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A STOREY ELEMENT FOR ANALYZING FRAME-SHEAR WALL STRUCTURES

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Abstract

A storey element, which can simulate one storey of the frame-shear wall structure, is established by simply summing the rigidity matrixes of the frame and wall. The internal forces and displacements of the frame-shear wall structures for various rigidity characteristics with the constant stiffness under various kinds of loads are calculated by the beam element method, storey element method and analytical formulas. The internal forces and displacements with the variable stiffness along the height under uniformly distributed load are calculated by the beam element method and storey element method. In comparison with the beam element method, the storey element method can remarkably reduce the volume of input, output data and calculation, the error is less than 8.3%.

Keyword: Frame-shear wall structure; storey element; beam element; portal method

1. Introduction

There exist two main methods to analyze the frame-shear wall structure shown in Figure 1. One is the finite element method [1] (FEM) to discretize the structure by beam element, it's a kind of numerical method, also an accurate method defined in this paper, the advantage of the method is that it can be used to analyze various structures, the disadvantage is the requirement of the large volume of input, output data and calculation. Another is the continuum approach [2], it's a kind of analytical method to solve the governing differential equation, also an approximate method defined in this paper, the advantage of the method is the simplicity and hand analysis, the disadvantage is the strict assumption, consequently, it can only be used to analyze the structure subjected to several special loads, with constant stiffness along the height.

In modern frame–shear wall structures, the rigidity varies along the height to satisfy the different architectural requirements [3,5]. The transfer matrix method [6,7] derived on the basis of the continuum assumption, is a numerical method, can be used to analyze the structures with variable rigidity along the height. The transfer matrix method overcomes the

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disadvantages of previous two methods and remains their advantages, therefore it is widely used especially during the preliminary design stage. Two disadvantages of the transfer matrix method still exist, firstly, the continuum assumption is adopted, which results in some error; secondly, the expression is different to FEM, so many engineers is unfamiliar to its solving procedure.

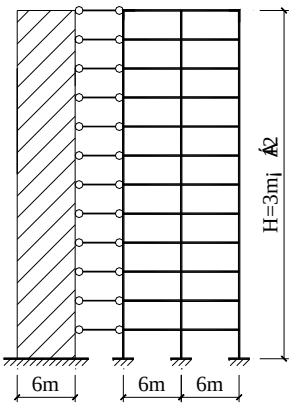

Figure 1. Computational Model for frame-shear wall structure

A storey element, which can simulate one storey of the frame-shear wall structure, is put forward. The storey element method significantly reduces the number of elements and nodes, so overcomes the disadvantage of beam element method. The storey element method is actually a kind of FEM, and the continuum assumption is abandoned, hence the storey element method overcomes two disadvantages of the transfer matrix method.

2. Storey Element

In order to establish the governing differential equation, the continuum assumption has to be adopted. Now using FEM to discretize the structure instead of the differential equation, the continuum assumption would be abandoned logically.The storey element is shown in Figure 2, which can simulate one storey of the frame-shear wall structure. $\{\delta\}^e = \{y_i, \theta_i, y_j, \theta_j\}^T$, $\{F\}^e = \{F_{si}, M_i, F_{sj}, M_j\}^T$ are the nodal displacement and force vectors respectively, the directions of $\{\delta\}^e$ in the Figure 2 are positive, and the positive directions of $\{F\}^e$ are the same as the directions of $\{\delta\}^e$. In storey element, the frame and shear wall deflect identically only at the floors, which conforms to the practical deflection, so the storey element method is more accurate than the analytical formulas.

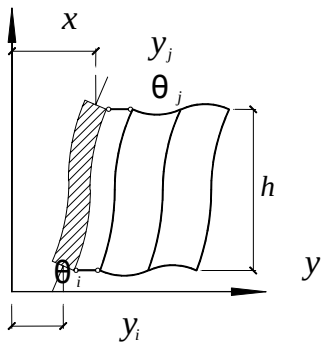

Figure.2 Storey element

The wall is represented by a flexural cantilever, which deforms in bending only, so its rigidity matrix $[K_W]$ is the same as the rigidity matrix of the beam element:

$$[K_W] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \tag{1}$$

Where

$k_{11} = \dfrac{12EI_W}{h^3}$, $k_{21} = \dfrac{-6EI_W}{h^2}$, $k_{22} = \dfrac{4EI_W}{h}$, $k_{31} = -k_{11}$, $k_{32} = -k_{21}$, $k_{33} = k_{11}$, $k_{41} = k_{21}$, $k_{42} = \dfrac{2EI_W}{h}$, $k_{43} = -k_{21}$, $k_{44} = k_{22}$; h is height of *i*th storey element, EI_W is the flexural rigidity of the wall.

The frame is represented by a shear cantilever, which deforms in shear only, its rigidity matrix $[K_F]$ is:

$$[K_F] = C_F \begin{bmatrix} C_F & 0 & -C_F & 0 \\ 0 & 0 & 0 & 0 \\ -C_F & 0 & C_F & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

C_F is *i*th storey shear rigidity of the frame; according to the portal method [1]:

$$C_F = \frac{12ER_bR_c}{h(R_b + R_c)} \tag{3}$$

Where $R_b = \sum (I_b / l)$ for all the beams of span l across *i*th floor, $R_c = \sum (I_c / h)$ for all the columns in *i*th storey; *E* is the elastic modulus; I_b and I_c are the moments of inertia of the beams and columns, respectively. The rigidity equation of the storey element is:

$$[K_{WF}]\{\delta\}^e = \{F\}^e \tag{4}$$

$[K_{WF}] = [K_W] + [K_F]$. The n-storey structure is discretized into n storey elements, in each element, the rigidities of both the frame and shear wall are constant. According to FEM procedure, the displacements and internal forces are obtained for given rigidity matrix. Therefore, the storey element method significantly reduces the number of elements and nodes, overcomes the disadvantage of beam element method. By simply summing the rigidity matrix $[K_F]$ and $[K_W]$ of *i*th frame and wall, then the rigidity matrix $[K_{WF}]$ of storey element is obtained, it facilitates the computer programming. The storey element method is actually a kind of FEM, and the continuum assumption is abandoned, hence the storey element overcomes two disadvantages of the transfer matrix method.

3. Calculation Results of Storey Element

In Figure 1 structure, beam section: 0.25 m×0.6 m; column section: 0.45 m×0.45 m; the wall section: 0.2 m×6 m; elastic modulus: $E = 3.25 \times 10^7 kN/m^2$. Calculate C_F of *i*th storey by Eq. (3); Calculate the rigidity characteristic value $\lambda = H\sqrt{C_F / EI_W} = 1.23$, H is the height of whole wall. Change the inertia moment of the wall, obtain different λ value from 0.75~2.5, which covers the range of the rigidity characteristic values for most frame-shear wall structures.

In Tables 1 and 2, F denotes the concentrated load 100 kN applying on the top the structure; q denotes the uniformly distributed load 10 kN/m along the height of the structure, q_0 denotes the inverted triangularly distributed load, the maximum intensity at the top $q_0 = 12kN/m$. BEM, SEM and AM represent the beam element method, storey element method and analytical method respectively. $M_{\max}$, F_{smax} and $f_{\max}$ are the bending moment, shear force at the bottom of the wall, and the top displacement. In Table 1 and 2, the value in parentheses is the error percentage in comparison with the value of BEM regarded as the accurate value. The variation of the rigidity in Table 2 is that: the rigidity of 7-9 storey wall is 75% of the rigidity of 1-6 storey wall, the rigidity of 10-12 storey wall is 50% of the rigidity of 1-6 storey wall, λ value is calculated according to the rigidity of 1-6 storey wall. For the variable rigidity, there is no analytical solution, so in Table 2, only the results of BEM and SEM methods are listed. The accuracy of the storey element is directly related to the accuracy of the portal method, almost not related to the load type, it is also verified by the results in Table 1, therefore in Table 2, only uniformly distributed load is calculated.

Table 1. Calculation results of 12-storey structure with constant rigidity

Load	λ	$M_{\max}$ $kN \cdot m$			$F_{s\max}$ kN			$f_{\max}$ mm		
		BEM	SEM	AM	BEM	SEM	AM	BEM	SEM	AM
F	0.75	3106	3047(1.9%)	3049(1.8%)	97.9	98.0(0.1%)	100(2.1%)	4.2	4.1(2.4%)	4.1(2.4%)
	1.23	2560	2470(3.5%)	2472(3.4%)	95.2	95.9(0.7%)	100(5.0%)	8.7	8.3(4.6%)	8.3(4.6%)
	1.75	2043	1935(5.3%)	1937(5.2%)	92.9	93.5(0.6%)	100(7.6%)	13.1	12.3(6.1%)	12.3(6.1)
	2.5	1498	1418(5.3%)	1421(5.1%)	88.3	90.4(2.4%)	100(13.3%)	17.5	16.1(8.0%)	16.1(8.0%)
q	0.75	5806	5729(1.3%)	5732(1.3%)	356.0	356.4(0.1%)	360(1.1%)	5.6	5.5(1.8%)	5.5(1.8%)
	1.23	5047	4931(2.3%)	4933(2.3%)	350.3	351.8(0.4%)	360(2.8%)	11.9	11.4(4.2%)	11.4(4.2%)
	1.75	4305	4165(3.3%)	4167(3.2%)	344.3	346.4(0.6)	360(4.6%)	18.3	17.1(6.6%)	17.1(6.6%)
	2.5	3472	3374(2.8%)	3379(2.7%)	332.3	337.9(1.7%)	360(8.3%)	24.9	22.9(8.0%)	22.9(8.0%)
q_0	0.75	4593	4524(1.5%)	4526(1.5%)	212.9	213.1(0.1%)	216(1.5%)	5.0	4.9(2.0%)	4.9(2.0%)
	1.23	3930	3827(2.6%)	3828(2.6%)	208.5	209.7(0.6%)	216(3.6%)	10.5	10.0(4.8%)	10.0(4.8%)
	1.75	3288	3162(3.8%)	3164(3.8%)	204.1	205.5(0.7%)	216(5.8%)	16.0	14.9(6.9%)	14.9(6.9%)
	2.5	2579	2489(3.5%)	2493(3.3%)	195.5	199.4(2.0%)	216(10.5%)	21.7	19.9(8.3%)	19.9(8.3%)

Table 2. Calculation results of 12-storey structure with variable rigidity

Load	λ	$M_{\max}$ $kN \cdot m$		$F_{s\max}$ kN		$f_{\max}$ mm	
		BEM	SEM	BEM	SEM	BEM	SEM
q	0.75	5796	5721(1.3%)	355.9	356.5(0.2%)	5.7	5.6(1.8%)
	1.23	5050	4928(2.4%)	350.7	352.1(0.4%)	12.0	11.5(4.2%)
	1.75	4308	4172(3.2%)	344.0	346.4(0.7%)	18.3	17.0(7.1%)
	2.5	3486	3392(2.7%)	332.2	337.8(1.7%)	24.8	22.8(8.1%)

4. Conclusions

The above investigation has led to the following conclusions:

1. Comparing with the beam element method, the storey element method can remarkably reduce the volume of input, output data and calculation. Taking the structure in Figure1 as example, there are 84 elements and 52 nodes for beam element method, only 12 elements and 13 nodes for the storey element method. The efficiency becomes more significant for the structure with more stories and spans.
2. The error of the storey element method is acceptable, less than 8.3% for different loads, different λ values, constant or variable rigidities. The error is caused by the portal method, with increasing λ value, the rigidity of the wall decreases, the error of the portal method becomes more significant, hence the total error increases slowly.

3. The accuracy of analytical method is less than that of the storey element due to continuum assumption. The error of Fsmax is larger than others, because the shear force of the frame at the bottom is zero in the analytical method, and Fsmax is equal to the external load

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