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# MULTI-OBJECTIVE OPTIMIZATION OF TIME-COST-QUALITY USING MULTI-COLONY ANT ALGORITHM

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## ABSTRACT

Construction planners often face the challenge of optimum resource utilization to compromise between different and usually conflicting aspects of projects. Time, cost and quality of project delivery are among the crucial aspects of each project. Emergence of new contracts that place an increasing pressure on maximizing the quality of projects while minimizing its time and cost, requires the development of models considering quality in addition to time and cost which has modeled extensively. In this paper, a new metaheuristic multi-colony ant algorithm is developed for the optimization of three objectives time-cost-quality as a trade-off problem. An example is analyzed to illustrate the capabilities of the present method in generating optimal/near optimal solutions. The model is also applied to two objective time-cost trade-off problem, and the results are compared to those of the existing approaches.

**Keywords**: time-cost-quality trade-off, multi-objective optimization, multi-colony, ant colony optimization

## 1. INTRODUCTION

The traditional time-cost trade-off problem has been the subject of intensive research since the development of the Critical Path Method (CPM) in the late 1950s. Construction planners face the challenge of optimum resource utilization to compromise between different aspects of projects, especially time and cost. Recent contracts consider the quality performance of projects in addition to time and cost. These new and emerging contracts impose an increasing pressure on decision makers in the construction industry to search for an optimal/near-optimal resource utilization plan that minimizes the construction cost and the time, while maximizing its quality. This creates new and pressing need for advanced resource utilization models that are capable of optimizing the multiple and conflicting objectives of construction time, cost and quality. El-Rayes et al. [1].

If durations of the activities are compressed, the cost will increase due to more resources

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allocated to their rapid accomplishment. On the other hand, using fewer resources will result in extended duration of activities. In addition to time and cost of activities, every resource utilization option will yield a specific performance quality. Trade-off between these conflicting aspects of project is a challenging job and as such planners are faced with numerous possible combinations for project delivery. As an example, the number of possible combinations in a project with 18 activities and 4 possible resource utilization options for each activity will be more than 6 billion. A novel searching tool would then be worthwhile for comprehensive yet efficient time-cost-quality trade-off problem.

Variety of methods has been used for modeling bi-objective Time-Cost Trade-off Problems (TCTP). Existing models can be classified as heuristic approaches and mathematical programming methods (Feng et al. [2]) Considering the objective of models these methods may also be categorized as shown in Table 1. The weaknesses of the heuristics and mathematical methods are widely documented in the literature (e.g. Zheng et al. [3]), but the major deficiency with most of the mathematical models is their inability to handle more than one objective. In addition, these methods often employ the hill climbing algorithms, which has only one randomly generated solution exposed to some kind of variation to create a better solution.

Table 1. Existing models for construction trade-offs classified by their objectives

| **Minimize project time and/or improve resource utilization** | **Time-cost trade-off for nonrepetitive construction** | **Minimize time and/or cost for repetitive construction** | **Minimizing time and/or cost while maximizing quality** |
|---|---|---|---|
| | Burns et al. 1996 | | |
| | Feng et al. 1997 | Elrayes and Moselhi 2001 | Bubu and Suresh 1996 |
| Easa 1989 | Li and Love 1997 | | |
| Chan et al. 1996 | Maxwell et al. 1998 | Hegazy and Ersahin 2001 | Khang an Myint 1999 |
| Hegazy 1999 | Li et al. 1999 | Hegazy and Wassef 2001 | Elrayes and Kandil 2005 |
| Gomar et al. 2002 | Feng et al. 2000 | | |
| | Zheng 2004 | Leu and Hwang 2001 | |
| | Zheng et al 2005 | | |

There have been extensive studies on time-cost trade-off problem but except two mathematical approaches used by Bubu et al. [4] and then by Khang et al. [5] there has been no proper approach for such three objective time-cost-quality trade-off problem until 2005 when El-Rayes et al. [1] reported their research in this field using multi-objective genetic algorithm. In this paper, a new metaheuristic approach is applied for optimization of three objective time-cost-quality problem based on multi-colony ant algorithm. Pareto archiving is introduced which is very efficient in developing the Pareto front in multi-objective problems.

## 2. MULTI-OBJECTIVE ANT COLONY OPTIMIZATION ALGORITHM

In this section a Multi-Objective Ant Colony Optimization (MOACO) algorithm is presented.

### *2.1 Ant Colony Optimization Algorithms*

In recent years, evolutionary and meta-heuristic algorithms have been extensively used as search and optimization tools in various problem domains, including science, commerce, and engineering. Ease of use, broad applicability, and global perspective may be considered as the primary reason for their success. Ant colony optimization algorithms are inspired by the fact that ants are able to find the shortest route between their nest and a food source, even though they are almost blind (Dorigo et al. [6]). Researchers have reported the robustness of ACO and their capacity to efficiently search for and locate an optimum/near optimum especially in discrete optimization problems.

In general, ACO algorithms employ a finite size of artificial ants with defined characteristics which collectively search for good quality solutions to the problem under consideration. Starting from an initial state, selected according to some case-dependent criteria, each ant builds a solution which is similar to a chromosome in a genetic algorithm. While building its own solution, each ant collects information on its own performance and uses this information to modify the representation of the problem, as seen by the other ants (Dorigo et al.[6]). The ant's internal states store information about the ant’s past behavior, which can be employed to compute the goodness/value of the generated solution. Artificial ants are permitted to release pheromone while developing a solution or after a solution has fully been developed, or both. The amount of pheromone deposited is made proportional to the goodness of the solution an artificial ant has developed (or is developing). Rapid drift of all the ants towards the same part of the search space is avoided by employing the stochastic component of the choice decision policy and the pheromone evaporation mechanism. In order to simulate the pheromone evaporation, the pheromone persistence coefficient ($\rho$) is defined which enables greater exploration of the search space and minimizes the chance of premature convergence to suboptimal solutions. A probabilistic decision policy is also used by the ants to direct their search towards the most interesting regions of the search space. The level of stochasticity in the policy and the strength of the updates in the pheromone trail determine the balance between the exploration of new points in the state space and the exploitation of accumulated knowledge(Dorigo et al. [6]).

Let $\tau_{ij}(t)$ be the total pheromone deposited on path ij at time t, and $\eta_{ij}(t)$ be the heuristic value of path ij at time t according to the measure of the objective function. Transition probability from node i to node j at time period t may be defined as (Dorigo et al. [7]):

$$P_{ij}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}(t)]^{\beta}}{\sum\limits_{l \in allowed}[\tau_{il}(t)]^{\alpha}[\eta_{il}(t)]^{\beta}} & \text{if } j \in \text{allowed} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Where α and β are parameters that control the relative importance of the pheromone trail versus a heuristic value. Let q be a random variable uniformly distributed over [0, 1], and $q_0$ ∈ [0, 1] be a tunable parameter. The next node j that ant k chooses to go is (Dorigo et al. [6]):

$$j = \begin{cases} \arg \max\limits_{l \in allowed_k} \left\{[\tau_{il}(t)]^{\alpha}[\eta_{il}(t)]^{\beta}\right\} & if \;\; q \le q_0 \\ J & otherwise \end{cases} \tag{4}$$

Where J = a random variable selected according to the probability distribution of $p_{ij}(t)$. The pheromone trail is changed globally. Upon completion of a tour by all ants in the colony, the global trail updating is done as follows:

$$\tau_{ij}(t+1) \xleftarrow{iteration} \rho.\tau_{ij}(t) + \Delta\tau_{ij} \tag{5}$$

Where 0 ≤ ρ ≤ 1; ρ = evaporation (i.e., loss) rate, the symbol $\xleftarrow{iteration}$ is used to show the next iteration and $\Delta\tau_{ij}$ represents the updating value of

$$\Delta\tau_{ij} = \begin{cases} \dfrac{Q}{f(k)} & \text{if edge(i, j) is traversed by the } k_{\text{th}} \text{ ant} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Where Q is a constant, representing the amount of pheromone an ant put on the path after an exploitation, and f(k) is the value of objective in each iteration.

*2.2 Multi-objective optimization*

Many problems of the real-life are optimization of more than one objective function at the same time. The fact of optimizing several objectives simultaneously has made the problem solving more complicated in multi-objective optimization. The existence of many multi-objective problems in the real-world, their intrinsic complexity and the advantages of metaheuristic procedures to deal with them has strongly developed this research area in the last few years (Gandiblex et al. [8]; Goldberg [9]).

Some researchers have designed genetic algorithms to deal with multi-objective optimizations in construction such as mentioned time-cost trade-off problems and they have adapted genetic algorithms for optimizing construction bi-objective time and cost, but there has been little or no reported application of ACO to multi-objective construction problems.

*2.3 Pareto front*

As mentioned, the goal of multi-objective optimization problems is to find the best compromise between multiple and conflicting objectives. Considering all objectives in these problems there will be more than one solution that optimizes simultaneously all the objectives and there is no distinct superiority between these solutions. Usually there is not a single best solution being better than the remainder with respect to every objective. Therefore we face with a set of solutions which are better than remainder solutions called Pareto front. Among the feasible solutions, solutions belonging to Pareto front are known as nondominated solutions, while the remainder solutions are known as dominated. Since none of the Pareto set solutions is absolutely better than the other nondominated solutions, all of them are equally acceptable as regards the satisfaction of all the objectives.

## 3. DESCRIPTION OF THE PROBLEM AND FORMULATION

In the present problem each activity has some options for resource utilization and the goal is finding the optimal/near optimal ways of project completion in the search space of whole possible combinations of these resource utilization options to activities. In order to apply ACO algorithm to a specific problem, the problem should be represented as graph or similar structure easily covered by ants, as shown in Figure 1. In which a project with N activities and K resource utilization options is characterized. The horizontal axis represents the activity number and the vertical axis the resource utilization numbers. The path of arrows represents a typical solution which may be selected by ants (see Figure 1). For more illustration and future reference a vector is defined for each possible solution that demonstrates the options of resource utilization for all of the activities respectively. For example mentioned vector for the route identified in Figure 1 will be V= [2, 3, 1, k, k, …, 3].

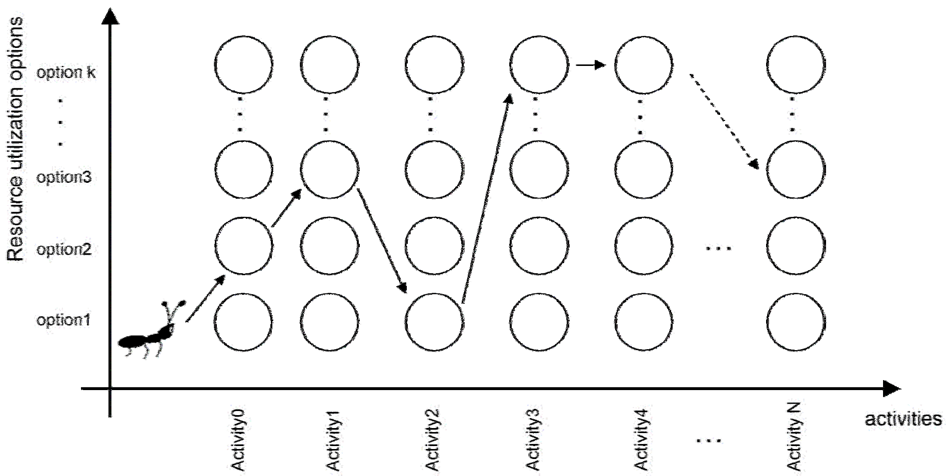


Figure 1. Graph representation of problem for a project with N activities and K resource utilization options

The problem mainly concentrates on selecting appropriate options for every activity to obtain the objective of time, cost and quality of a project. The objective of time may be expresses as

$$T = \max_{L_k \in L} \left[ \sum_{i \in L_k} t_i^{(k)} x_i^{(k)} \right] \tag{1}$$

Where $t_i^{(k)}$ represents the duration of activity i when performing the $k_{th}$ option; and $x_i^{(k)}$ stands for the index variable of activity i when performing the $k_{th}$ option. If $x_i^{(k)} = 1$ then the activity i perform the $k_{th}$ option, while $x_i^{(k)} = 0$ means not. The sum of index variables of all options should be equal to 1. $L_k$ means the activity sequence on the $k_{th}$ path, and $L_k = \{i1_k, i2_k, ..., in_k\}$ where $ij_k$ represents the sequence number of activity j on the $k_{th}$ path. $L$ stands for the set of all paths of a network, and $L = \{L_k | k = 1,2,...,m\}$, where $m$ symbolizes the number of all paths of a network. In other words, for each combination of selected options for activities, mentioned equation will calculate sum of the activity durations on the critical path of the project's network.

The total cost of a project consists of two parts: direct cost and indirect cost. Direct cost is determined as sum of the direct cost of all activities within a project network. On the other hand, indirect cost is composed of the expenditure on management during project implementation, which depends heavily upon the project duration, i.e. the longer the duration, the higher the indirect cost.

In a real construction project, it is feasible to evaluate indirect cost per time unit to calculating the total cost. Subsequently, Eq. (2) can be forwarded to compute the total cost of a project:

$$C = \sum_{i \in A} dc_i^{(k)} x_i^{(k)} + T \times ic_i^{(k)} \tag{2}$$

Where $dc_i^{(k)}$ direct cost of activity i under the $k_{th}$ option, which equals to the quantities of the activity multiplied by its price; $ic_i^{(k)}$ =indirect cost per time unit of activity under the $k_{th}$ option, which can be generated by experts through estimation or derived from division of the indirect cost of budget report according to contractual duration; and A = set of activities in a network.

Quantifying the construction quality as a function of different resource utilizations is a challenging work because of difficulty in measuring the impact of these strategies of performing activities on the quality of activities. Moreover, it is a complicated work to evaluate the proportion of the individual activities quality performance on total quality level of the project. Some indicators have been investigated and identified in recent studies that were aimed at developing quality-based contractor prequalification systems (Anderson and Russell [10]; Minchin and Smith [11]). The identified quality indicators were derived from performance based models that correlate the long-term performance of the end product of each activity to its quality indicators. The objective of quality may be evaluated with following function:

$$Q_T = \sum_{i \in A} wt_i \sum_{l=1}^{L} wt_{i,l} \times Q_{i,l}^k \tag{3}$$

Where $Q_{i,l}^k$ = performance of quality indicator ($l$) in activity ($i$) using resource utilization ($k$); $wt_{i,l}$ = weight of quality indicator ($l$) compared to other activities in the project (El-Rayes [1]). Aggregation of the estimated quality for all the considered activities to provide an overall quality at the project level is done by Eq. (3).

## 4. PROPOSED MULTI COLONY ANT ALGORITHM FOR TIME-COST-QUALITY TRADE-OFF

In the proposed Non-dominated Archiving ACO (NA-ACO) algorithm, for each objective a colony of agents is assigned. All the colonies have the same number of ants. All the ants in one colony try to find a solution at the same time according to the assigned objective. Solutions found for one objective in one cycle are not evaluated in the corresponding colony. The produced solutions are transferred to the next colony to be evaluated according to the assigned objective and the global trail of that colony is updated. The new solutions found based on the new pheromone trail in the second colony are transferred to the third colony. This process (finding set of solutions in each colony and having the following colony to use these produced solutions for updating) continues up to a predefined iteration called cycle iteration. In this step, the values of the objectives are calculated according to the generated solutions of third colony and the nondominated ones are moved to the external set called Archive. After the completion of a cycle, the global pheromone trails of all colonies are set to the initial value of $\tau_0$. In the next step, the second cycle is started and at the end of the cycle, derived nondominated solutions are moved to the same Archive. The dominated solutions of Archive are moved out and another pheromone updating is done for all colonies according to the existing solutions in archive. The whole process is repeated until all the nondominated solutions (Pareto set) of archive satisfying all the constraints or a predetermined number of iterations is met. The solutions of Archive are the final Pareto answers of the multiobjective optimization problem. To achieve better distributed Pareto solutions, in each step, all the produced solutions are evaluated according to the all three objectives and the nondominated ones are moved to the archive. If there is any solution which is dominated with newly arrived solutions, they will move out.

## 5. CASE STUDY

In order to illustrate the concept and performance of the proposed algorithm, a test project with detailed information as shown in Table 2 is used as a case study. The example was originally introduced by Feng et al. [12] and then the same used by Zheng et al. [13] for stochastic construction time-cost trade-off analysis. Table 2 includes the related data on different resource utilizations and their corresponding time, cost and quality. The project has an indirect cost equal to $500/day. Originally the example did not contain the information about quality level of resource utilization options. Herein they are presented based on the quality indicators and mentioned procedures. Results reported by Zheng [13] for multi-objective modified adaptive weighting approach (MAWA) is used by means of validation the results provided by the current approach.

The proposed Multi-Colony Ant Algorithm was fed with the project data as shown in Table 2. The number of ants in each colony, number of cycle iteration and number of total iteration are set to 50, 20 and 60, respectively. Other parameters of algorithm are set to $\rho = 0.97, \alpha = 2, \beta = 0$ and the $Q$ parameter for the colonies of time, cost and quality is equal to 10, 10000, 0.0005.

Table 2. Detalied data of the example

| **Activity** | **Preceding activity** | **Resource options** | **Duration (days)** | **Cost (dollars)** | **Activity weight (%)** | **Quality (%)** |
|---|---|---|---|---|---|---|
| 1 | | 1 | 14 | 23,000 | 8 | 98 |
| | | 2 | 20 | 18,000 | | 89 |
| | | 3 | 24 | 12,000 | | 84 |
| 2 | 1 | 1 | 15 | 3,000 | 6 | 99 |
| | | 2 | 18 | 2,400 | | 95 |
| | | 3 | 20 | 1,800 | | 85 |
| | | 4 | 30 | 1,200 | | 70 |
| | | 5 | 60 | 600 | | 59 |
| 3 | 1 | 1 | 15 | 4,500 | 14 | 98 |
| | | 2 | 22 | 4,000 | | 81 |
| | | 3 | 33 | 3,200 | | 63 |
| 4 | 1 | 1 | 12 | 45,000 | 19 | 94 |
| | | 2 | 16 | 35,000 | | 76 |
| | | 3 | 20 | 30,000 | | 64 |
| 5 | 2,3 | 1 | 22 | 20,000 | 17 | 99 |
| | | 2 | 24 | 17,500 | | 89 |
| | | 3 | 28 | 15,000 | | 72 |
| | | 4 | 30 | 10,000 | | 61 |
| 6 | 4 | 1 | 14 | 40,000 | 19 | 100 |
| | | 2 | 18 | 32,000 | | 79 |
| | | 3 | 24 | 18,000 | | 68 |
| 7 | 5,6 | 1 | 9 | 30,000 | 17 | 93 |
| | | 2 | 15 | 24,000 | | 71 |
| | | 3 | 18 | 22,000 | | 67 |

Running the model using mentioned data resulted in selection of 103 nondominated solutions (Pareto optimal). Each solution contains a specific optimum way of project delivery. The set of solutions provides an optimal trade-off between time, cost and quality. Accordingly, sample of selected solutions are shown in Table 3.

Table 3. Sample of 15 pareto optimal solutions from 103 selected nondominated solutions

| **Solution** | **Time (days)** | **Cost ($)** | **Quality (%)** | **Resource options for activities** | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
| 1 | 60 | 155,500 | 92 | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| 2 | 61 | 142,500 | 86 | 1 | 1 | 1 | 3 | 1 | 2 | 1 |
| 3 | 62 | 163,000 | 95 | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| 4 | 63 | 131,000 | 84 | 1 | 1 | 1 | 2 | 2 | 3 | 1 |
| 5 | 65 | 162,400 | 95 | 1 | 2 | 1 | 1 | 2 | 1 | 1 |
| 6 | 66 | 128,500 | 82 | 1 | 1 | 1 | 2 | 3 | 3 | 1 |
| 7 | 67 | 127,300 | 83 | 1 | 3 | 1 | 3 | 1 | 3 | 1 |
| 8 | 68 | 118,500 | 77 | 1 | 1 | 1 | 3 | 4 | 3 | 1 |
| 9 | 71 | 117,900 | 77 | 1 | 2 | 1 | 3 | 4 | 3 | 1 |
| 10 | 74 | 112,500 | 73 | 1 | 1 | 1 | 3 | 4 | 3 | 2 |
| 11 | 78 | 107,500 | 76 | 3 | 1 | 1 | 3 | 4 | 3 | 1 |
| 12 | 87 | 150,200 | 93 | 3 | 4 | 1 | 1 | 2 | 1 | 1 |
| 13 | 92 | 98,300 | 70 | 3 | 3 | 1 | 3 | 4 | 3 | 3 |
| 14 | 126 | 104,600 | 73 | 3 | 5 | 1 | 3 | 2 | 3 | 3 |
| 15 | 132 | 95,800 | 63 | 3 | 5 | 3 | 3 | 4 | 3 | 3 |

Results of the present approach for time-cost trade-off problem are compared to those reported by Zheng et al. [13]. The number of ants in each colony, number of cycle iteration and number of total iteration are set to 20, 10 and 30, respectively. Algorithm's parameters for time-cost trade-off problem are set to $\rho = 0.97, \alpha = 2, \beta = 0$ and the $Q$ parameter for the colonies of time, cost and quality is equal to 10, 10000, 0.0005. Running the model using mentioned parameters and detail data of the example led to the selection of 12 nondominated solutions. These data in addition to the reported results of Zheng [13] are shown comparatively in Table 4.

Table 4. Comparison of the results generated by MAWA model (Zheng, 2005) with the proposed MOACO model

| **Solution** | **50-MAWA*** | | **100-MAWA*** | | **30-MOACO** | |
|---|---|---|---|---|---|---|
| | **Time (days)** | **Cost ($)** | **Time (days)** | **Cost ($)** | **Time (days)** | **Cost ($)** |
| 1 | 61 | 173,000 | 61 | 173,000 | 61 | 173,000 |
| 2 | 63 | 164,000 | 62 | 172,000 | 62 | 171,000 |
| 3 | 67 | 157,000 | 63 | 162,500 | 63 | 162,500 |
| 4 | 68 | 152,500 | 66 | 161,500 | 66 | 161,500 |
| 5 | 74 | 150,500 | 67 | 157,000 | 67 | 157,000 |
| 6 | 77 | 150,400 | 68 | 152,500 | 68 | 152,500 |
| 7 | 78 | 146,500 | 74 | 149,500 | 74 | 149,500 |
| 8 | 90 | 143,900 | 77 | 149,000 | 77 | 149,000 |
| 9 | | | 78 | 146,500 | 78 | 146,500 |
| 10 | | | 84 | 143,500 | 84 | 143,500 |
| 11 | | | 87 | 143,000 | 87 | 143,000 |
| 12 | | | | | 60 | 173,500 |

***This model is based on adaptive weighting method and genetic algorithm with 50 population size and 50,100 generations respectively.**

This comparison not only confirms the present model's capability in generating the set of nondominated solutions but also yields one more nondominated solution as well as dominating one of the 100-MAWA solutions with only 30 total iterations.

## 6. CONCLUSIONS

A Multi-Objective Ant Colony Optimization is developed to analyze the advanced time-cost-quality trade-off problem. The model is capable of compromising between important aspect of construction projects meaning that minimizing time and cost of projects while maximizing its quality. The efficiency of the proposed algorithm is verified by an example which confirms the capability of model in considering quality and generating Pareto optimal. Moreover, the model was used to optimize time-cost trade-off for the same example and compared to the results of the example modeled by MAWA approach (Zheng [13]) which validated the capabilities of the present model. The present algorithm provides an attractive alternative for the solution of the construction multi-objective optimization problems.

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## NOTATION

The following symbols are used in this paper:

$T =$ Total duration of project;

$t_i^{(k)} =$ The duration of activity i when performing the $k_{th}$ option;

$x_i^{(k)} =$ The index variable of activity i when performing the $k_{th}$ option;

$L_k =$ The activity sequence in the $k_{th}$ path;

$L =$ The set of all paths of a network;

$m$ = Number of all paths of a network;

$C$ = Total cost of project;

$dc_i^{(k)} =$ Direct cost of activity i under the $k_{th}$ option;

$ic_i^{(k)} =$ Indirect cost per time unit of activity under the $k_{th}$ option;

$A$ = Set of activities in a network;

$Q_T =$ Total Quality performance of project;

$Q_{i,l}^k =$ Performance of quality indicator (*l*) in activity (*i*) under the $k_{th}$ option;

$wt_{i,l} =$ Weight of quality indicator (*l*) compared to other activities in the project;

$p_{ij}(t) =$ Transition probability from node i to node j;

$\tau_{ij}(t) =$ The heuristic value of path ij at time t according to the measure of the objective function;

$\eta_{ij}(t) =$ The heuristic value of path ij at time t according to the measure of the objective function;

$\alpha, \beta =$ Parameters thet control the relative importance of the pheromone trail versus a heuristic value;

q= Random variable uniformly distributed over[0 ,1], and $q_0 \in [0\ ,\ 1]$ be a tunable parametr;

j = The node which will be chosen by ant *k;*

l = The allowed paths for ant *k;*

J= Random variable selected according to the probability distribution of $p_{ij}(t)$ ;

$\rho =$ Evaporation rate;

$\Delta\tau_{ij} =$ The updating value of edge (i,j);

$Q$ = Constant representing the amount of pheromone an ant put on the path after an exploration;

f(k)= The value of objective in each iteration;

$\tau_0 =$ Initial value of pheromone trails of all three colonies.