ASIAN JOURNAL OF CIVIL ENGINEERING (BUILDING AND HOUSING) VOL. 7, NO. 6 (2006)
PAGES 639-649

A METHOD FOR FREE VIBRATION ANALYSIS OF STIFFENED MULTI-BAY COUPLED SHEAR WALLS

K.B. Bozdogan*
Ege University, Civil Engineering Department, Izmir, Turkey

ABSTRACT

In this study an approximate method based on the continuum approach and transfer matrix method for free vibration analysis of multi bay coupled shear walls is presented. In this method the whole structure is idealized as sandwich beam in this method. Initially the differential equation of this equivalent sandwich beam is written then shape functions for each storey can be obtained by the solution of differential equations. By using boundary conditions and storey transfer matrices which are obtained by these shape functions, system modes and periods can be calculated. Reliability of the study is shown with a few examples. A computer program has been developed in MATLAB and numerical samples have been solved for demonstration of the reliability of this method. The results of the samples display the agreement between the present method and the other methods given in literature.

Keywords: Coupled shear wall, transfer matrix, free vibration

1. INTRODUCTION

Shear walls are commonly used in tall buildings with the aim to increase the resistance to lateral loads. They are formed as coupled shear walls because of the rows of openings constituted for the architectural aspects such as windows, doors etc. The behaviour of coupled shear walls can be improved by incorporating stiffening beams at various level. A number of methods are available for the analysis of tall buildings with coupled shear wall systems. These include the finite element method [1-4], continuous connection method [5-12], the equivalent frame method [13-14], and the boundary element method [15].

In this study, an approximate method based on continuum system model and transfer matrix approach is suggested for the free vibration analysis of coupled shear walls.

2. ANALYSIS

Under the horizontal loads, coupled shear walls demonstrate neither Timoshenko beam, nor

* Email-address of the corresponding author: kanat.burak.bozdogan@ege.edu.tr

Euler-Bernouilli beam behavior. The behavior of coupled walls is equivalent sandwich beam which denotes the total of these two types of behavior (Figure 1). Initially the differential equation of this equivalent sandwich beam can be written. The flexural rigidity of sandwich beam contains of the sum of the flexural rigidity of shear walls near the openings. The shear rigidity of the sandwich beam is equal the sum of the connecting beam shear rigidities. The global flexural rigidity of the structural system can be calculated with the help of axial deformation of shear walls near the openings.

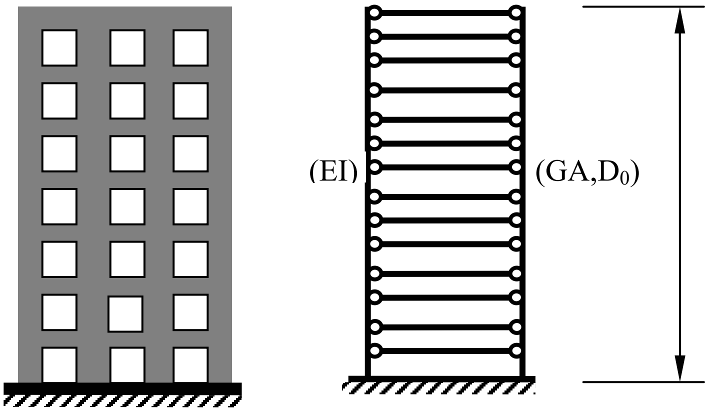

Figure 1. Mathematical model of equivalent sandwich beam

2. STOREY TRANSFER MATRICES

Under the horizontal loads equation of coupled shear wall of ith storey can be written as,

$$EI_i \frac{\partial^4 y_i}{\partial z^4} - GA_i \frac{\partial^2 y_i}{\partial z^2} + GA_i \frac{\partial \psi_i}{\partial z} = 0 \tag{1}$$

$$GA_i \frac{\partial y_i}{\partial z} + D_i \frac{\partial^2 \psi_i}{\partial z^2} - GA_i \psi_i = 0 \tag{2}$$

where y_i are the total shape functions, z are the vertical axis, ψ_i is the rotation angles of coupled shear wall because of bending, EI_i are the total bending rigidities of shear wall and D are the bending rigidities which represent the axial deformation and can be calculated as the equation below,

$$D_i = \sum E A d^2 \tag{3}$$

GA_i are the equivalent shear rigidities of connecting beams and can be calculated as in

Eq. (4) [16],

$$GA_i = \Sigma \frac{6EI_{bj}[(d_j + s_j)^2 + (d_j + s_{j+1})^2]}{d_j^3 h(1 + \frac{12\rho EI_{bj}}{GA_{bj} d_j^2})} \tag{4}$$

where d_j are the clear span lengths of coupling beam; s_j are the wall lengths; EI_{bj} and GA_{bj} are the flexural rigidities of connecting beam and the shear rigidity of connecting beams respectively; and ρ is the poisson ratio.

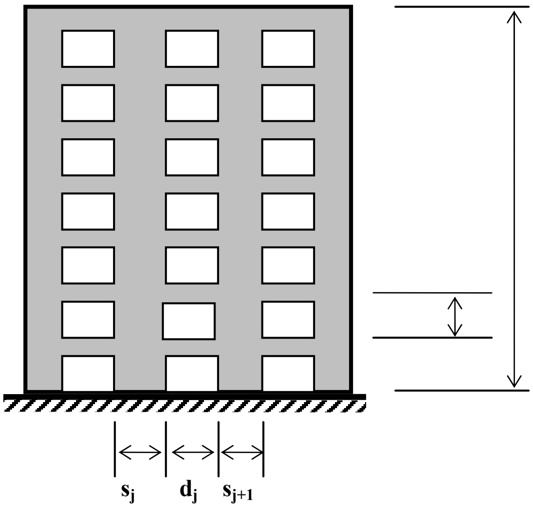

Figure 2. Coupled shear wall

With the solution of Eqs. (1) and (2) with respect to the z, total shape function and rotation angle can be obtained as,

$$y_i(z) = c_1 + c_2 z + c_3 z^2 + c_4 z^3 + c_5 \cosh(a_i z) + c_6 \sinh(a_i z) \tag{5}$$

$$\psi_i(z) = c_2 + 2c_3 z + (3z^2 + 6b_i)c_4 + \left(-\frac{EI_i}{GA_i} a_i^2 + a_i\right) c_5 \sinh(a_i z) + \left(-\frac{EI_i}{GA_i} a_i^2 + a_i\right) c_6 \cosh(a_i z) \tag{6}$$

where c_1, c_2, c_3, c_4, c_5, c_6 are integral constants. a_i and b_i can be calculated by using formula given below.

$$a_i = \sqrt{(1 + \frac{D_i}{EI_i}) \frac{GA_i}{D_i}}\ ,\ b_i = \frac{D_i}{GA_i} \tag{7}$$

With the help of Eq. (5), the total rotation angle, bending moment of shear wall, bending moment because of the axial deformation and the total shear force can be obtained as follows.

$$y'(z) = c_2 + 2c_3 z + 3c_4 z^2 + c_5 a_i \sinh(a_i z) + c_6 a_i \cosh(a_i z) \tag{8}$$

$$M_{wi}(z) = EI_i y_i'' = EI_i (2c_3 + 6c_4 z + c_5 a_i^2 \cosh(a_i z) + c_6 a_i^2 \sinh(a_i z)) \tag{9}$$

$$M_{a_i}(z) = -D_i \psi_i' = -D_i (2c_3 + 6c_4 z + c_6 f_i a_i \sinh(a_i z) + c_5 f_i a_i \cosh(a_i z)) \tag{10}$$

$$\begin{aligned} V_i(z) &= EI_i \frac{d^3 y_i}{dz^3} - GA_i \frac{dy_i}{dz} + GA_i \psi_i = c_4 (6EI_i + 6GA_i b_i) \\ &+ c_5 \sinh(a_i z)(GA_i (f_i - a_i) + EI_i a_i^3) + c_6 \cosh(a_i z)(GA_i (f_i - a_i) + EI_i) \end{aligned} \tag{11}$$

where f_i,

$$f_i = \frac{-EI_i}{GA_i} a_i^3 + a_i \tag{12}$$

When Eq. (5), (8), (6), (9), (10) and (11) have written in a matrix form as,

$$\begin{bmatrix} y_i(z) \\ y_i'(z) \\ \psi_i(z) \\ M_{wi}(z) \\ M_{axi}(z) \\ V(z) \end{bmatrix} = \begin{bmatrix} 1 & z & z^2 & z^3 & \cosh(a_i z) & \sinh(a_i z) \\ 0 & 1 & 2z & 3z^2 & a_i \sinh(a_i z) & a_i \cosh(a_i z) \\ 0 & 1 & 2z & (3z^2 + 6b_i) & f_i \sinh(a_i z) & f_i \cosh(a_i z) \\ 0 & 0 & 2EI_i & 6EI_i z & EI_i a_i^2 \cosh(a_i z) & EI_i a_i^2 \sinh(a_i z) \\ 0 & 0 & -2D_{0i} & -6D_{0i} z & -D_{0i} f_i a_i \cosh(a_i z) & -D_{0i} f_i a_i \sinh(a_i z) \\ 0 & 0 & 0 & (6EI_i + 6GA_i b_i) & \sinh(a_i z)(GA_i (f_i - a_i) + EI_i a_i^3) & \cosh(a_i z)(GA_i (f_i - a_i) + EI_i a_i^3) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} \tag{13}$$

At the initial point of the storey for z=0, equation (13) can be written as

$$\begin{bmatrix} y_i(0) \\ y_i'(0) \\ \psi_i(0) \\ M_{wi}(0) \\ M_{axi}(0) \\ V_i(0) \end{bmatrix} = A_i(0) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} \tag{14}$$

$$\underline{c} = \left[c_1 \; c_2 \; c_3 \; c_4 \; c_5 \; c_6 \right]^t \tag{15}$$

When vector $\underline{c}$ is solved out from formula (14) and is substituted ton the Eq. (13), Eq. (16) is obtained.

$$\begin{bmatrix} y_i(z) \\ y'_i(z) \\ \psi_i(z) \\ M_{wi}(z) \\ M_{axi}(z) \\ V_i(z) \end{bmatrix} = \mathbf{A_i A_i^{-1}(0)} \begin{bmatrix} y_i(0) \\ y'_i(0) \\ \psi_i(0) \\ M_{wi}(0) \\ M_{axi}(0) \\ V_i(0) \end{bmatrix} \tag{16}$$

where $\underline{T}_i = \underline{A}_i \underline{A}_i(0)^{-1}$ is the storey transfer matrix for z=h.

3. DYNAMIC ANALYSIS

For free vibration analysis of coupled shear wall the structure is considered as a discrete lumped mass system. The values of m_i can be approximated by

$$\mathrm{m_1} = \frac{\mathrm{M_T}}{\mathrm{n}} * 1.5 \tag{17}$$

$$m_i = \frac{M_T}{n} \quad \text{(i=2, 3...n–1)} \tag{18}$$

$$m_n = \frac{M_T}{2n} \tag{19}$$

where n is the number of storey and M_T is the total mass of the system.

The storey transfer matrices obtained in Eq. (16) can be used for dynamic analysis of coupled shear wall. Therefore, when considering the inertial forces in storey, the relationship between, *i*th and (*i+1*)th Storey can be written with the following matrix equation.

$$\begin{bmatrix} y_{(i+1)} \\ y'_{(i+1)} \\ \psi_{(i+1)} \\ M_{w(i+1)} \\ M_{ax(i+1)} \\ V_{(i+1)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ m_i w^2{}_i & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{T_i} \begin{bmatrix} y_i \\ y'_i \\ \psi_i \\ M_{wi} \\ M_{axi} \\ V_i \end{bmatrix} \tag{20}$$

where, m_i is the mass of *i*th storey and ω are the natural frequencies. Dynamic transfer matrix can be shown as T_{di} below.

$$\mathbf{T_{di}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \mathbf{m_i}\omega^2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{T_i} \tag{21}$$

The displacements and internal forces relationship between base and top of structures can be found as follows.

$$\begin{bmatrix} y_{top} \\ y^1_{top} \\ \psi_{top} \\ M_{wtop} \\ M_{axtop} \\ V_{top} \end{bmatrix} = T_{dn} T_{d(n-1)} \ldots\ldots T_{d1} \begin{bmatrix} y_{base} \\ y^1_{base} \\ \psi_{base} \\ M_{wbase} \\ M_{axbase} \\ V_{base} \end{bmatrix} \tag{22}$$

When the boundary condition is considered in Eq. (22), for nontrivial solution in $\mathbf{t_d} = \mathbf{T_{dn}} \mathbf{T_{dn-1}} \mathbf{T_{dn-2}} \ldots\ldots \mathbf{T_{d1}}$ is gained as:

$$f = \begin{bmatrix} t_{44} & t_{45} & t_{46} \\ t_{54} & t_{55} & t_{56} \\ t_{64} & t_{65} & t_{66} \end{bmatrix} \tag{23}$$

The values ω which set the determinant to zero are the natural frequencies of coupled shear wall.

4. EFFECT OF STIFFENING BEAM

The effect of the stiffening beam can be considered by

$$GA_{si} = r_i * GA_i \tag{24}$$

where r_i is a reduction factor and can be calculated by follow formula

$$r_i = \frac{I_s + I_b}{I_b} \tag{25}$$

5. PROCEDURE OF COMPUTATION

Procedure of computation of transfer matrix method is presented below step by step

1. Calculation of the structural properties of each storey (GA, EI, m, ...)
2. Computation of storey transfer matrices for each storey using the structural properties obtained in step 1.
3. Computation of system transfer matrix with the help of storey transfer matrices.
4. Applying the boundary conditions and obtaining the nontrivial equation.
5. Determination of the angular frequencies by using numerical method.
6. Using the angular frequencies determination of modes with the help of storey transfer matrices.

6. NUMERICAL EXAMPLES

In this part of this study, to verify the present method three numerical examples have been solved by a program written in MATLAB [17]. The results are compared with the ones which had been given in literature.

Example 1. A typical single bay coupled shear wall structure is analysed as an example. This shear wall rests on rigid foundation has 0.30m thickness and the following properties: d_l=2m, s_1= s_2= m, H= 95m, h=3.8m, coupling beam section: 0.3×0.3m, p(density)= 24.00 kN/m^3 and E=2.76×10^7 kN/m^2 .The location of the stiffening beam is located at nid-level of the structural height and. has section: 0.3×1.5m. The first five natural frequencies are calculated in this method and are compared with those found in the literature [18] (Table 1).

Table 1. Comparison of natural frequencies in Example 1 (Hz)

Mode	Kuang and Chau [18]	Present Method
1	0.76	0.75
2	2.93	2.84
3	8.12	8.03
4	13.30	12.90
5	22.46	22.01

Example 2. Consider the coupled shear wall shown in Figure 3. The coupled shear wall with three bays consists of 12 storey, and has the following properties: $E=2\times10^7$ kN/m^2, p=24.05 kN/m^3.The height of connecting beams is 80 cm, the cross-sectional area of stiffening beams is 0.5136 m^2, second moment area of stiffening beams is 0.441 m^{4}m and the thickness of the wall is 16 cm everywhere. The same coupled shear wall was considered in Ref. (Bikce and Aksogan.) and analyzed using both continous connection and finite element methods. Free vibration analysis was carried out and is compared with those found in the literature [19] (Table 2).

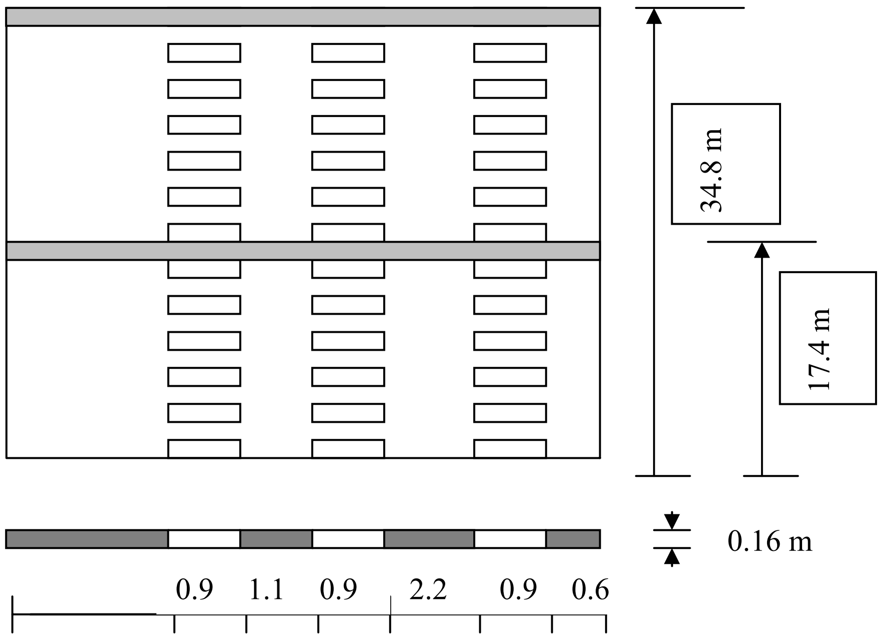

Figure 3. 12 storey coupled shear wall

Table 2. Comparison of natural frequencies in Example 2 (Hz)

Mode	SAP 2000 [19]	Bikce and Aksogan [19]	Present Method
1	3.1926	3.2138	3.1891
2	12.0504	12.2186	12.1494
3	30.7974	30.7603	33.7450
4	45.5791	45.6712	46.4534
5	71.4034	71.4489	72.2198

Example 3. The building with coupled shear walls considered is a part of a 24 story office building (Figure 4). It is designed for seismic loadings. The known dimensions are, H=67.2m, h=2.8 m, The height of connecting beams is 0.35 m, the cross-sectional area of stiffening beams is $0.3m^2$, the height of stiffening beams is 1 m, the thickness of the wall is 0.3m everywhere, $E=2.00\times10^7$ kN/m^2 and p=24.05 kN/m^3. .In this example the first five natural frequencies of the buildings can be found and compared by the literature [20] in Table 3.

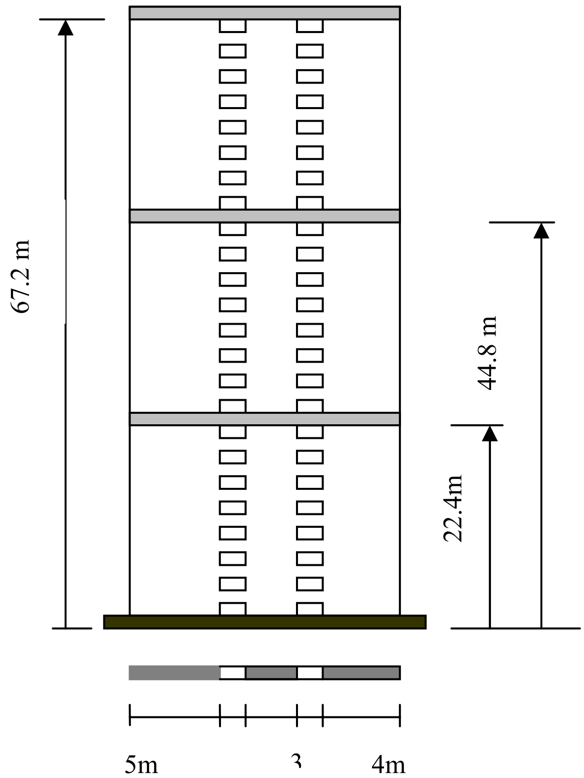

Figure 4. Third example Structure

Table 3. Comparison of natural frequencies in Example 3 (Hz)

Mode	SAP 2000 (Finite Element)	Emsen and Aksogan [20]	Present Method
1	1.32823	1.33192	1.3501
2	5.02593	5.01227	5.1362
3	10.3234	10.2786	10.3279
4	20.8250	20.8289	21.6247
5	29.7554	29.6079	30.1175

7. CONCLUSION

In this study, an approximate method based on continuum system model and transfer matrix approach is suggested for the static and dynamic analysis of coupled shear walls. In this method the whole structure is idealized as a sandwich beam. Examples demonstrate a good agreement with the finite element method solution given with literature. The proposed method is simple and accurate enough to be used both for at the concept design stage and final analyses. This method is also suitable for implementation on any computer programs.

REFERENCES

1. Kwan, A.K.H., Mixed finite elements for analysis of coupled shear/core walls., *ASCE Journal of Structural Engineering*, No. 59, **119**(1993) 1388-1408.
2. Kwan, A.K.H., Equivalence of finite elements and analogous frame modules for shear/core wall analysis. *Computers and Structures*, No. 2, **57**(1995):193-203.
3. Chaallal, O., Finite element model for seismic RC coupled walls having slender coupling beams, *ASCE Journal of Structural Engineering*, No. 10, **118**(1992) 2936-2943.
4. Kim, H.S, Lee, D.G, Analysis of shear wall with openings using super elements, *Engineering Structures*, **25**(2003) 981-991.
5. Rosman, R., Approximate analysis of shear walls subject to lateral loads. *Proceedings of the American Concrete Institute*, No. 6, **61**(1964), 717-734.
6. Basu, A., Nagpal, A.K., Bajaj, R.S. and Guliani, A., Dynamic characteristics of coupled shear walls, *ASCE, Journal of Structural Division*, **105**(1979) 1637-1651.
7. Coull, A., Smith, B.S., Tall Buildings 1. In Handbook of Structural Concrete. Edited by F.K., Kong, R.H., Evans, E. Cohen and F. Roll. Pitman Books Limited, London, England, 1983.
8. Li, Gq., Choo Bs., , A continuous-discrete approach to the free vibration analysis of stiffened pierced walls on flexible foundations. International *Journal of Solids and Structures*, No. 2, **33**(1996) 249-263.
9. Aksogan, O., Arslan, H.M. and Choo, B.S., Forced vibration analysis of stiffened coupled shear walls using continuous connection method, *Engineering Structures*, **25**(2003) 499-506.
10. Ha, H.H., and Tan, T.M.H., An efficient analysis of continuum shear wall models, *Candial Journal of Civil Engineering*, **26**(1999) 425-433.
11. Bozdogan, K.B, Ozturk, D. and Nuhoglu, A., Static and dynamic analysis of coupled shear walls by using transfer matrix method, 14th National Mechanics Congress, Hatay-Turkey, 2005 (In Turkish).
12. Zeidabadi, N.A., Mirtalae, K. and Mobasher, B., Optimized use of the outrigger system to stiffen the coupled shear walls in tall buildings, *The Structural Design of Tall and Special Buildings*, **13**(2004) 9-27, DOI:10.1002/tal.228.
13. Chaallal, O., Ghamallal, N., Seismic response of flexibly supported coupled shear walls, *ASCE Journal of Structural Engineering*, No. 10, **122**(1996) 1187-1197.

14. Smith, B.S., Modified beam method for analyzing symmetrical shear walls. *Journal of American Concrete Institute*. No. 2, **67**(1970) 977-980.
15. Rashed, Y.F., Analysis of building shear walls using boundary elements, *Engineering Analysis with Boundary Elements*, **24**(2000) 287-293.
16. Potzta, G., Kollar, L.P., Analysis of building structures by replacement sandwich beams, *International Journal of Solids and Structures*, **40**(2003) 535-553.
17. Matlab V7.1 2004, Users Manual, Mathworks Inc., USA.
18. Kuang, J.S., Chau, C.K., Free Vibration of stiffened coupled shear walls, *The Structural Design of Tall Buildings*, **7**(1998) 135-145.
19. Bikce, M., Aksogan, O. and Arslan, H.M, Dynamic analysis of coupled shear walls, *C.U.J., Fac. Eng. Arch*. No.1-2, **15**(2000) 45-71.
20. Emsen, E., Aksoğan, O., Dynamic Analysis of Coupled Shear Walls Using Various Methods, Congress on The Civil Engineering Problems of Antalya., September 2005 (In Turkish), pp. 22-24.