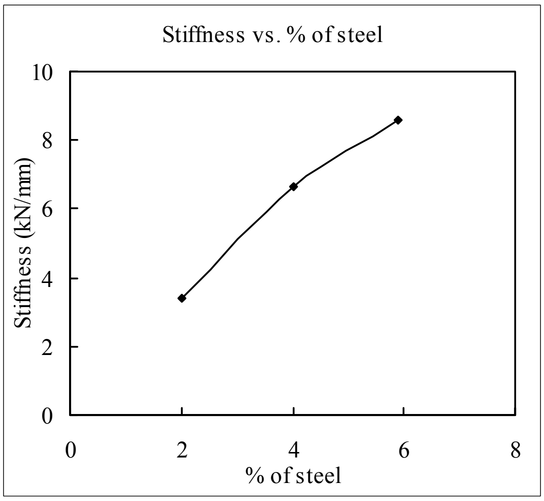

Figure 8. Effect of percentage of steel on stiffness

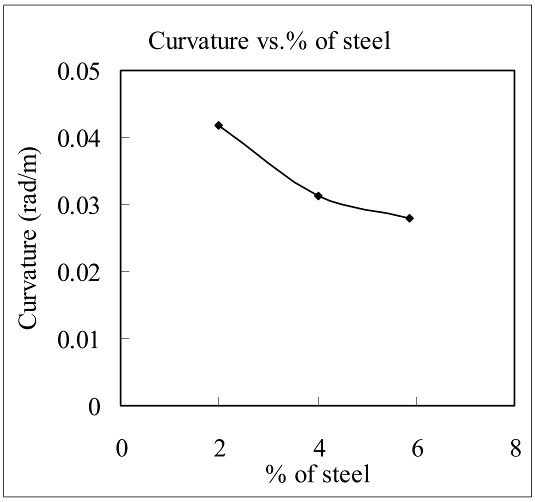

Figure 9. Effect of percentage of steel on curvature

From the above figures it is observed that an increase in tension steel increases the ultimate load carrying capacity and stiffness but decreases the deflection and curvature of the section.

5. CONCLUSIONS

The objective of this paper is to predict the behavior of over-reinforced concrete beams. The numerical model which included the combined stiffnesses of concrete and reinforcing steel and cracking of the concrete can predict the behavior of the over-reinforced concrete beams. The model also included the nonlinear stress-strain relationships of constituents of reinforced concrete. From the analysis carried out by the model it is observed that for the case of high strength concrete beams an increase in the reinforcing steel increases the strength and stiffness of the members but decrease the deflection and curvature of the members and the curvature of over-reinforced concrete beams increases as the strength of concrete is increased.

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