ASIAN JOURNAL OF CIVIL ENGINEERING (BUILDING AND HOUSING) VOL. 12, NO. 2 (2011)
PAGES 245-254

Technical Note

DYNAMIC STABILITY OF THIN-WALLED MEMBER WITH VARIABLE STIFFNESS CONSIDERING DAMPING

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ABSTRACT

The dynamic stability is studied for thin-walled structural elements with variable stiffness subjected to periodically alternating axial force in this paper. Here, the variation stiffness means that it changes with periodically alternating axial force as for nonlinear geometry stiffness matrix of thin-walled member. Damping is considered and the governing equations are expressed in terms of a system of two second-order differential equations of the *Mathieu* type, with periodic coefficients. *MATLAB* package is used to determine the stability boundary. Numerical example is presented for the dynamic stability boundary of a simply supported beam with I-shaped cross section. Comparison is made with finite element analysis. Considered damping, some conclusions are drawn out: Excited zone of thin-walled member is continuous, the dynamic instability is highly dominant in the first region while the second and third instability regions are of much less practical importance; The larger the ratio of damp, the less the dynamic instability region; The larger the ratio of damp, the more time dependent components of the load wanted, absorption of damping is commonly of no effect to prevent parametrically excited vibration from dynamic instability; Parametrically excited vibration considering damping is much more different from damped forced vibration in nature.

Keywords: Dynamic stability; variation stiffness; thin-walled member; finite element method; parametrically excited vibration

1. INTRODUCTION

From viewpoints of engineering, all thin-walled members should keep steady first, that is, keep balance and stability under disturbing forces, based on it, vibration characteristics can be carried out later. Dynamic stability is one of the three criteria to dynamic design of structures [1]. Therefore, researches on dynamic stability of thin-walled member is

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becoming more prevalent nowadays. Although history of research on dynamic stability of thin-walled member, e.g., on parametrically excited vibration is not long, many researchers devote themselves to the area [2-4]. But a few papers dealt to the parametrically excited vibration with damping. Damp ratio value ties up dynamic stability zone. Only if damp ratio is less than a certain point (about 0.25), i.e., time dependent components of the load (excited parameter) is larger than a certain value, it is possible that the variation stiffness thin-walled member loses its stability. In fact, as for steel structure, damp ratio is about 0.005~0.05 (less than 0.25), and for concrete structure, damp ratio is about 0.05 (less than 0.25). Therefore, dynamic instability of thin-walled member is general, popular and universal, damping should be considered here.

MATLAB programs are written to work out graph of dynamic instability regions. Two non-dimensional parameters are introduced: one is the proportion of dynamic load to what fundamental static buckling load minus static load leaves and another is the proportion of load frequency to fundamental natural frequency. Therefore, the length, size of thin-walled members as well as boundary conditions have no influence upon boundary of dynamic instability regions. The programs are also valid for variable cross section thin-walled members, even if it is not revised and enlarged. What is more, fundamental static buckling load, fundamental natural frequency and frequency under axial force are obtained once done and for all. If different model of static buckling and vibrating are to be considered, changing finite element is not a tough issue. Finite element method (FEM) has not been noted in any works on dynamic stability of variation stiffness thin-walled member yet. The paper intends to be of some help for engineers in those areas of dynamic analysis and design.

2. BASIC ASSUMPTIONS

The usual assumptions in the field of strength of materials are made, i.e., Hooke’s law holds and plane sections remain plane. As in the case of the applied theory of vibrations, the influences of longitudinal inertia forces and the inertia forces associated with the rotation of the cross sections of the member with respect to its own principal axes are not included.

3. PHYSICAL SYSTEMS

If a thin-walled member is subjected to a periodical longitudinal load, and if the amplitude of the load is less than that of the static buckling value, in general, the member experiences only longitudinal vibrations. However, it can be shown that for certain relationships between the disturbing frequency θ and the natural frequency of transverse vibration ω, thin-walled member becomes dynamically unstable and transverse vibrations occur, the amplitude of these vibrations rapidly increases to large values.

A thin-walled member subjected to periodically alternating axial force is essentially variation stiffness. Linear stiffness matrix of thin-walled member remains constant, while nonlinear geometry stiffness matrix changes with periodically alternating axial force. So, problem discussed is essentially dynamic stability of variation stiffness thin-walled member.

4. MATHEMATICAL MODEL

With damping regarded, thin-walled member with variation stiffness can be represented by an assembly of finite elements connected together at the nodes.The matrix equation for discrete system axially loaded is

$$Mx'' + Cx' + Kx = 0 \tag{1}$$

where M is global mass matrix, C is damping matrix and K is global stiffness matrix.

For thin-walled member subjected to a periodic longitudinal force $P = P_o + P_t Cos\theta\, t$, where θ is the disturbing frequency, the static and time dependent components of the load P_o and P_t can be represented as a fraction of the fundamental static buckling load P^*. Hence, putting $P = \alpha P^* + \beta P^* Cos\theta\, t$, with α and β as percentages of the static buckling load P^*.

A periodic longitudinal force P is used to modify nonlinear geometry stiffness matrix of thin-walled member in Equation(1), thus oscillation equation of thin-walled member with variation stiffness is obtained:

$$Mx'' + Cx' + \left\{K_e - \left(\alpha P^* + \beta P^* Cos\theta t\right)K_G\right\}x = 0 \tag{2}$$

Equation(2) is essentially a second-order differential equation with periodic coefficients, where K_e is linear stiffness matrix which reflects strain energy and K_G is nonlinear geometry stiffness matrix which reflects the influence of P_o and P_t.

I representing the unit matrix, equation (2) may be written again as:

$$x'' + M^{-1}Cx' + M^{-1}\left\{K_e - \left(\alpha P^* + \beta P^* Cos\theta t\right)K_G\right\}x = 0$$

$$x'' + M^{-1}Cx' + M^{-1}\left\{K_e - \alpha P^* K_G\right\}\left\{I - \left\{K_e - \alpha P^* K_G\right\}^{-1}\beta P^* Cos\theta t K_G\right\}x = 0 \tag{3}$$

where

$$M^{-1}C = M^{-1}(\alpha M + \beta K) = \left(\alpha I + \beta M^{-1}K\right) = 2\varepsilon \tag{4}$$

$$M^{-1}\left\{K_e - \alpha P^* K_G\right\} = \Omega^2 \tag{5}$$

$$\frac{1}{2} \times \left\{K_e - \alpha P^* K_G\right\}^{-1}\beta P^* K_G = \left\{2K_e - 2\alpha P^* K_G\right\}^{-1}\beta P^* K_G = \mu \tag{6}$$

The above equation (3) becomes a second-order differential equation with periodic coefficients of the Mathieu type.

$$x'' + 2\varepsilon x' + \Omega^2\left(I - 2\mu Cos\theta t\right)x = 0 \tag{7}$$

It can be written as another form of Mathieu type:

$$x'' + \left(\Omega^2 - \varepsilon^2\right)\left(I - 2\frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2}\mathrm{Cos}\theta t\right)x = 0 \tag{8}$$

Mathieu equation is called to be periodic in the sense that it satisfies equation (8) for every positive $T = \frac{2\pi}{\theta}$ $2T = \frac{4\pi}{\theta}$.

The periodic solution with a period 2T in the form is sough

$$x_i(t) = \sum_{k=1,3,5}^{\infty}\left(a_k \sin\frac{k\theta t}{2} + b_k \cos\frac{k\theta t}{2}\right) \text{ s} \tag{9}$$

Letting the coefficients in the congeneric terms of $\sin\frac{k\theta t}{2}$ and $\cos\frac{k\theta t}{2}$ are equal respectively, and substituting the series (9) into Eq.(8) leads to the following system of linear homogeneous algebraic equation in terms of a_k and b_k

$$\begin{cases} \left(I + \frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2} - \left[\frac{\theta^2}{4\left(\Omega^2 - \varepsilon^2\right)}\right]\right)a_1 - \frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2}a_3 = 0 \\ \left(I - \left[\frac{k^2\theta^2}{4\left(\Omega^2 - \varepsilon^2\right)}\right]\right)a_k - \frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2}\left(a_{k-2} + a_{k+2}\right) = 0 \\ \left(I - \frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2} - \left[\frac{\theta^2}{4\left(\Omega^2 - \varepsilon^2\right)}\right]\right)b_1 - \frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2}b_3 = 0 \\ \left(I - \left[\frac{k^2\theta^2}{4\left(\Omega^2 - \varepsilon^2\right)}\right]\right)b_k - \frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2}\left(b_{k-2} + b_{k+2}\right) = 0 \end{cases} \quad (k=3, 5, 7, \ldots\ldots)$$

The necessary condition for the existence of the periodic solution of Eq. (9) is that the determinants of the homogeneous systems obtained be equal to zero.Considering the two conditions under the $\pm$ sign, we obtain

$$\begin{vmatrix} I \pm \frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2} - \left[\frac{\theta^2}{4\left(\Omega^2 - \varepsilon^2\right)}\right] & -\frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2} & 0 & \cdots \\ -\frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2} & I - \left[\frac{9\theta^2}{4\left(\Omega^2 - \varepsilon^2\right)}\right] & -\frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2} & \cdots \\ 0 & -\frac{\mu\Omega^2}{\Omega^2 - \varepsilon^2} & I - \left[\frac{25\theta^2}{4\left(\Omega^2 - \varepsilon^2\right)}\right] & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix} = 0 \tag{10}$$

This equation relating the frequencies of external loading with the natural frequency of the member and the magnitude of the external force makes it possible to find regions of instability that are bounded by the periodic solutions with a period 2T.

To determine the regions of instability bounded by the periodic solutions with a period T, we proceed in an analogous manner. By substituting the series

$$x_i(t) = b_{i0} + \sum_{k=2,4,6}^{\infty}\left(a_{ik}\sin\frac{k\theta t}{2} + b_{ik}\cos\frac{k\theta t}{2}\right) \tag{11}$$

into Eq. (9), the following systems of algebraic equations are given:

$$\begin{cases} \left(I - \left[\frac{\theta^2}{(\Omega^2-\varepsilon^2)}\right]\right)a_2 - \frac{\mu\Omega^2}{\Omega^2-\varepsilon^2}a_4 = 0 \\ \left(I - \left[\frac{k^2\theta^2}{4(\Omega^2-\varepsilon^2)}\right]\right)a_k - \frac{\mu\Omega^2}{\Omega^2-\varepsilon^2}(a_{k-2} + a_{k+2}) = 0 \\ b_0 I - \frac{\mu\Omega^2}{\Omega^2-[\varepsilon^2]}b_2 = 0 \qquad (k=4, 6, 8,\ldots\ldots) \\ \left(I - \left[\frac{\theta^2}{(\Omega^2-\varepsilon^2)}\right]\right)b^2 - \frac{\mu\Omega^2}{\Omega^2-\varepsilon^2}(2b_0 + b_4) = 0 \\ \left(I - \left[\frac{k^2\theta^2}{4(\Omega^2-\varepsilon^2)}\right]\right)b_k - \frac{\mu\Omega^2}{\Omega^2-\varepsilon^2}(b_{k-2} + b_{k+2}) = 0 \end{cases} \tag{12}$$

Let the determinants of the homogeneous system to zero, we arrive at the following equations for the critical frequencies:

$$\begin{vmatrix} \left(I - \left[\frac{\theta^2}{(\Omega^2-\varepsilon^2)}\right]\right) & -\frac{\mu\Omega^2}{\Omega^2-\varepsilon^2} & 0 & \cdots \\ -\frac{\mu\Omega^2}{\Omega^2-\varepsilon^2} & \left(I - \left[\frac{\theta^2}{4(\Omega^2-\varepsilon^2)}\right]\right) & -\frac{\mu\Omega^2}{\Omega^2-\varepsilon^2} & \cdots \\ 0 & -\frac{\mu\Omega^2}{\Omega^2-\varepsilon^2} & \left(I - \left[\frac{9\theta^2}{4(\Omega^2-\varepsilon^2)}\right]\right) & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix} = 0 \tag{13}$$

and

$$\begin{vmatrix} I & -\dfrac{\mu\Omega^2}{\Omega^2-\varepsilon^2} & 0 & 0 & \cdots \\ -\dfrac{2\mu\Omega^2}{\Omega^2-\varepsilon^2} & I-\left[\dfrac{\theta^2}{(\Omega^2-\varepsilon^2)}\right] & -\dfrac{\mu\Omega^2}{\Omega^2-\varepsilon^2} & 0 & \cdots \\ 0 & -\dfrac{\mu\Omega^2}{\Omega^2-\varepsilon^2} & I-\left[\dfrac{\theta^2}{4(\Omega^2-\varepsilon^2)}\right] & -\dfrac{\mu\Omega^2}{\Omega^2-[\varepsilon^2]} & \cdots \\ 0 & 0 & -\dfrac{\mu\Omega^2}{\Omega^2-\varepsilon^2} & I-\left[\dfrac{9\theta^2}{4(\Omega^2-\varepsilon^2)}\right] & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix} = 0 \qquad (14)$$

for determining the regions of instability bounded by the periodic solutions with a period T of the thin-walled member.

5. DYNAMIC STABILITY OF THE MEMBER WITH DAMPING

5.1 Numerical examples

An open I-shaped cross-section of thin-walled member with both ends simply supported is shown in Figure 1.

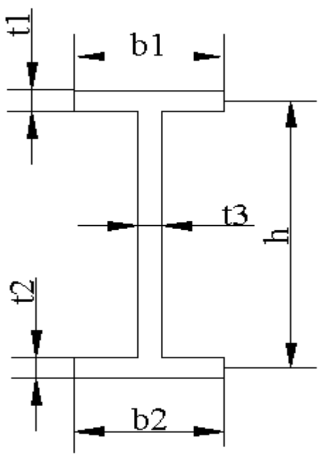

Figure 1. I-shaped

The following properties were taken for numerical computations: length $L = 2m$, cross-sectional dimension:

$t1 = 11.05mm$, $b1 = 203mm$, $t2 = 7.24mm$, $b2 = 203mm$, $h = 191.95mm$.

Young's modulus: $E = 2.0e11 N/m^2$; Shear modulus: $G = 0.8e11 N/mm^2$;

Poission's ratio: $\gamma = 0.25$; Rayleigh damping: $[C] = \overline{\alpha}[M] + \overline{\beta}[K]$.

Assuming damping ratio $\xi_i = \xi_j = \xi$, then there is $\overline{\alpha} = \dfrac{2w_i w_j}{w_i + w_j}\xi$, $\overline{\beta} = \dfrac{2\xi}{w_i + w_j}$.

Dynamic stability of open-section thin-walled member in the conditions of $\xi = 0.01$ and $\xi = 0.05$, while $\alpha = 0$ is calculated,respectively.

We are much obliged to S. Kiipornchai and S. L Chan for their nonlinear thin-walled open section finite element [5]. The MATLAB program developed here adopts their element but eliminates axial deformation, that is, we take into account of bending coupled with twist and warping deformation of thin-walled member. The results are shown in Table.1.

Table 1: Boundary of dynamic stability of variation stiffness thin-walled member

Serial number of Dynamic instability regions	$\frac{\theta}{2\Omega}$	ξ	β					
			0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
The first upper boundary	Ref.[2]	0.01	——	1.0477	1.0949	1.1398	1.1829	1.2245
	This paper		——	1.0430	1.0863	1.1275	1.1669	1.2045
	Ref.[2]	0.05	——	——	1.0808	1.1304	1.1757	1.2186
	This paper		——	——	1.0785	1.1211	1.1617	1.2007
The first lower boundary	Ref.[2]	0.01	——	0.9496	0.8949	0.8369	0.7748	0.7072
	This paper		——	0.9489	0.8973	0.8418	0.7815	0.7154
	Ref.[2]	0.05	——	——	0.9065	0.8439	0.7795	0.7107
	This paper		——	——	0.8741	0.8210	0.7629	0.6986
The second upper boundary	This paper	0.01	——	——	0.4987	0.4995	0.4997	0.499800.4961
	This paper	0.05	——	——	——	——	0.4927	
The second lower boundary	This paper	0.01	——	——	0.4809	0.4533	0.412600.4210	0.3538
	This paper	0.05	——	——	——	——		0.3590
The third upper boundary	This paper	0.01	——	——	——	0.3084	0.2741	0.1997
	This paper	0.05	——	——	——	——	——	0.2921
The third lower boundary	This paper	0.01	——	——	——	0.3190	0.3113	0.3015
	This paper	0.05	——	——	——	——	——	0.2133

5.2 Discussions

Instability of thin-walled member is a parametric excitation problem.A boundary of the instability regions for variation stiffness thin-walled member is shown in Table 1. Compared to results of Ref. [2], the results of MATLAB program developed in this paper are with more precision and efficiency. And, it reveals some interesting features. First, regions of stability are larger than regions of instability. Second, judging from the magnitude of the relative width parameter, the first region of instability is large and reduces rapidly for the second and third regions, which indicates that the instability is highly dominant in the first region. Therefore, the first region is always called the principal region and is generally most important while the second and third instability regions are of much less practical importance.

Using FEM, the static and time dependent components of the load for thin-walled member of arbitrary section can be determined out when dynamic instability of thin-walled member occurs. It can be seen that the results of Table 1 will not change with the size, length and restriction of variation stiffness thin-walled member due to adoption of ratio method, which shows the commonness of dynamic stability of variation stiffness thin-walled member.

Only if damp ratio is less than a certain point, i.e., time dependent component of the load (excited parameter) is larger than a certain value, variation stiffness thin-walled member loses its stability. Compared to the results between $\xi = 0.01$ and $\xi = 0.05$ in Table 1 indicates that the larger the damp ratio is, the more time dependent components of the load wanted. Therefore, the effect of damping on dynamic instability of thin-walled member with varying stiffness is general and should not be neglected. So dynamic instability of variation stiffness thin-walled member is general, popular and universal.

From Table 1 it can be seen that, as dynamic load factor β increases beyond the limit point, the eigenvalue solution of the instability region boundary becomes imaginary which implies that the periodic solution of the Mathieu equation does not exist in that region. The stability behavior in that region is definitely unstable, and this may be due to the large lateral displacement of the thin-walled member due to increasing values of dynamic load factors. Indeed, the thin-walled member loses its stability because nonlinear geometry stiffness matrix decreases to a lowest point with dynamic load on it. At $\beta = 0.0$, the dynamic load takes no action on the thin-walled member which should be understood as a critical state.

From Table 1 it also can be seen that both damp ratio value and parametrically excited vibration coefficient are small. And, parametrically excited vibration coefficient seems to increase in proportion to damp ratio. The first dynamic instability region takes up most of parametric plan, occurring in all probability and causing a lot of harm. So it's more difficult to inspire the first dynamic instability than the third dynamic instability where damp exists. The larger the damp ratio value, the less dynamic instability region.

Both parametrically excited vibrations and forced vibrations could lead to instability phenomena, and they are similar in appearance. But they are not the same dynamic response. Forced vibration occurs when disturbing frequency is close to the natural frequency of thin-walled member while parametrically excited vibrations take place at many cases. It is pointed out that to avoid dynamic instability of thin-walled member is more difficult than prevent it from sympathetic vibration.

6. SUMMARY AND CONCLUSIONS

The dynamic stability of variation stiffness thin-walled member subjected to periodically alternating axial force is analyzed considering damping in this paper.Here, the variation stiffness means that it changes with periodically alternating axial force as for nonlinear geometry stiffness matrix of thin-walled member.Dynamic stability of variation stiffness thin-walled member with damping can be transformed into a system of second-order differential equation of Mathieu type with periodic coefficients.Using MATLAB package, a computer program is developed to calculate regions of dynamic instability corresponding to bending vibration, torsion and warping coupling vibration. It is the same as neglecting damping, as for the same dynamic instability mode, the larger the load swing, the wider the dynamic instability zone. Considered damping, some conclusions are drawn out: Excited zone of thin-walled member is continuous, the dynamic instability is highly dominant in the first region while the second and third instability regions are of much less practical importance ; The larger the damp ratio value, the less the dynamic instability region; The larger the damp ratio, the more time dependent components of the load wanted, absorption of damping is commonly of no effect to prevent parametrically excited vibration from dynamic instability; Parametrically excited vibration considering damping is much more different from damped forced vibration in nature .

MATLAB programs are written to work out the boundary of dynamic instability regions. Two non-dimensional parameters are introduced: one is the proportion of dynamic load to what fundamental static buckling load minus static load leaves and another is the proportion of load frequency to fundamental natural frequency.Therefore, the length, size of thin-walled members as well as boundary conditions have no influence upon graph of dynamic instability regions.Need not revised and enlarged, the programs are also valid for variable cross section thin-walled members. What's more, fundamental static buckling load, fundamental natural frequency and frequency under axial force are obtained once done and for all. The finite element method has not been noted in works on dynamic stability of variation stiffness thin-walled member yet.

The paper intends to be of some help for engineers in those areas of dynamic analysis and design.

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