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SYSTEM RELIABILITY ASSESSMENT OF REDUNDANT TRUSSES USING IMPROVED ALGEBRAIC FORCE METHOD AND ARTIFICIAL INTELLIGENCE

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ABSTRACT

Calculation of system failure probability in large redundant structures is a time consuming process. Therefore in many researches it has been approximated in a conservative manner directly from probability of the failure of the members. This paper proposes three strategies to speed up the calculation of P_f of indeterminate trusses. In the first strategy based on the principles of probability, a criterion is established to discard some correlated paths. In the second strategy the force method is developed and applied due to its efficiency and speed in analyzing large redundant trusses. In the force method, the number of equations to be solved is the same as the degree of static indeterminacy which is usually smaller than the total degree of freedom used in conventional displacement method. Another advantage of the force method is the immediate access to member forces, which is required to be used in reliability analysis. In this research the force method formulation is improved and made it possible for the analysis of trusses of different topologies. The third strategy corresponds to employing an Artificial Intelligent agent to identify and control the repeated failure paths to avoid the use of extra computational time.

Keywords: Reliability analysis; structural system probability of failure; reliability index; improved force method; branch and bound method; artificial intelligence

1. INTRODUCTION

The performance of a structure depends on the actions, the strength of the material being used and workmanship. However, fluctuations of the loads, changeability of the material properties, and the mathematical models are associated with some uncertainty**.** Due to many sources of uncertainty, inherent in structural design, there is the risk of unacceptable performance for structures, known as the failure of the structure.

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Uncertainties in applied loads, materials, real kind of connection of members etc require a probabilistic analysis and design of structures. In this regard by using reliability concepts, failure probability of structures can be determined. First *failure* should be defined and then failure paths and probability of failure can be calculated.

Considerable progress has been made in recent years in the reliability estimation of structures under uncertain loads. Many algorithms have been developed to estimate the reliability at the element and system levels. These methods can be categorized into three methods, namely: (a) numerical integration method; (b) failure path method; and (c) simulation**.** Among these methods, those methods based on failure paths and simulation are more common [1].

Monte Carlo simulation has often been used to calculate the probability of failure and to verify the results of other reliability analysis methods (Ang and Tang [2], Grimmelt and Schueller [3], Schueller and Stix [4]). In this method, the random loads and random resistance of a structure are simulated and these simulated data are utilized to find out if the structure fails or not, according to pre-determined limit states. The probability of failure is the relative ratio between the number of failure occurrence and the total number of simulations. This approach is easy to employ but when encountered low failure probability which often happens in real structural systems, the number of simulation becomes large so that the method becomes unpractical for most of the realistic problems [1].

In the failure path methods such as the branch and bound method [5] and the truncated enumeration method [6], failure paths with low probability are bounded and dominant failure paths are chosen. Then upper and lower bounds are determined for system failure probability. These methods have been developed and used in later studies [7,8]. There are two obstacles on these methods; firstly, there are so many failure paths in large structures and processes would be time-consuming. Secondly, the obtained upper and lower bounds of probability of failure are usually not narrow. The failure path method and simulation method have been used simultaneously in Ref. [9].

Reliability analysis of a structural system in the failure path methods consists of two steps: (a) identification of failure modes and (b) estimation of failure probabilities of individual modes and the overall system. The failure modes can be identified utilizing the standard structural analysis and can be generated automatically as described by Watwood [10] and Gorman [11]. However, real structures can have many different failure modes and it is not possible to enumerate all the possible failure modes. Many studies are performed on the possibility of employing dominant failure modes based on the assumption that safety of a structural system can be estimated efficiently using these dominant failure modes (Moses and Stahl [12], Ang and Ma [13], Murotsu et al. [14], Ranganathan and Deshpande [15], Tung and Kiremidjian [16]). The final step of reliability analysis is to find the failure probability of a given system using the combination of failure modes and bound techniques [1].

Though the analytical methods provide elegant approaches for handling the system reliability problems, one of the short comings of most of them is that for practical structures there are numerous failure sequences or stable configurations that need to be enumerated. Furthermore, the system failure probability computation involves unions and intersections of multiple failure events and yet only approximate bounds can be computed [9].

Due to various difficulties involved, many researchers have made simplifying

assumptions and estimated the system failure probability based on failure probability of its members. Gharaibeh, McCartney and Frangopol quantified the impact of each individual component on the performance of the system by ranking structural members and identifying critical components in the structural system [17]. Toğan and Daloğlu [18, 19] made a simplifying assumption to estimate structural system failure probability. They estimated the probability of failure of trusses as the summation of failure probability of its members. Also a faster method was established in ref. [1] to estimate the structural system failure probability of truss structures directly from its members.

All efforts and simplifying assumptions denote the importance of computational time. Therefore finding a faster method to determine the failure paths would be helpful. In this research three strategies are proposed to increase the speed of analyses.

In the first strategy, probability of the failure of a path is estimated by minimum joint failure probability of each member on the path along with the first member. Then next paths which include these members and are correlated to this path are discarded. Discarding these paths results in reduction of the upper bound of failure probability as well as the time required for the calculations.

In the second strategy an efficient method is presented to analyze the trusses. In this research the force method is developed and used for analyses of trusses with different topologies. The force method has two advantages over the conventional stiffness method. Firstly, unlike stiffness method, force method calculates member forces before nodal displacements. In reliability analysis, member forces are required. Secondly, the number of equations to be solved is the same as the degree of static indeterminacy of the structure which is often less than the total degrees of freedom for trusses, thus increasing the speed of optimization [20].

The advantage of using this method lies in the fact that the matrices corresponding to the particular solution B_0 matrix and the complementary solutions B_1 matrix are formed independent of the mechanical properties of members. These matrices are utilized several times in the process of a sequential analysis [20]. This can be done provided the topology of the truss under investigation does not change. Whereas in reliability analysis of indeterminate trusses, it is required to analyze trusses in which some member(s) has/have failed. Therefore, it has not yet been possible to use the force method for such trusses.

In this research, formulation has been developed such that one can use the force method for analysis of trusses with different topologies. Thus by avoiding the repetition of the analysis, the speed of computation increases.

The third strategy is using an artificial intelligent agent in saving failure paths and identifying repeated ones and thus avoiding extra calculations and saving the computational time.

2. RELIABILITY OF STRUCTURAL SYSTEMS

In the reliability analysis of a structural system we investigate the probability of the structure not to fail under the applied loads. For calculating the probability of the failure of a structural system one should specify the probability of the failure of its members and those of the failure paths.

Most of the structural models are a combination of series and parallel systems. In a statically indeterminate truss which has some failure paths the bars consisting in a failure path are parallel since only if all the bars of this path fail the truss will fail. By the failure of each path member, redistribution of the internal forces occurs among the remaining members and the next member to fail is determined [21]. On the other hand, all these failure paths are in series and if one fails the truss will fail.

Due to correlation, exact calculation of the paths failure probabilities and thus the structural system failure probability is very difficult for practical problems. Accordingly, some common bounds such as Cornell, Ditlevsen etc are usually used to evaluate these probabilities [5,22].

2.1 Automatic generation of the safety margins

Consider an indeterminate truss having n members and l loads applied on its joints. For statically indeterminate truss structure, failure in a member does not necessarily lead to structural failure. Failure event $F_i^{(1)}$ occurs if safety margin of member i in intact structure becomes smaller than zero, and it is mathematically defined by

$$F_i^{(1)} \equiv (M_i^{(1)} < 0) \equiv R_i - S_i < 0 \tag{1}$$

Where the superscript 1 designates the intact structure in which no member has failed. R_i is the strength of member i and S_i is its internal force. The strength of the member is determined by

$$R_i = A_i.\sigma_{y_i} \tag{2}$$

Here A_i and σ_{y_i} are cross-section area and yielding stress of the member, respectively. Internal force in the member due to applied forces can be calculated by analyzing the structure as Eq. (3)

On the other hand, the internal force S_i is not easy to evaluate, and thus it is derived by applying a matrix method [5].

$$S_i = \sum_{j=1}^{l} a_{ij}^{(1)}.L_j \tag{3}$$

Where $a_{ij}^{(1)}$ denotes the internal force of member due to unit load applied in direction of load L_j in the intact structure. Substituting the outcomes of Eqs. (2) and (3) for strength and force in Eq. (1), safety margin for each member can be derived.

Now the member which has the highest probability of failure is chosen as the candidate to fail and its stiffness matrix is removed from the assembled stiffness matrix. Then if the member is perfectly elastic-plastic and ductile, its strength is treated as a force and new truss is reanalyzed. This process is repeated until the determinant of the assembled stiffness matrix becomes zero, indicating the collapse of the structure.

The internal force of the ith bar after redistribution of the bar forces r_1 to r_{p-1} can be calculated from

$$S_i^{(p)} = \sum_{j=1}^{l} a_{ij}^{(p)}.L_j - \sum_{k=1}^{p-1} b_{ir_k}^{(p)} R_{r_k} \tag{4}$$

Where $b_{ir_k}^{(p)}$ denotes the internal force in i^{th} member due to applying unit force in direction of the member r_k in p^{th} stage. Safety margin in this stage is determined by Eq. (5) as

$$M_i^{(p)} = A_i \sigma_{y_i} + \sum_{k=1}^{p-1} b_{ir_k}^{(p)} R_{r_k} - \sum_{j=1}^{l} a_{ij}^{(p)}.L_j \tag{5}$$

The correlation coefficient between the events of jth stage (truss without members r_1 to r_{j-1}) and k^{th} stage (truss without members r_1 to r_{k-1}) is calculated by

$$\rho(F^{(j)}, F^{(k)}) = \rho_{jk} = \frac{COV(F^{(j)}, F^{(k)})}{\sigma_{F^{(j)}}.\sigma_{F^{(k)}}} \tag{6}$$

Where $F^{(j)}$ and $F^{(k)}$ denote events of jth and k^{th} stages. The exact value of the joint probability of the two events, assuming random variables are normal and their correlation coefficient and reliability indices are given, can be calculated by converting the two dimensional integral of Eq. (7) to the one dimensional Eq. (8), as described in [23].

$$P_{jk} = \int_{-\infty}^{-\beta_j} \int_{-\infty}^{-\beta_k} \varphi_2(x_1, x_2, t) dx_1 dx_2 \tag{7}$$

$$P_{jk} = \Phi(-\beta_j)\Phi(-\beta_j) + \int_0^{\rho_{jk}} \varphi_2(-\beta_j, -\beta_k, t) dt \tag{8}$$

Where φ_2 and Φ indicate bivariate standard normal density and standard normal distribution functions which are determined by Eqs. (9) and (10), respectively.

$$\varphi_2(x_1, x_2, \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{x_1^2 + x_2^2 - 2\rho x_1 x_2}{2(1-\rho^2)}\right) \tag{9}$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-t^2/2) dt \tag{10}$$

2.2 Branch and bound method

Dominant failure paths in this research have been determined using the branch and bound method.

2.2.1 Branching operations

This is an operation which selects a combination of members to form a mechanism such that a failure path with a large failure probability is found. The member r_i, with the largest value of the joint failure probability along with the first member of the path r_1, is selected in the ith failure stage as defined in following equation.

$$P(F_{r_1}^{(1)} \cap F_{r_i}^{(i)}) = \max\{P(F_{r_1}^{(1)} \cap F_j^{(i)})\} \quad j = \{working \;\; members\} \tag{11}$$

Where the superscript i, denotes a truss in which members r_1 to r_{i-1} have failed. The probability of a path occurring at stage i with members r_1 and r_{i-1} being deleted can be found from

$$P_i = P\{\bigcap_{j=1}^{i}(F_{r_j}^{(j)})\} \tag{12}$$

The above operation is continued until the failure of the structure occurs at stage F.

It is difficult to evaluate exactly the failure path probability when the path length p exceeds 3 members [5]. Therefore in this research a more conservative and approximate relation as Eq. (13) is employed for finding the probability of each failure path [21].

$$P_{f_F} = Min\{P_{f_{1,2}}, P_{f_{1,3}}, ..., P_{f_{1,F}}\}\ ;\ P_{f_{1,j}} = P(F_{r_1}^{(1)} \cap F_{r_j}^{(j)})\ ; j = 2, ..., F \tag{13}$$

The advantage of this approximate equation is that it does not need to save data of all the stages, and it is only needed to save the data of the first stage.

2.2.2 Adjusting operations

Upper and lower bounds in the branch and bound method are generally evaluated by these operations which are based on the Cornell's bounds. In the Cornell's method, system failure probability is equal or less than the sum of the probability of the dominant failure path, and also is equal or more than the P_f of the path whose failure probability is the most.

During branching operation, the probability of the failure of each path is estimated using Eq. (13). Then this value is compared to the probability of the most probable failure path obtained so far, i.e. $MaxP_f$. If the probability of the new path is more than $MaxP_f$, then $MaxP_f$ is upgraded. After the completion of the branch and bound operations, the upper bound is obtained using the sum of the failure probabilities of the paths. Also the lower bound is approximated from Eq. (13) as $MaxP_f$.

2.2.3 Bounding operations

This is an operation for selecting members to be discarded [21]. Before describing this operation, it should be noted that the probability of the occurrence of a path gradually (haphazardly) decreases during the branching operation. In other words the failure probability of members *1* to *p* is equal or less than that of the members *1* to *p–1*.

Now during the branching operation and when a new path is supposed to be selected, we can compare the probability of the occurrence of this path with $MaxP_f \times 10^{-\eta}$. If this occurrence probability is less than the above mentioned value, then it will become smaller during the branching operation and we can discard such an incomplete path, before the formation of the mechanism. Discarding incomplete paths with low probability of occurrence is called *bounding*.

η is a constant and depends on the accuracy of the problem and designer's opinion. The greater η is considered, the less the number of bounded paths and the more accurate the probability of failure. On the other hand one can be certain that the discarded failure paths have the probabilities less than $MaxP_f \times 10^{-\eta}$ [5]. The effects of η are evaluated through an example presented in the 6th section. The branch and bounding operations are repeated until no member is left to be selected [21].

3. ALGEBRAIC FORCE METHOD FOR STATIC ANALYSIS OF STRUCTURES

Primary Formulation and history of the force method can be found in references [24, 25]. Kaveh and Kalatjari improved the force method and used it for optimizing trusses by genetic algorithm [20, 26]. Then Rahami, Kaveh and Gholipour [27] used the integrated force method and energy methods to optimize trusses topology.

In the force method the equilibrium equations of the structure, independent of mechanical property of members, can be expressed as

$$P_{n\times 1} = H_{n\times m} \cdot r_{m\times 1} \tag{14}$$

where P is the n-dimensional load vector, H is an $n\times m$ equilibrium matrix and r is an m-dimensional member forces vector. For truss structures, n is equal to the active degrees of freedom for joints and m is the number of members. Each row in H contains the coefficients corresponding to the equilibrium of member forces connected to a joint with external loads applied in the directions of the degrees of freedom of the joint. Therefore for an active joint in *3-D* space, the equilibrium conditions $\sum Fx=0$, $\sum Fy=0$ and $\sum Fz=0$ establish the relationships between the member forces connected to a joint and the applied loads at that joint. These relations fill three consequent rows of H [20].

For a rigid truss, with $t=m-n\geq 0$, the matrix H has full row rank, i.e. rank $(H)=n$, and t specifies the degree of static indeterminacy (redundancy) of the structure [20].

Force method contains two main steps. In the first step, matrices B_0 and B_1 are constructed such that the member force vector can be expressed as

$$r_{m\times 1} = B_{0_{m\times n}} \cdot P_{n\times 1} + B_{1_{m\times t}} \cdot q_{t\times 1} \tag{15}$$

In this equation, B_0 is an $m\times n$ matrix such that HB_0 is an $n\times n$ unit matrix and HB_1 is a $n\times t$ null matrix. Each column of B_0 contains the forces produced in the members of the

selected rigid primary structure if unit load is applied in the direction of the degrees of freedom of each joint, and each column in B_1, with dimensions of $m \times t$, is a self-equilibrating stress system, known as a null vector. In Eq. (15), the vector q has dimension t corresponding to the degree of static indeterminacy, and the columns of B_1 form the base of a null space, known as the null basis.

Methods for the formation of special patterns for B_1 leading to sparse, banded and well conditioned flexibility matrices are developed by Kaveh [24]. In the second step, the following equations are solved [20]:

$$\left(B_1^t F_m B_1 \right) \cdot q = -\left(B_1^t F_m B_0 \right) \cdot P \tag{16}$$

Where F_m is an $m \times m$ diagonal matrix containing the member flexibility, and $G = B_1^t F_m B_1$ is known as the flexibility matrix of the structure. For a banded B_1, the matrix G also becomes banded [24]. The joint displacements can be obtained from

$$d_{n\times 1} = B^t_{0_{n\times m}} \cdot F_{m_{m\times m}} \cdot r_{m\times 1} \tag{17}$$

In this paper the LU method is used to determine the B_0 and B_1 matrices. In this method, the calculated flexibility matrix $G = B_1^t F_m B_1$ becomes a symmetric and positive-definite matrix. Therefore the matrix q can easily be determined by the Cholesky method. Thus there it is not necessary to control and avoid division by zero or the root of negative numbers.

3.1 LU factorization method

This method is well described in Kaneko et al. [28]. Kaveh and Kaltjari presented new relationships to increase its efficiency and speed. In this method, the equilibrium matrix $H=[H_1;H_2]$ is decomposed such that $H_1=LU$. Matrix H_1 is a non-singular n×n square matrix and its columns are obtained from independent columns of H. Matrix L is a lower triangular non-singular matrix, and U is a non-singular upper triangular matrix having unit entries on its main diagonal.

Using the Eq. (18), the columns of U and L are formed simultaneously. If zero occurs in the diagonal entry of L, for avoiding singularity of H_1, the corresponding dependent column in H which is related to the member number of redundant is skipped and operations are continued from the next column. Thereby dependent columns of H forming an $n \times t$ matrix H_2 are obtained.

$$\begin{gathered} \text{For} \qquad j=1, 2, \ldots, n \\ u_{mj} = \frac{h_{mj} - \sum_{i=1}^{m-1} l_{mi} u_{ij}}{l_{mm}} \; ; m = 1,2,,\ldots, j-1 \\ l_{ij} = h_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj} \; ; i = j, j+1, \ldots, n \qquad u_{jj} = 1.0 \end{gathered} \tag{18}$$

In these relations, h_{ij}, l_{ij} and u_{ij} are the entries of row I and column j of H, L and U matrices, respectively. Forming $H_1=LU$ and H_2, the following matrices are constructed:

$$B_1 = \begin{bmatrix} -H_1^{-1}H_2 \\ I \end{bmatrix}_{m\times t} \quad ; \quad B_0 = \begin{bmatrix} H_1^{-1} \\ O \end{bmatrix}_{m\times n} \tag{19}$$

H_1^{-1} is an $n\times n$ matrix and can be obtained using different numerical methods such as LU decomposition, Chapra and Raymond [28]. The submatrix O in B_0 is a null matrix of dimension $t\times n$ and the unit matrix I in B_1, is of dimension $t\times t$. The rows for these two submatrices are placed in rows corresponding to redundants, after appropriate row operations. The matrix B_1 obtained in this way has all its zero entries at the lower left corner of the matrix. With this pattern the flexibility matrix will not be banded. The next stage of calculation consists of using Eqs. (15) and (16) for calculating internal forces of the members and the redundants vector, respectively. The nodal displacements can be calculated employing the Eq. (17); however this has no application in the reliability analysis.

4. SUGGESTED METHODS

Three strategies are proposed in this study to save calculation time to determine the failure paths:

- Proposed strategy of reliability,
- Improved formulation of the algebraic force method,
- Using artificial intelligence techniques.

4.1 The proposed method of reliability

As mentioned before failure probability of a path like $i\rightarrow j\rightarrow k\rightarrow l$, is estimated by Eq. (13), in this research. Therefore failure probability of this path is estimated by minimum joint failure probability of members j, k and l to i. This path is shown by solid line in Figure 1.

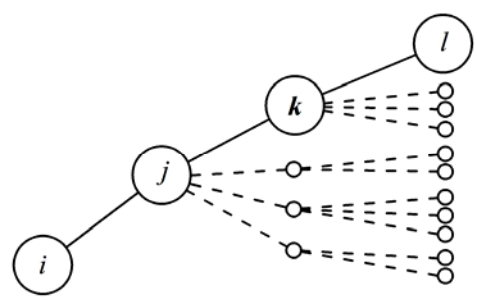

Figure 1. Schematic diagram of the paths branched from $i\rightarrow j$

Now, suppose minimum joint probability of these members is associated with i and j as

shown in Venn diagram of Figure 2. Therefore failure probability of the path $i \rightarrow j \rightarrow k \rightarrow l$, i.e. $P(F_i \cap F_j \cap F_k \cap F_l)$ is estimated by $P(F_i \cap F_j)$. However, it is obvious from Figure 2 that the event $F_i \cap F_j \cap F_k \cap F_l$ is a subset of the event $F_i \cap F_j$.

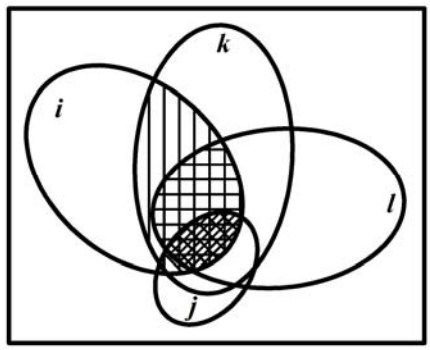

Figure 2. Minimum intersection in Venn diagram

We can generalize this to other paths branching from $i \rightarrow j$ and so these paths are also subsets of the event $F_i \cap F_j$. These paths are shown by dotted line in Figure 1.

Now, if the event of failure path $i \rightarrow j \rightarrow k \rightarrow l$ is estimated by event $F_i \cap F_j$ using Eq. (13), then the probability of this event will be added to the probabilities of the previous paths according to Cornell's upper bound.

In this case, we should prevent producing other failure paths branched from $i \rightarrow j$, because as mentioned before all these paths are subsets of $F_i \cap F_j$ and union of events which are all subsets of an event is also a subset of that event. Producing such paths not only does not add any information to the failure space, but also causes fatal errors in result by accumulating their probabilities according to Cornell's upper bound and results in a very large amount for the upper bound of system failure probability. In addition, producing these paths results in an unnecessary increasing of the computational time.

Discarding correlated paths increases the accuracy and leads to saving computational time. However we can still discard more paths and obtain better results and save more time. For this purpose, consider the case where the members i and l have lower joint failure probability than the other members in the path $i \rightarrow j \rightarrow k \rightarrow l$. In such a case, there will be no path to discard based on the strategy proposed, since the smallest probability corresponds to the last member of the path.

Now, again consider the member j which does not have minimum intersection with i anymore. As indicated above, the probability of all paths branching from a path like $i \rightarrow j$ is certainly a subset of the event $F_i \cap F_j$. This means the probability of the failure of these paths should not exceed the probability of $F_i \cap F_j$.

Therefore during branching operation whenever the sum of probability of previous paths

exceeds the probability of $P(F_i \cap F_j)$, then we should stop branching and choose $P(F_i \cap F_j)$ instead of all paths branched from $i \rightarrow j$. By discarding the remaining branches not only the accuracy of the upper bound but also the calculation speed increases.

4.2 Improved formulation of the algebraic force method

Before introducing our improved formulation, we discuss on implementation of the traditional stiffness method in the reliability analysis.

Consider a 10-member truss as shown in Figure 3. This truss has eight active degrees of freedom and two degrees of indeterminacy. To calculate internal forces of the members, first we must determine the assembled stiffness matrix. Then the nodal displacements vector is calculated using as

$$K.a = F \tag{20}$$

where K is the stiffness matrix of structure and F is the external loads vector. According to this equation, 8 equations with 8 unknowns (number of active degrees of freedom for joints) should be solved for the considered truss.

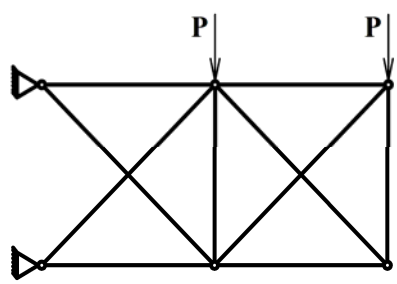

Figure 3. A statically indeterminate 10-member truss

During the branching operation, when a new member fails, redistribution of forces should be performed. For this purpose the stiffness of the members should be nullified. In addition the stiffness matrix of structure must be modified. For this purpose, the stiffness of the new failed member must be considered as zero and corresponding arrays on the assembled stiffness matrix should be modified. Then the new structure must be analyzed to calculate the failure probability of the members and select the most probable member to fail. This operation is repeated until the collapse of structure or up to the stage where the stiffness matrix becomes singular. Several analyses should be performed for each reliability analysis, and the number of equations for each analysis is equal to the number of the active degrees of freedom (8 for the above mentioned truss).

Now we can improve the formulation of the algebraic force method in order to increase the speed of reliability analysis. We can modify the H matrix similar to the stiffness method. However in this case, all processes like decomposition of H matrix, determination of B_0 and B_1 matrices and solution of equation (15) should be performed from the start. Instead, we can consider the flexibility of the failed member as infinite or a very large number. Thus the flexibility matrix F_m will be modified and it is not necessary to change the H matrix, since

its arrays do not depend on the stiffness or the flexibility of the members. Now only the equation (15) must be solved with the modification performed on F_m. Solution of this equation does not consume much time, because only 2 equations with 2 unknowns must be solved in place of 8 equations.

In other words, the number of the equations in the force method decreases from the number of active degrees of freedom of nodes to the number of the degree of static indeterminacy. The latter is usually less for truss structures. Considering that these equations should be solved several times in a reliability analysis, using the proposed method can saves considerable amount of computational time. Another advantage of the force method is that the internal forces of the members are calculated directly and before the nodal displacement vector, resulting in saving the calculation time.

It should be noted that the existing force in each failed member, indicates the collapse of the structure in the proposed force method.

4.3 Application of artificial intelligence techniques

Each failure path in a truss denotes a sequence of members. Consider a truss with n members. If the path $i \rightarrow j \rightarrow k$ is a failure path, then the path $i \rightarrow k \rightarrow j$ or each combination of these three members certainly causes the collapse of the structure. In other words, failure of these members simultaneously (independent of their sequence) results in the collapse of structure.

Now if in the truss under investigation, two members i and k are failed, then the failure of the third member j will definitely result in total instability of the truss, and controlling the stability condition would not be necessary. As mentioned before, we can control instability by checking singularity of stiffness matrix in stiffness method and existence of the force in the failed members.

There are two points here that should be mentioned. Firstly, in order to calculate the probability of failure path $i \rightarrow k \rightarrow j$, we should analyze the truss in which member i and then member k have failed and there is no need to eliminate member j. Secondly, for a structure with S degrees of redundancy, each path containing $S+1$ *members* is certainly a failure path and it would be unnecessary to control instability condition or to use an AI technique.

In the present study, in order to save the failure path $i \rightarrow j \rightarrow k$ for a truss with n members, we use a string of length n in which all its characters corresponding to the failed members, i.e. i, j and k are zero, and the remaining characters are 1.

$$\overbrace{\underset{i\ \ j\qquad k}{111101011110111}}^{n} \tag{21}$$

The number of paths formed by the failure of the members i, j and k is equal to 3! =6. The above string may be similarly produced during the branching operation by the formation of 5 other paths. These paths are identified immediately by AI agent as failure paths, and there is no need to control the instability requirement, resulting in saving computational time.

In general, a string of length n and containing n_0 zero characters denotes an unstable truss with n members in which n_0 members have failed. In this situation, up to n_0! different sequences may form the abovementioned string. However, only the first sequence is analyzed and saved and the other sequences are identified by searching in unstable truss bank and consequently extra analyses are avoided.

The type of search has a major effect on the calculation time. In the *sequential search*, we must compare in sequence, starting with the first string, until the new string is located. Consequently the maximum number of search is the same as the number of the members in the bank. In this approach if the new string is not found, then it is added to the bank. During the branching operation, the number of search increases and results in wasting considerable amount of computational time. Instead, we can use a faster approach, namely the *binary search* [30]. However we must sort the bank members in descending or ascending order, in advance.

The comparison of the strings is similar to the comparison of binary numbers. If the entries of the bank are ordered in descending form, then for finding a string one should compare it with the middle entry of the bank. If the considered string is bigger than the middle string, the answer will be in the top half of the bank, and otherwise it will be in the bottom half.

An algorithm for performing a binary search of an array which is sorted in the non-decreasing order is similar to thumbing back and forth in a phone book. This means, when we search for x, the algorithm first compares x with the middle item of the array. If they are equal, the algorithm has done the job. If x is smaller than the middle item, then x must be in the first half of the array (in case is present at all), and the algorithm repeats the search procedure on the first half of the array, i.e. x is compared to the middle item of the first half of the array. If they are the same, then the algorithm is done, etc.) If x is larger than the middle item of the array, the search will be repeated on the second half of the array. This process is repeated until x is found or it is determined that x is not in the array.

In this method, the search space decreases into half in the first step and then decreases to one fourth. This operation is repeated until the new string is found. In this method the maximum required number of search $n_{\max}$, in a bank with N strings, is determined by Eq. (22).

$$n_{\max} = \text{int}[\log_2 N] + 1 \tag{22}$$

In this equation, *int* picks the integer part of the result and $n_{\max}$ is usually much smaller than N. Thus for a large search space, using this method is unavoidable. Sorting bank members should be performed in a manner that each string is saved on its own place, so that there is no need to sort the bank after entering each new string. In this research, the *quick sort* has been used to find the right location of a new string [30].

During performing the operations, some of the calculation time is associated with the control of the instability condition. Using the AI techniques, the repeated failure paths are identified, resulting in decreasing the time required to calculate the failure probability of entire structure.

In this research the AI acts as that whenever the algorithm comes to a new path, first it

examines whether this combination of members exists in the previously selected failure paths or not. In case the answer becomes positive then it avoids checking the collapse condition and responds immediately. Otherwise, it sends the truss to the analysis unit and waits for result. Then the intelligent agent examines the result. If the new path is a failure path, then the agent saves it to be used later if it becomes necessary.

As it is mentioned before by forming the failure of the path $i \rightarrow j \rightarrow k$, its corresponding string like the string obtained by Eq. (21) is saved. Then the intelligent agent, used in present study, identifies the other 3!-1 failure paths containing these members in order to avoid the extra processing time required for their identification.

However there are still more failure paths, which can be identified, considering string equation 21. For example, in the considered truss, if $j \rightarrow m \rightarrow k$ is not a failure path, then the branching operation is continued.

Now, after the reliability analysis of truss, if the next member to fail is selected as i, then a new path $j \rightarrow m \rightarrow k \rightarrow i$ forms. The corresponding obtained string can be shown by

$$\overbrace{1111\underset{i}{0}1\underset{j}{0}1111\underset{k}{0}1\underset{m}{0}1}^{n} \tag{23}$$

The new path is definitely a failure path, since it contains members *i, j* and *k.* This means that the number of failure paths which can be formed and identified by string equation 21 are more than 3!. In general, one can say that *all the paths containing these three members are definitely failure paths and there is no need to check the collapse requirement for them.*

It should be noted that the failure of the new string (Eq. (23) is predictable, comparing with the existing strings of Eq. (21). Since all the ith, j^{th} and k^{th} characters of the both paths (Eqs. (21 and 23)) are zero and only their m^{th} characters are different.

Now, if we use the "&" operator (conjunction operator) for each character of these strings, the result will be the string 23. Since this operator responds to 0 in between two events 0 and 1. In general, this operator will respond to 1 only if both events are 1. This comparison is presented mathematically by the following:

$$\begin{array}{r} \underset{i}{1}\underset{j}{1}1\underset{k}{0}1\underset{m}{0}11101111 \\ \& \; 11101011101011 \\ \hline 11101011101011 \end{array} \tag{24}$$

It can therefore be concluded that whenever the conjunction of the new string characters with those of one existing member of the bank becomes identical to the new string characters, then the new path is a failure path and extra analyses and controls can be discarded.

As mentioned before in this study, after saving the first string like 21, the binary search method has been used for identifying the next 3!-1 failure paths (n_0!–1 in general). Comparison criterion used in this case is equality of strings and the search space decreases

into half in each step based on largeness or smallness and the operation is repeated until the required string is found.

However for identifying more failure paths like string 23 containing more failed members, the comparison criterion is not equality of strings anymore. As mentioned, the comparison criterion is equality of the new failure path and its conjunction with one member in the bank. Therefore, the smallness or largeness of the new string compared to the previously used strings is not a criterion and one should develop a new ordering method or binary search algorithm.

5. NUMERICAL EXAMPLES

In this section four examples are presented to evaluate the effects of the proposed strategies.

Example 1. Consider a statically indeterminate 6-member truss with one degree of redundancy as shown in Figure 4. Dimensions are in cm. The allowable yield stress and applied loads are uncorrelated normal random variables. Mean value and coefficient of variation of yielding stress of members are 2760 kg/cm^2 (39.2563 ksi or 270.662 MPa) and 0.05, respectively. Statistics parameters of applied loads are given in Table 1. In this example, failure due to buckling is not considered. The behavior of the material in compression and tension are considered to be identical.

Cross section area of all the members are 2.3 cm^2 (0.3565 in^2). Five first failure paths and their corresponding safety indices are compared with those of Ref. [31] in Table 2. Positive and negative signs denote failing members in tension and compression, respectively. Since in this example, the degree of redundancy is equals to one, the first and second strategies are not applicable and despite using approximate Eq. (13), the answer will be exact.

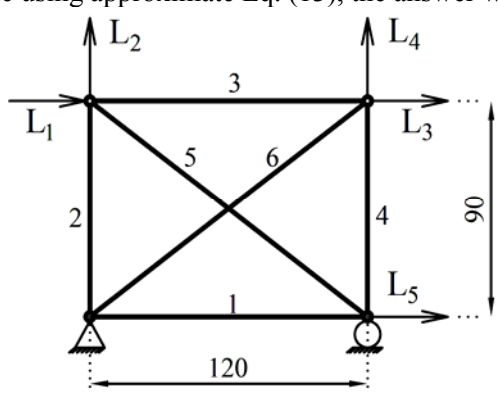

Figure 4. Statically indet erminate 6-member truss in example 1

It can be observed from Table 3 that the calculation time using the developed force method (second strategy) is less than that of the stiffness method, while results are identical.

Table 1: Statistic data of loads (6-member truss)

Random variable	kips	Coefficient of variation
L1	11023.1	0.1
L2	6613.86	0.1
L3	4409.24	0.1
L4	6613.86	0.1
L5	4409.24	0.1

Table 2: Results of force and stiffness methods (6-member truss)

Failure path	Reliability index	
	Ref. [31]	Present study
$6^+ \longrightarrow 2^+$	3.05279	3.05279
$6^+ \longrightarrow 1^+$	3.44798	3.447975
$6^+ \longrightarrow 5^-$	4.87714	4.87714
$2^+ \longrightarrow 3^-$	6.99763	6.99763
$1^+ \longrightarrow 3^-$	8.12484	8.12483

Table 3: Comparison of the stiffness and the developed force method (6-member truss)

	Developed force method	Stiffness method
Upper bound of system failure probability	1.7475×10^{-6}	1.7475×10^{-6}
Time (sec)	1.321×10^{-2}	1.654×10^{-2}

Example 2. Consider a 16-member statically indeterminate truss as shown in Figure 5. Dimensions of the truss members in this figure are shown in centimeter. The yield stress and

applied loads are uncorrelated normal random variables. Mean value of the load P and the yield stress are 4445kg (9799.54kips) and 2760 kg/cm^2 (39.2563 ksi or 270.662MPa), respectively. Coefficients of variation of both loads and member strengths are considered 0.1. Modulus of elasticity of members is considered as $2.06\times10^6\, kg/cm^2$ (29300 psi or 202015.96 MPa) . Cross section areas are presented in Table 4. Failure due to buckling is not included. Effects of using of each proposed strategy on the number of the dominant failure paths, the reliability analyses and the calculation time are presented in Table 5. All the magnitudes in this table are based on $\eta = 4$ (see subsection 2.2.3).

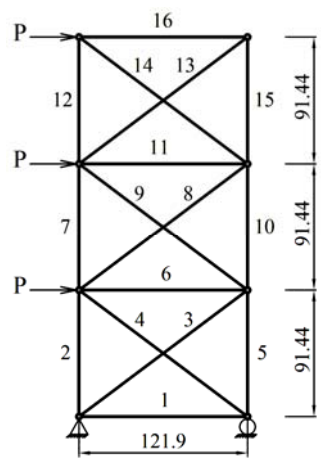

Figure 5. The statically indeterminate 16-member truss of example 2

Table 4: Cross section areas of 16-member truss

Member	Area (cm^2)
1	3.35
2,5	8.64
3,4,14	5.76
6	2.29
7,8,10	4.03
9	7.35
11,12,15	1.58
13,16	2.29

Table 5: Comparison of the proposed strategies on example 2

	No	**St. 1**	**St. 2**	**St. 3**	**All**
Calculation time (sec)	0.41	0.18	0.29	0.37	0.11
Number of dominant failure paths	383	32	383	383	32
Number of reliability analyses	163	61	163	121	54

Based on results of Table 5, calculation time is decreased by using each proposed strategy; however, the effect of the first strategy is more apparent. In this example, there are 383 probable path failures. Using the first strategy, 351 failure paths (of 383 paths) are discarded due to some correlation between them and only 32 failure paths are constructed.

In this example, without using the first and third strategies, the reliability analysis subroutine is called 163 times. This subroutine analyzes the truss whenever one of its members fails as many times as the number of its joint loads. In this truss, on which three loads are applied, at the third stage of a path (two members have failed), then the subroutine analyzes the truss 3+2=5 times. Also, this subroutine in each step, determines the joint probability of failure of all survived members with the first member of each path.

Calling the reliability subroutine has decreased to 61 times, by using the first strategy. The intelligent agent of the third strategy also has avoided calling this subroutine 42 times by identifying the repeated failure paths. Calling the subroutine has decreased to 54 by using the first and third strategies simultaneously.

Upper bound of the structural system failure probability in this example is 6.29×10^{-5}. This value without using first strategy and discarding correlated paths is 2.65×10^{-4}. However, both of these values are smaller than the overestimated value in Ref. [1] i.e. 5.04×10^{-4}. Lower bound, i.e. the most probable failure path is estimated as 2.02×10^{-5} using Eq. (13). Therefore, the bounds which are determined using the first strategy are fairly narrower.

Example 3. Consider the statically indeterminate 25-member truss as shown in Figure 6. Member cross section areas, nodal coordinates and the mean values of the applied loads are presented in Tables 6, 7 and 8. The coefficient of variation of the applied loads is 0.1. The mean value and the coefficient of variation of members strength are 2760 kg/cm^2 (39.2563 ksi or 270.662MPa), and 0.03, respectively. The yield stress and the applied loads are considered as uncorrelated normal random variables and the behavior of the material in tension and compression are considered identical.

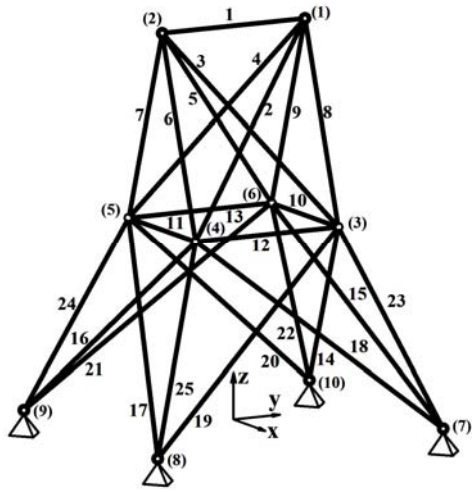

Figure 6. The statically indeterminate 25-member truss in example 3

Table 6: Cross section area of truss in example 3

Member	Area (cm^2)
1	2.86
2,5	2.83
3,4	6.02
6,9	2.35
7,8	5.76
10,11	1.21
12,13	1.10
14,17	3.43
15,16	1.45
18,21	0.84
19,20	1.28
22,25	2.42
23,24	4.84

Table 7: Nodal coordinates of example 3 (in^2)

Node	x	y	Z
1	0	37.5	200
2	0	–37.5	200
3	37.5	37.5	100
4	37.5	–37.5	100
5	–37.5	–37.5	100
6	–37.5	37.5	100
7	100	100	0
8	100	–100	0
9	–100	–100	0
10	–100	100	0

Table 8: Statistic data of the loads for example 3 (kips)

Node	X	y	Z
1	19.599	19.599	−4.982
2	−19.599	−19.599	−4.982
3	19.599	19.599	0
5	−19.599	−19.599	0

Reliability analysis of this structure without using the first strategy consumes about three hours, considering millions of dominant failure path which many of them are somehow correlated and the resulting upper bound for structural system failure probability will be very conservative. Therefore, the application of the first strategy the calculation time decreases from 3 hours to 15.05 seconds. For $\eta=2$, the comparison of this strategy with the second and third strategies are presented in Table 9.

Table 9: Comparison of the proposed strategies on example 3

	St. 1	**St. 1,2**	**St. 1,3**	**All**
Calculation time (sec)	15.05	5.66	14.04	5.27
Number of dominant failure paths	340	340	340	340
Number of reliability analyses	1022	1022	845	845

According to Table 9, using of the proposed strategies results in considerable reduction of the calculation time. By using the second strategy i.e. the developed force method, the calculation time is decreased, while the number of analyses stayed unchanged. The third strategy has saved time by identifying the repeated failure paths and avoiding the additional calculations. In [5,18] the system failure probability has been estimated conservatively as the sum of the failure probability of the members and thus is estimated as 1.00×10^{-5}. While in this research the upper bound of system failure probability is evaluated as 6.685×10^{-7} (for $\eta=2$), which is much better than the overestimated value of the above mentioned references.

The structure is also analyzed by different value of η, in order to evaluate the mentioned parameter's effects. Calculation time, number of the dominant failure paths and the upper bound of the structural failure probability obtained in each situation are presented in Table 10. Based on the results of Table 10, all the mentioned parameters are obviously increased by raising the η parameter.

Table 10: Effect of η on example 3

	$\boldsymbol{\eta = 1}$	$\boldsymbol{\eta = 2}$	$\boldsymbol{\eta = 3}$	$\boldsymbol{\eta = 4}$	$\boldsymbol{H \geq 5}$
Calculation time (sec)	3.31	15.05	25.66	36.00	≥48.34
Number of dominant failure paths	159	340	660	903	≥1219
Structural failure probability upper bound ($\times10^{-7}$)	6.288	6.685	6.776	6.786	6.787

Based on Table 10, for $\eta \geq 5$, the upper bound of the structural failure probability is calculated as 6.685×10^{-7}, i.e., considering more failure path not only does not make any contribution to the outcome, but also results only in wasting time. Also as indicated in Table 10, the result error for $\eta = 2$ is evaluated about 1.5 percent, while the time required is only 15 seconds which is much smaller than 48 seconds. That is, for problems such as reliability-

based optimization ones, in which the calculation time is an important parameter, predetermination of η, may be unavoidable. It is clearly evident that the suitable value of η may vary slightly from problem to problem.

Example 4. Consider a statically indeterminate 72-member truss as shown in Figure 7. In this figure the dimensions are shown in centimeters. The cross section areas of the members are presented in Table 11. Statistical parameters of the loads are provided in Table 12. The mean value and the coefficient of variation of the member strength are 2760 kg/cm^2 (39.2563 ksi or 270.662 Mpa), and 0.05, respectively. The yield stress and the applied loads are considered as uncorrelated normal random variables and behavior of the material in tension and compression are considered identical. Results of the reliability analysis of this truss based on the proposed strategies are presented in Table 13, considering $\eta=2$.

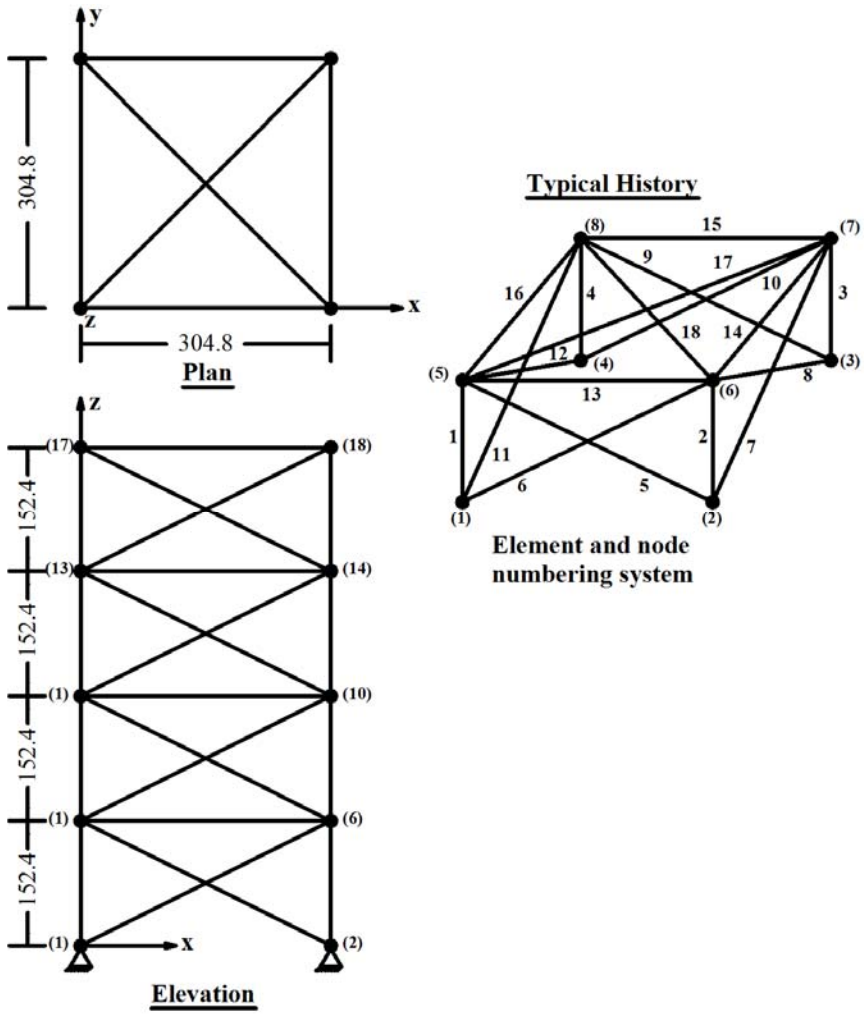

Figure 7. Statically indeterminate 72-member truss of example 4

Table 11: Member cross section area of truss in example 4

Member	Area (cm^2)
1-4	2.1
5-12	0.7
13-16	0.7
17-18	0.7
19-22	1.4
23-30	0.7
31-34	0.7
35-36	0.7
37-40	1.4
41-48	0.7
49-52	0.7
53-54	0.7
55-58	2.1
59-66	0.7
67-70	0.7
71-72	0.7

Table 12: Statistic data of the loads of example 4 (kips)

Node number and load direction	Mean	Variation coefficient
17-z	–4900.9	0.2
18-z	–4900.9	0.2
19-z	–4900.9	0.2
20-z	–4900.9	0.2

Table 13: Comparison of the proposed strategies for example 4

	St. 1	St. 1,2	St. 1,3	All
Calculation time (sec)	33.18	21.42	31.40	19.41
Number of dominant failure paths	69	69	69	69
Number of reliability analyses	631	631	579	579

Considering the results presented in Table 12, the calculation time is reduced by using each of the proposed strategy. In this example, without using the first strategy, determination of the dominant failure paths and the bounds of the system failure probability consumes hours. Thus decreasing the calculation time to 33.18 seconds is quite considerable. Also, similar to the third example, decreasing the dominant failure paths from millions to 69 is quite remarkable. Consequently this strategy is utilized in all the cases along with the others strategies.

Using the developed force method has resulted to saving time about 35 percents, without affecting the number of the dominant failure paths and the number of the analyses. The number of time for calling the reliability subroutine is decreased from 631 to 579 by using the third strategy i.e. the artificial intelligence.

The lower and upper bounds of the system failure probability are evaluated as 3.08×10^{-8} and as 1.284×10^{-6}, respectively.

Example 5. Consider a statically indeterminate 120-bar dome truss as shown in Figure 8. All the dimensions are shown in centimeters in this figure. The cross section areas of the groups are presented in Table 14. The yield stress and the applied loads are considered as uncorrelated normal random variables. All the unsupported joints are subjected to vertical loads. The mean value of these loads are taken as −13.46 kips (−60 kN) at the highest node, −6.744 kips (–30kN) at nodes of elevation 585 cm and −2.248 kips (−10 kN) at the rest of the unsupported nodes. The coefficient variation of all the loads is considered as 0.1. The mean value and the coefficient of variation of the member strength are 2760kg/cm^2 (39.2563 ksi or 270.662MPa), and 0.03, respectively. Behavior of the material in tension and compression are considered identical. Results of the reliability analysis of this dome truss considering $\eta = 2$ and using the proposed strategies are presented in Table 15.

Table 14: Group cross-section area of truss in example 5

Group	Area (cm^2)
1	2.20
2	5.40
3	2.20
4	4.80
5	4.00
6	4.60
7	3.00

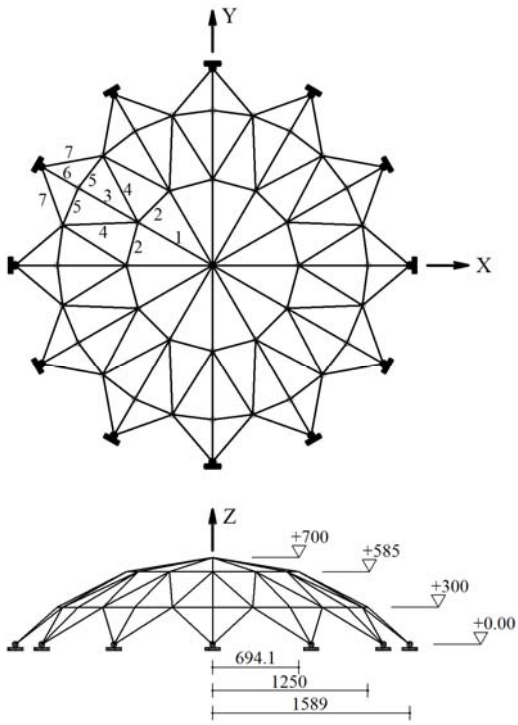

Figure 7. Statically indeterminate 120-bar dome truss of example 5

Table 15: Comparison of the proposed strategies for example 5

	St. 1	**St. 1,2**	**St. 1,3**	**All**
Calculation time (sec)	2447.91	125.34	2266.05	116.03
Number of dominant failure paths	480	480	480	480
Number of reliability analyses	2473	2473	2347	2347

Similar to the previous examples, several hours should be consumed in order to form millions of failure paths and determine the structural failure probability, whereas by using the first strategy these numbers decrease to only 2473 and 480 seconds, respectively. Also, the calculation time is reduced by using each of the proposed strategy as indicated in Table 15.

The dome structure, under investigation, has 111 active degree of freedom and only 9 degree of static indeterminacy. Therefore, the calculation time is reduced about 95 percent by using the developed force method (second strategy) in place of the traditional stiffness method, while the outcomes are absolutely identical, as predicted in the mentioned table.

Number of the required reliability analyses is reduced from 2473 to 2347 by using the third strategy, since many failure paths are immediately identified by the intelligent agent without any extra stability control.

The upper bounds of the system failure probability are evaluated as 2.0794×10^{-6} which is considerably less than the sum of the component failure probabilities i.e. 3.296×10^{-4}.

6. CONCLUSIONS

In this paper, three strategies are presented to speed up the determination of the dominant failure paths and the structural system failure probability. The second strategy is useful for truss structures and the first one is the most effective for estimating the value of the upper bound for the system failure probability.

In the first strategy, the occurrence probability of a path is estimated as the minimum joint probability of each member failure in the path and failure event of the first member. Then other paths which are branched from this member (which has the minimum joint probability to first one) are discarded, since these paths are somehow correlated and all are subset of the first path, and producing them adds nothing to the total failure space. Avoiding producing such paths has two advantages. Firstly, many of the correlated paths do not added to the upper bound of the system failure probability and more accurate result is obtained. Secondly, a large amount of computational time will obviously be saved. Using this strategy results in a narrower bounds as well as saving time about 50 to 100 percents computer time.

The second strategy corresponds to the implementation of a fast method to analyze trusses of different topology, by improving the force method and its use in the reliability analysis. Truss analysis by the standard stiffness method requires the solution of equations as many as the active degrees of freedom for the nodes, while in the force method, the number of equations to be solved is the same as the degree of static indeterminacy. In most of the practical cases the number of the static indeterminacy is less than the active degrees of freedom. Another advantage of this method is that the member forces are obtained immediately. Considering the number of required analyses to determination of the dominant failure paths, using this method can save a considerable amount of time about 30 to 95 percent.

In this study, as the third strategy the artificial intelligent agent is used to save and identify repeated failure paths and avoid extra controls. This resulted in saving time about 5 to 10 percents.

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