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ENGINEERING OPTIMIZATION WITH HYBRID PARTICLE SWARM AND ANT COLONY OPTIMIZATION

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ABSTRACT

This article presents a heuristic particle swarm ant colony optimization algorithm to solve engineering optimization problems. Although PSO has simple principle and ease to be implemented and can eventually locate the desired solution, however, its practical use in solving engineering optimization problems is severely limited by the high computational cost of the slow convergence rate. Here, ant colony and harmony search principles are employed to speed up local search and improve precision of the solutions. A modified feasible-based mechanism is described which handles the problem-specific constraints. Benchmark optimization problems are used to illustrate the reliability of the proposed algorithm.

Keywords: Optimization algorithm; particle swarm; ant colony; harmony search; engineering design problems

1. INTRODUCTION

Many engineering design problems can be formulated as constrained optimization problems. Generally, a constrained optimization problem can be described as follows

$$
\begin{aligned}
&\text{find } \{x\} \text{ to minimize } f_{\text{cost}}(\{x\}) \\
&\text{subject to:} \\
&\qquad g_j(\{x\}) \le 0 \qquad j = 1,2,\ldots,n_g \\
&\qquad h_k(\{x\}) = 0 \qquad k = 1,2,\ldots,n_h \\
&\qquad x_{i,\min} \le x_i \le x_{i,\max} \qquad i = 1,2,\ldots,d
\end{aligned}
\tag{1}
$$

where $\{x\} = [x_1, x_2, ..., x_d]^T$ denotes the decision solution vector; f_{cost} is a cost function

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(objective function); $x_{i,\min}$ and $x_{i,\max}$ are the minimum and the maximum permissible values for the *i*th variable, respectively; n_g is the number of inequality constraints and n_h is the number of equality constraints. In a common practice, equality constraint $h_k(\{x\}) = 0$ can be replaced by an inequality constraint $|h_k(\{x\})| - \varepsilon \leq 0$ where ε is a small tolerant amount.

Heuristic methods are quite suitable and powerful for obtaining the solution of engineering optimization problems. These methods do not require the derivatives of the objective function and constraints. Having in common processes of natural evolution, these algorithms share many similarities: each maintains a population of solutions which are evolved through random alterations and selection. The differences between these procedures lie in the representation technique utilized to encode the candidates, the type of alterations used to create new solutions, and the mechanism employed for selecting new patterns [1].

Particle Swarm Optimization (PSO) is a relatively new heuristic approach utilized for engineering optimization problems due to its simple principle and ease of implementation [2]. The PSO algorithm is initialized with a population (swarm) of random potential solutions (particles). Each particle iteratively moves across the search space and is attracted to the position of the best fitness historically achieved by the particle itself (local best) and by the best among the neighbors of the particle (global best), [3].

Although PSO does eventually locate the desired solution, however, its practical use in solving engineering optimization problems is severely limited due to the high computational cost of the slow convergence rate [4]. The convergence rate of PSO is also typically slower than those of local direct search techniques (e.g., Hooke and Jeeves method [5] and Nelder–Mead simplex search method), as they do not utilize much local information to determine the most promising search direction [6]. In actuality, PSO had difficulties in controlling the balance between exploration (global investigation of the search place) and exploitation (the fine search around a local optimum), [7]. In order to deal with the slow convergence of PSO, various hybrid methods are developed [8-16]. Adding some abilities of one method to the PSO algorithm improves the performance of the resulted algorithm. Recently, heuristic particle swarm ant colony optimization (HPSACO) [1,17] is proposed by the authors. HPSACO utilizes a particle swarm optimization with passive congregation (PSOPC) algorithm as a global search, the idea of ant colony approach (ACO) worked as a local search and the harmony search (HS) utilized to handle the boundary constraints.

The most common approach adopted to deal with constrained search spaces is the use of penalty functions. Despite the popularity of penalty functions, these require a careful fine tuning of the penalty factors that accurately estimates the degree of penalization to be applied in order to approach efficiently to the feasible region [18]. Therefore, several other techniques have been incorporated to handle the constraints. The feasible-based constrained approach is one of the powerful and reliable approaches used by many researchers [19]. In this paper, the problem-specific constraints are handled by using a modified feasible-based mechanism. HPSACO is utilized for solving engineering problems. The ACO and HS principles are used in the HPSACO as a helping factor to guide the exploration and to increase the control of exploitation. HPSACO utilizes an efficient terminating criterion considering exactitude of the solutions. Simulation results and comparisons based on various constrained engineering design problems demonstrate the reliability of the algorithm.

2. A BRIEF INTRODUCTION TO THREE META-HEURISTIC ALGORITHMS

2.1 Particle swarm optimization

The pseudo-code of the PSO algorithm can be summarized as follows [3]:

Step 1: *Initialization*. Initialize an array of particles with random positions and their associated velocities.
Step 2: *Local best updating.* Evaluate the fitness function of the particles and update local best position P_i^k according to the best current value of the fitness function.
Step 3: *Global best updating.* Determine the current global minimum fitness value among the current positions and update P_g^k , the global best position.
Step 4: *Solution construction*. Change the velocities and move each particle to the new position considering the related velocity.
Step 5: *Terminating criterion controlling.* Repeat Steps 2–4 until a terminating criterion is satisfied.

2.2 Ant colony optimization

The general procedure of the ACO algorithm manages the scheduling of four steps [20]:

Step 1: *Initialization*. Initialize the ACO parameters and the initial positions of the ants.
Step 2: *Solution construction*. Each ant constructs a complete solution to the problem according to a probabilistic state transition rule. The state transition rule depends mainly on the state of the pheromone and visibility of ants.
Step 3: *Pheromone updating rule*. When every ant has constructed a solution, the intensity of pheromone trails on each edge is updated by the pheromone updating rule.
Step 4: *Terminating criterion controlling.* Steps 2 and 3 are iterated until a terminating criterion.

2.3 Harmony search algorithm

The HS optimization procedure consists of the following steps [21]:

Step 1: *Initialization.* Initialize the optimization operators of HS algorithm includes the harmony memory (HM), the harmony memory size (HMS), the harmony memory considering rate (HMCR), and the pitch adjusting rate (PAR).
Step 2: *Solution construction*. Generate A new harmony vector from the HM, based on memory considerations, pitch adjustments, and randomization.
Step 3: *Harmony memory updating.* If a new harmony vector is better than the worst harmony in the HM, judging in terms of the objective function value, the new harmony is included in the HM and the existing worst harmony is excluded from the HM.
Step 4: *Terminating criterion controlling.* Repeat steps 2 and 3 until the terminating criterion is satisfied.

3. A MODIFIED FEASIBLE-BASED MECHANISM

The aim of constraint optimization is to search for feasible solutions with better objective values. $\{x\}$ is a feasible solution if it satisfies all the problem-specific constraints and the limits of the variables. Due to the simplicity, the penalty function method has been considered as the most popular technique to handle problem-specific constraints. However, since the objective function and the constraint violation are simultaneously considered in the penalty function, the performance of this kind of approach is significantly affected by the penalty factors while determining the suitable penalty factors is usually difficult.

In this paper, a modified feasible-based mechanism is used to handle the problem-specific constraints which can be described as follows:

Level 1: Any feasible solution is preferred to any infeasible solution.
Level 2: Infeasible solutions containing slight violates of the constraints (from 0.01 in the first iteration to 0.001 in the last iteration) are considered as feasible solutions.
Level 3: Between two feasible solutions, the one having better objective function value is preferred.
Level 4: Between two infeasible solutions, the one having smaller sum of constraint violation is preferred. This sum is calculated as

$$Viol = \sum_{j=1}^{n_g} \max\left(0, g_j(\{x\})\right) + \sum_{j=1}^{n_h} \max\left(0, |\, h_j(\{x\})\,| - \delta\right) \tag{2}$$

By using feasible-based rule in the first and fourth levels, the search tends to the feasible region rather than infeasible region, and in the third level the search tends to the feasible region with good solutions [9]. For most of the engineering optimization problems, the global minimum locates on or close to the boundary of a feasible design space. Applying the level 2, the particles can approach to the boundaries and can fly to the global minimum with a high probability.

4. A HEURISTIC PARTICLE SWARM ANT COLONY OPTIMIZATION ALGORITHM

The heuristic particle swarm ant colony optimization (HPSACO), a hybridized approach based on HS, PSO and ACO, is described in this section.

4.1 Combining PSO with ACO

The method based on hybrid PSO and the ACO, is called particle swarm ant colony optimization (PSACO), which has been originally introduced by Shelokar et al. [10] for solving the continuous unconstrained problems and recently utilized for the design of structures by the authors [12,13]. We have applied PSOPC instead of PSO to improve the performance of the new method. The relation of standard deviation in ACO stage is different from that of Ref. [10], and the inertia weight is changed in PSOPC stage.

The implementation of PSACO algorithm consists of two stages [12]. In the first stage, it applies PSOPC, while ACO is employed in the second stage. ACO works as a local search. Through updating process in the PSOPC stage, each particle moves by adding a change velocity V_i^{k+1} to the current position X_i^k as follows

$$X_i^{k+1} = X_i^k + V_i^{k+1} \tag{3}$$

$$V_i^{k+1} = \omega V_i^k + c_1 r_1 (P_i^k - X_i^k) + c_2 r_2 (P_g^k - X_i^k) + c_3 r_3 (R_i^k - X_i^k) \tag{4}$$

where ω is an inertia weight to control the influence of the previous velocity; r_1, r_2 and r_3 are three random numbers uniformly distributed in the range of (0,1); c_1 and c_2 are two acceleration constants; c_3 is the passive congregation coefficient.

The ACO stage handles *M* ants equal to the number of particles in PSOPC, and each ant generates a solution around P_g^k which can be expressed as

$$Z_i^k = N(P_g^k, \sigma) \tag{5}$$

where, Z_i^k is the solution constructed by ant *i* in the stage *k*, $N(P_g^k, \sigma)$ denotes a random number obtained by Gaussian function with mean value P_g^k and variance σ, where

$$\sigma = (x_{\max} - x_{\min}) \times \eta \tag{6}$$

η is the step size. The ACO stage of the algorithm is based on a continuous ant colony optimization (ACO_R) which was introduced by Socha and Dorigo [22]. They have considered a weighted sum of several one-dimensional Gaussian functions. Here, since ACO works as an auxiliary tool to guide the exploration and to increase the control in exploitation, we utilize a simple Gaussian functions. The Gaussian functions with mean P_g^k can be considered as a continuous pheromone. In ACO algorithms, the probability of selecting a path with more pheromone is greater than other paths. Similarly, in the Gaussian functions, the probability of selecting a solution in the neighborhood of P_g^k is greater than the others.

Finaly, between Z_i^k and X_i^k, the best one based on the value of the objective function is is selected as the current position of particle *i*.

4.2 HS Added to PSACO

The heuristic particle swarm ant colony optimization algorithm (HPSACO) is resulted from combining PSACO and HS [1]. A particle in the search space may violate either the problem-specific constraints or the limits of the variables. Here, the harmony search-based approach is employed to deal with this problem. According to this mechanism, any component of the solution vector (particle) violating the variable boundaries can be regenerated as

$$X_{i,j} = \begin{cases} \text{w.p. HMCR} \Longrightarrow \text{select a new value for a variable from } P_i^k \\ \qquad\qquad\qquad\quad \Longrightarrow \text{w.p. } (1-\text{PAR}) \text{ do nothing} \\ \qquad\qquad\qquad\quad \Longrightarrow \text{w.p. PAR choose a neighboring value} \\ \text{w.p. } (1-\text{HMCR}) \Longrightarrow \text{select a new value randomly} \end{cases} \tag{7}$$

where $X_{i,j}$ is the *j*th component of the particle *i*; The HMCR varying between 0 and 1 sets the rate of choosing a value in the new vector from the historic values stored in the P_i^k, and (1−HMCR) sets the rate of randomly choosing one value from the possible range of values. The pitch adjusting process is performed only after a value is chosen from P_i^k.

4.3 Terminating Criterion

The necessity for an exact definition of the terminating criterion in heuristic algorithms is vital. The following terminating criterion is considered to fulfill this goal. This terminating criterion is defined by using a pre-fixed value denoted by A^* which is considered as the required exactitude of the solutions with a reverse relation [23]. According to this criterion, if in an iteration of search process, the absolute value of the component *i* in all of the particles' velocity vectors is less than $A^*/2$, continuation of the search process can not change the amount of variable *i*; then the variable *i* reaches an optimum value and can be deleted from the virtual list of design variables. When this list is emptied, the search process stops. With these alterations, the number of iterations decreases [1].

5. OPTIMAL DESIGN OF THE ENGINEERING PROBLEMS

Several well-studied engineering design problems taken from the optimization literature are used to show the efficiency of the proposed approach. These examples have been previously solved using a variety of other techniques, which is useful to show the validity and effectiveness of the proposed algorithm. For each example, 30 independent runs are carried out using the HPSACO and compared to other algorithms.

For the HPSACO algorithm, a population of 50 individuals consisting of 25 particles and 25 ants are used; the value of constants c_1 and c_2 are set to 0.8 and the passive congregation coefficient c_3 is taken as 0.6. The value of inertia weight decreases linearly from 0.9 in the first iteration to 0.4 in the last iteration. The amount of step size (η) in ACO stage is recommended as 0.01, [12]. HMCR is set to 0.95 and PAR is taken as 0.10, [1].

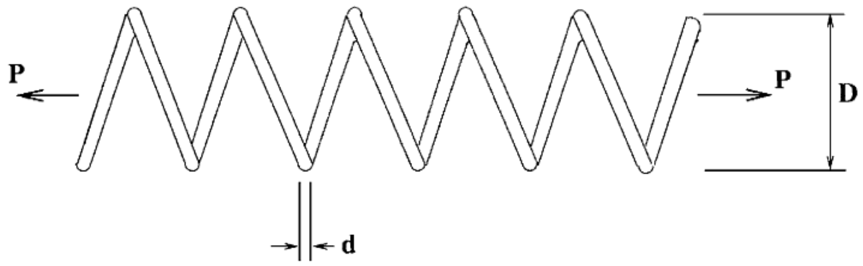

Figure 1. Tension/compression spring

5.1 A Tension/Compression String Design Problem

This problem consists of minimizing the weight of a tension/compression spring subject to constraints on shear stress, surge frequency and minimum deflection as shown in Figure 1.

The design variables are the mean coil diameter D (=x_1); the wire diameter d (=x_2) and the number of active coils N (=x_3). The problem can be stated as:

Cost function

$$f_{\text{cost}}(\{x\}) = (x_3 + 2)x_2 x_1^2 \tag{8}$$

Constraint functions

$$\begin{aligned} g_1(\{x\}) &= 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0 \\ g_2(\{x\}) &= \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} - 1 \le 0 \\ g_3(\{x\}) &= 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0 \\ g_4(\{x\}) &= \frac{x_1 + x_2}{1.5} - 1 \le 0 \end{aligned} \tag{9}$$

Variable regions

$$0.05 \le x_1 \le 2,\ 0.25 \le x_2 \le 1.3,\ 2 \le x_3 \le 15 \tag{10}$$

This problem has been solved by Belegundu [24] using eight different mathematical optimization techniques (only the best results are shown). Arora [25] also solved this problem using a numerical optimization technique called a constraint correction at the constant cost. Coello [26] as well as Coello and Montes [27] solved this problem using GA-based method. Additionally, He and Wang [11] utilized a co-evolutionary particle swarm optimization (CPSO). Recently, Montes and Coello [28] and Kaveh and Talatahari [29] used evolution strategies and an improved ant colony optimization to solve this problem, respectively. Table 1 presents the best solution of this problem obtained using the HPSACO algorithm and compares the HPSACO results with solutions reported by other researchers.

Table 2 shows the statistical simulation results. From Table 1, it can be seen that the best feasible solution obtained by HPSACO is better than those previously reported.

Table 1. Optimum results for the tension/compression spring design

Methods	Optimal design variables			f_{cost}
	$x_1(d)$	$x_2(D)$	x_3(N)	
Belegundu [24]	0.050000	0.315900	14.250000	0.0128334
Arora [25]	0.053396	0.399180	9.185400	0.0127303
Coello [26]	0.051480	0.351661	11.632201	0.0127048
Coello & Montes [27]	0.051989	0.363965	10.890522	0.0126810
He & Wang [11]	0.051728	0.357644	11.244543	0.0126747
Montes & Coello [28]	0.051643	0.355360	11.397926	0.012698
Kaveh & Talatahari [29]	0.051865	0.361500	11.000000	0.0126432
Present work	0.051432	0.351062	11.609791	0.0126391

Table 2. Statistical results of different methods for the tension/compression spring

Methods	Best	Mean	Worst	Std Dev
Belegundu [24]	0.0128334	N/A	N/A	N/A
Arora [25]	0.0127303	N/A	N/A	N/A
Coello [26]	0.0127048	0.012769	0.012822	3.9390e-5
Coello & Montes [27]	0.0126810	0.0127420	0.012973	5.9000e-5
He & Wang [11]	0.0126747	0.012730	0.012924	5.1985e-5
Montes & Coello [28]	0.012698	0.013461	0.16485	9.6600e-4
Kaveh & Talatahari [29]	0.0126432	0.012720	0.012884	3.4888e-5
Present work	0.0126391	0.012657	0.012737	2.1825e-5

5.2 Welded Beam Design Problem

The objective is to find the minimum fabricating cost of the welded beam subjected to constraints on shear stress (τ), bending stress (σ), buckling load (P_C), end deflection (δ), and side constraint. There are four design variables, namely $h(=x_1)$, $l(=x_2)$, $t(=x_3)$ and $b(=x_4)$, as shown in Figure 2.

The mathematical formulation of the cost function $f_{\text{cost}}(\{x\})$, which is the total fabricating cost mainly comprised of the set-up, welding labor, and material costs, is as follows:

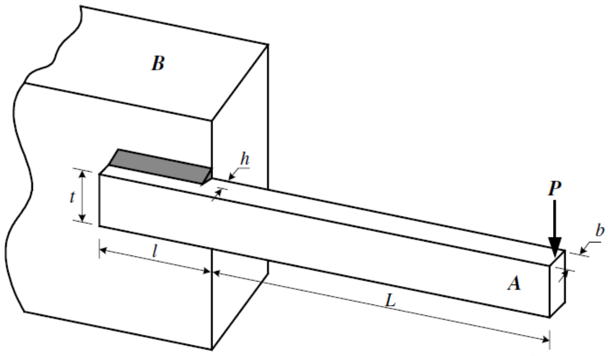

Figure 2. Welded beam structure

Cost function:

$$f_{\text{cost}}(\{x\}) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2) \tag{11}$$

Constraint functions

$$\begin{aligned}
g_1(\{x\}) &= \tau(\{x\}) - \tau_{\max} \le 0 \\
g_2(\{x\}) &= \sigma(\{x\}) - \sigma_{\max} \le 0 \\
g_3(\{x\}) &= x_1 - x_4 \le 0 \\
g_4(\{x\}) &= 0.10471 x_1^2 + 0.04811 x_3 x_4 (14.0 + x_2) - 5.0 \le 0 \\
g_5(\{x\}) &= 0.125 - x_1 \le 0 \\
g_6(\{x\}) &= \delta(\{x\}) - \delta_{\max} \le 0 \\
g_7(\{x\}) &= P - P_c(\{x\}) \le 0
\end{aligned} \tag{12}$$

where

$$\tau(\{x\}) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}\ ,\ \tau' = \frac{P}{\sqrt{2}x_1x_2}\ ,\ \tau' = \frac{MR}{J}$$

$$M = P(L + \frac{x_2}{2})$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}\ , J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}$$

$$\sigma(\{x\}) = \frac{6PL}{x_4x_3^2}\ ,\ \delta(\{x\}) = \frac{4PL^3}{Ex_3^3x_4}$$

$$P_c(\{x\}) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$$

$$P = 6000lb\ ,\ L = 14in\ ,\ E = 30\times10^6\,psi\ ,\ G = 12\times10^6\,psi$$

Variable regions

$$0.1 \le x_1, x_4 \le 2\ ,\ 0.1 \le x_2, x_3 \le 10 \tag{13}$$

The welded beam structure is a practical design problem that has been often used as a benchmark problem for testing different optimization methods. Deb [30], Coello [26] and Coello and Montes [27] solved this problem using GA-based methods. Radgsdell and Phillips [31] compared optimal results of different optimization methods that were mainly based on mathematical optimization algorithms. These methods, are APPROX (Griffith and Stewart's successive linear approximation), DAVID (Davidon–Fletcher–Powell with a penalty function), SIMPLEX (Simplex method with a penalty function), and RANDOM (Richardson's random method) algorithms. Also, He and Wang [11] using CPSO, Montes and Coello [28] employing evolution strategies and the Kaveh and Talatahari [29] using the ACO solved this problem. The comparison of results, are shown in Table 3. The statistical simulation results are summarized in Table 4. From Table 4, it can be seen that, the standard deviation of the results by HPSACO in 30 independent runs is very small.

Table 3. Optimum results for the welded beam design

Methods	**Optimal design variables**				
	$x_1(h)$	$x_2(l)$	$x_3(t)$	$x_4(b)$	f_{cost}
Regsdell & Phillips [31]					
APPROX	0.2444	6.2189	8.2915	0.2444	2.3815
DAVID	0.2434	6.2552	8.2915	0.2444	2.3841
SIMPLEX	0.2792	5.6256	7.7512	0.2796	2.5307
RANDOM	0.4575	4.7313	5.0853	0.6600	4.1185
Deb [30]	0.248900	6.173000	8.178900	0.253300	2.433116
Coello [26]	0.208800	3.420500	8.997500	0.210000	1.748309
Coello & Montes [27]	0.205986	3.471328	9.020224	0.206480	1.728226
He & Wang [11]	0.202369	3.544214	9.048210	0.205723	1.728024
Montes & Coello [28]	0.199742	3.612060	9.037500	0.206082	1.737300
Kaveh & Talatahari [29]	0.205700	3.471131	9.036683	0.205731	1.724918
Present work	0.205729	3.469875	9.036805	0.205765	1.724849

Table 4. Statistical results of different methods for the welded beam design

Methods	**Best**	**Mean**	**Worst**	**Std Dev**
Regsdell & Phillips [31]	2.3815	N/A	N/A	N/A
Deb [30]	2.433116	N/A	N/A	N/A
Coello [26]	1.748309	1.771973	1.785835	0.011220
Coello & Montes [27]	1.728226	1.792654	1.993408	0.074713
He & Wang [11]	1.728024	1.748831	1.782143	0.012926
Montes & Coello [28]	1.737300	1.813290	1.994651	0.070500
Kaveh & Talatahari [29]	1.724918	1.729752	1.775961	0.009200
Present work	1.724849	1.727564	1.759522	0.008254

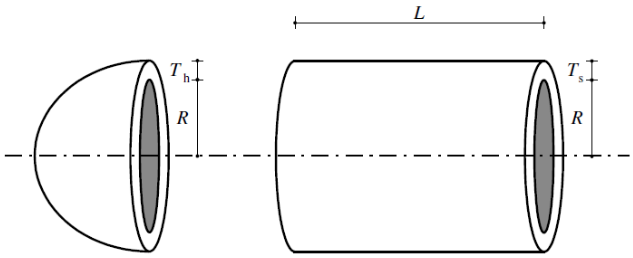

Figure 3. Schematic of pressure vessel

5.3 A Pressure Vessel Design Problem

A cylindrical vessel is capped at both ends by hemispherical heads as shown in Figure 3. The objective is to minimize the total cost, including the cost of material, forming and welding, Kannan and Karmer [33]:

$$f_{\text{cost}}(\{x\}) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3 \tag{14}$$

where x_1 is the thickness of the shell (T_s), x_2 is the thickness of the head (T_h), x_3 is the inner radius (R) and x_4 is the length of cylindrical section of the vessel, not including the head (L). T_s and T_h are integer multiples of 0.0625 inch, the available thickness of rolled steel plates, and R and L are continuous.

The constraint functions can be stated as follows

$$\begin{aligned} g_1(\{x\}) &= -x_1 + 0.0193x_3 \le 0 \\ g_2(\{x\}) &= -x_2 + 0.00954x_3 \le 0 \\ g_3(\{x\}) &= -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1{,}296{,}000 \le 0 \\ g_4(\{x\}) &= x_4 - 240 \le 0 \end{aligned} \tag{15}$$

Variable regions

$$0 \le x_1, x_2 \le 99, \; 10 \le x_3, x_4 \le 200 \tag{16}$$

The approaches applied to this problem include a branch and bound technique [32], an augmented Lagrangian multiplier approach [33], genetic adaptive search [34], a GA-based co-evolution model [26], a feasibility-based tournament selection scheme [27], a co-evolutionary particle swarm optimization [11], an evolution strategy [28] and an improved ant colony optimization [29]. The best solutions obtained by the above mentioned approaches are listed in Table 5, and their statistical simulation results are shown in Table 6.

Table 5. Optimum results for the pressure vessel

Methods	Optimal design variables				f_{cost}
	$x_1(T_S)$	$x_2(T_h)$	x_3(R)	$x_4(L)$	
Sandgren [32]	1.125000	0.625000	47.700000	117.70100	8129.1036
Kannan & Kramer [33]	1.125000	0.625000	58.291000	43.690000	7198.0428
Deb & Gene [34]	0.937500	0.500000	48.329000	112.67900	6410.3811
Coello [26]	0.812500	0.437500	40.323900	200.00000	6288.7445
Coello & Montes [27]	0.812500	0.437500	42.097398	176.65405	6059.9463
He & Wang [11]	0.812500	0.437500	42.091266	176.74650	6061.0777
Montes & Coello [28]	0.812500	0.437500	42.098087	176.64051	6059.7456
Kaveh & Talatahari [29]	0.812500	0.437500	42.098353	176.63775	6059.7258
Present work	0.812500	0.437500	42.103566	176.57322	6059.0925

Table 6. Statistical results of different methods for the pressure vessel

Methods	Best	Mean	Worst	Std Dev
Sandgren [32]	8129.1036	N/A	N/A	N/A
Kannan & Kramer [33]	7198.0428	N/A	N/A	N/A
Deb & Gene [34]	6410.3811	N/A	N/A	N/A
Coello [26]	6288.7445	6293.8432	6308.1497	7.4133
Coello & Montes [27]	6059.9463	6177.2533	6469.3220	130.9297
He & Wang [11]	6061.0777	6147.1332	6363.8041	86.4545
Montes & Coello [28]	6059.7456	6850.0049	7332.8798	426.0000
Kaveh & Talatahari [29]	6059.7258	6081.7812	6150.1289	67.2418
Present work	6059.0925	6075.2567	6135.3336	41.6825

From Table 5, it can be seen that the best solution found by HPSACO is better than the best solutions found by other techniques. From Table 6, it can be observed that the average searching quality of HPSACO is better than those of other methods.

5.4 A Pressure Vessel Design Problem

As the final example, a four-storey, two-bay frame is selected from [35]. The frame has 15 nodes and 20 elements constructed from I-beam sections and the elements are grouped into five different dimensional sets. The objective is to minimize the weight of the frame through finding the optimum cross-section dimensions b, h, t_w and t_f for each group of elements. Since there are four design variables for each group, 20 sizing variables are considered. The material density is 7850 kg/m^3 and the Young's modulus is 210 kN/mm^2. The frame dimensions, configuration, loading, and grouping of the members are shown in Figure 4.

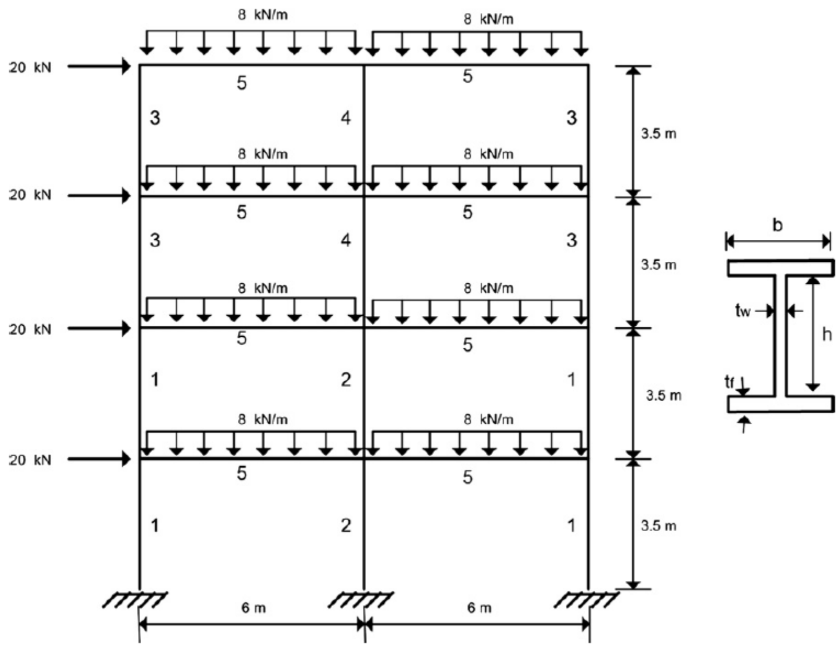

Figure 4. A four-storey, two-bay steel frame

The objective is to minimize the weight of the frame. This optimum design also has to satisfy the stress and the displacement constraints. Therefore, the problem can be stated as:

Cost function

$$f_{\text{cost}}(\{x\}) = \sum_{i=1}^{20} L_i \cdot (h \cdot t_w + 2b \cdot t_f).\gamma \tag{17}$$

Design constraint functions

$$\sigma(\{x\}) - \sigma_{\max} \le 0\,,\ \delta(\{x\}) - \delta_{\max} \le 0$$

$$0.3 \le b/h \le 1.0\,,\ 0.03 \le t_w/b \le 0.10\,,\ 0.03 \le t_f/h \le 0.10 \tag{18}$$

Variable regions

$$6 \le b \le 30\,,\ 0.7 \le t_f \le 2.5\,, 0.7 \le h \le 60\,,\ 0.4 \le t_w \le 2.5 \tag{19}$$

where the maximum allowable stress ($\sigma_{\max}$) is 160 MPa and the only displacement constraint is the maximum top storey sway ($\delta_{\max}$) limited to 2.0 cm.

Table 7 gives the best solution vectors and the corresponding weight using the proposed method, and compares the obtained results in this research with the outcomes of the harmony search and a hybrid harmony search algorithm (HHSA), Ref. [34]. An optimal structural weight of 3,564.25 kg is achieved by the HPSACO algorithm while it was 3,733.9 kg and 3,845.2 kg for HS and HSSA, respectively. The optimum result is obtained after approximately 10,500 fitness function evaluations which is less than 41,000 function evaluations for the HSSA.

Table 7. Optimum results of the HPSACO algorithm for the four-storey, two-bay steel frame

Group no.	**1**	**2**	**3**	**4**	**5**
b	15.02268	9.996847	12.46830	10.97384	12.07308
t_f	1.295927	2.163955	1.207647	1.703614	1.266929
h	43.0489	33.3357	39.7650	36.8659	41.4340
t_w	0.45767	0.61512	0.40000	0.40000	0.40000
Max. displacement (cm)			1.993		
Max. stress (MPa)			70.879		
Weight for HPSACO (kg)			3,564.25		
Weight for HS (kg) [35]			3,733.9		
Weight for HHSA (kg) [35]			3,845.2		

6. CONCLUSIONS

Heuristic algorithms are suitable tools to determine the optimum solutions of the engineering problems. However, their applications are limited by the high computational cost of the slow convergence rate. This paper tackles two major problems of optimization. The first problem is the relative deficiency of the global optimization method in refining optimum solutions with relatively small computational overhead. The second problem tackled arises from the arbitrary selection of penalty coefficients in constraint optimization. In order to deal with the deficiency of the global optimization methods, a hybrid algorithm based on the particle swarm optimization with passive congregation (PSOPC), the ant colony algorithm (ACO), and the harmony search (HS) approach, so-called HPSACO, is developed. HPSACO utilizes a PSOPC algorithm as a global search, and the idea of the ACO functions as a local search, and updating the positions of the particles is performed by a pheromone-guided mechanism. The HS-based approach is utilized to handle the boundary constraints. These principles are used in the HPSACO as helping factors to guide the exploration and to increase the control of the exploitation.

In the HPSACO algorithm, a modified feasible-based mechanism is presented to handle the problem-specific constraints. Using this mechanism, the particles can approach to the boundaries and can fly to the global minimum with a high probability. The harmony search strategy is utilized to handle the variable limits. According to this mechanism, any component of the solution vector violating the variable boundaries can be regenerated from the harmony memory.

The efficiency of the algorithm is demonstrated using several test problems and its performance is compared to those of the other conventional methods. The results reveal that HPSACO not only decreases the computational cost, but it is a reliable and efficient algorithm and the hybrid approach is a powerful tool for obtaining optimum solutions not only in terms of the quality of the solutions but also in terms of the accuracy.

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