

## AN IMPROVED HPSACO FOR ENGINEERING OPTIMUM DESIGN PROBLEMS

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### ABSTRACT

An improved heuristic particle swarm ant colony optimization (HPSACO) is presented to solve engineering optimization problems. This new algorithm follows the HPSACO levels; however some modifications are performed in global and local searching levels to improve its performance. Here, the properties of the big bang–big crunch algorithm are added to PSO and ACO in global and local searching levels, respectively and these changes improve precision of the solutions and the reliability of the algorithm. Benchmark engineering optimization problems are used to illustrate the reliability of the proposed algorithm.

**Keywords:** Optimization methods; particle swarm ant colony optimization; big bang–big crunch algorithm; engineering design problems

### 1. INTRODUCTION

Heuristic particle swarm ant colony optimization (HPSACO) is introduced by Kaveh and Talatahari [1] to enhance the searching ability of the particle swarm optimization (PSO). Due to some disadvantages of the standard PSO algorithm such as the high computational cost of the slow convergence rate [2] and existing difficulties in controlling the balance between exploration and exploitation [3], HPSACO as a hybrid algorithm is developed. This algorithm is utilized for different optimization problems [4-6].

In the HPSACO, particle swarm optimization with passive congregation (PSOPC) is acted as the main optimizer. In this algorithm, each particle iteratively moves across the search space based on the position of the best fitness historically achieved by the particle itself (local best), by the best among the neighbors of the particle (global best) and by the position of a particle selected randomly. It is possible to improve the location of the global best by employing a local optimizer. Since the global best affects on the moving of all the agents in one hand, and it is considered as the final reported solution on the other hand, adding a local optimizer can improve the performance of the algorithm. HPSACO utilizes

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the idea of ant colony approach (ACO) as a local optimizer. To handle the boundary constraints, HPSACO also employs the advanced algorithm based on the local best matrix and the harmony search (HS) methodology.

Here, an improved HPSACO algorithm is presented. In the improved algorithm, the PSO and ACO levels are improved by using the principles of the big bang–big crunch (BB–BC) optimization [7]. BB–BC utilizes the center of mass to direct the searching process and here this term is added to HPSACO. The resulted method is then tested by some benchmark engineering examples to estimate its potential for solving optimization problems.

## 2. ENGINEERING OPTIMIZATION PROBLEMS

In general, engineering design problems can be formulated as constrained optimization problems which can be described as follows:

$$\begin{aligned} &\text{find } \mathbf{X} \text{ to minimize } f_{\text{cost}}(\mathbf{X}) \\ &\text{subject to :} \\ &\quad g_j(\mathbf{X}) \leq 0 \quad j = 1, 2, \dots, n_g \\ &\quad h_k(\mathbf{X}) = 0 \quad k = 1, 2, \dots, n_h \\ &\quad x_{i,\min} \leq x_i \leq x_{i,\max} \quad i = 1, 2, \dots, d \end{aligned} \tag{1}$$

where  $\mathbf{X} = [x_1, x_2, \dots, x_d]^T$  denotes the decision solution vector;  $f_{\text{cost}}$  is a cost function (objective function);  $x_{i,\min}$  and  $x_{i,\max}$  are the minimum and the maximum permissible values for the  $i$ th variable, respectively;  $n_g$  is the number of inequality constraints and  $n_h$  is the number of equality constraints. In common practice, equality constraint  $h_k(\{x\}) = 0$  can be replaced by an inequality constraint  $|h_k(\{x\})| - \varepsilon \leq 0$ , where  $\varepsilon$  is a small tolerant amount.

## 3. IMPROVED HEURISTIC PARTICLE SWARM ANT COLONY OPTIMIZATION

### 3.1 Review to heuristic particle swarm ant colony optimization

The heuristic particle swarm-ant colony optimization (HPSACO) algorithm applies particle swarm optimizer with passive congregation (PSOPC) for global optimization which involves a number of particles initialized randomly in the feasible space [1]. These particles fly through the search space and their positions are updated based on the best positions of individual particles, the best position among all particles in the search space, and the position of a particle selected randomly from the swarm in each iteration, as

$$\mathbf{X}_i^{k+1} = \mathbf{X}_i^k + \mathbf{V}_i^{k+1} \quad (2)$$

$$\mathbf{V}_i^{k+1} = \omega \mathbf{V}_i^k + c_1 r_1 (\mathbf{P}_i^k - \mathbf{X}_i^k) + c_2 r_2 (\mathbf{P}_g^k - \mathbf{X}_i^k) + c_3 r_3 (\mathbf{R}_i^k - \mathbf{X}_i^k) \quad (3)$$

where  $\mathbf{X}_i^k$  and  $\mathbf{V}_i^k$  are the position and the velocity of particle  $i$  in the iteration  $k$ ;  $\omega$  is an inertia weight to control the influence of the previous velocity;  $r_1, r_2$  and  $r_3$  are three random numbers uniformly distributed in the range of (0,1);  $c_1$  and  $c_2$  are two acceleration constants;  $c_3$  is the passive congregation coefficient;  $\mathbf{P}_i^k$  is the best position of the  $i$ th particle up to iteration  $k$ ;  $\mathbf{P}_g^k$  is the best position among all particles in the swarm up to iteration  $k$ ; and  $\mathbf{R}_i$  is a particle selected randomly from the swarm.

In HPSACO, ant colony strategy (ACO) works as a local search where  $M$  ants (equal to the number of particles in PSOPC level) generate solutions around  $\mathbf{P}_g^k$  as follows

$$\mathbf{Z}_i^k = N(\mathbf{P}_g^k, \sigma) \quad (4)$$

where,  $\mathbf{Z}_i^k$  is the solution constructed by ant  $i$  in the stage  $k$ ;  $N(\mathbf{P}_g^k, \sigma)$  denotes a random vector normally distributed with mean value  $\mathbf{P}_g^k$  and variance  $\sigma$ .

### 3.2 Improved heuristic particle swarm ant colony optimization

In this section, an improved HPSACO is provided by using the positive characters of the BB-BC algorithm. This new algorithm follows the HPSACO levels and it has global searching, local searching and location controlling levels similar to the HPSACO. In global searching level of the new algorithm, instead of PSOPC, a hybrid PSO and the BB-BC methodology is utilized. This method is obtained by modifying the velocity formulation of the PSO algorithm by adding the term of the center of mass from the BB-BC algorithm instead of the location of a particle selected randomly from the swarm, which is formulated as follows

$$\mathbf{V}_i^{k+1} = \omega \mathbf{V}_i^k + c_1 r_1 (\mathbf{P}_i^k - \mathbf{X}_i^k) + c_2 r_2 (\mathbf{P}_g^k - \mathbf{X}_i^k) + c_3 r_3 (\mathbf{X}_c^k - \mathbf{X}_i^k) \quad (5)$$

The local searching level of the new algorithm employs the ACO stage of HPSACO and the big crunch level of the BB-BC. Therefore, in addition to  $\mathbf{P}_g^k$ , other points such as  $\mathbf{P}_i^k$  and  $\mathbf{X}_c^k$  are used to generate a new solutions on local level as

$$\mathbf{Z}_i^k = N(\alpha_2 \mathbf{X}_c^k + (1 - \alpha_2)(\alpha_3 \mathbf{P}_g^k + (1 - \alpha_3) \mathbf{P}_i^k), \sigma) \quad (6)$$

Location controller is similar to HS strategy of HPSACO.

## 4. NUMERICAL EXAMPLES

Some benchmark engineering optimization problems are presented in this section to evaluate the efficiency of the new algorithm. These examples have been previously solved using a variety of other techniques and contain a welded beam design problem, a speed reducer design problem, and a four-storey, two-bay frame design problem. For each example, 30 independent runs are carried out using the HPSACO and compared to other algorithms.

A population of 50 individuals consisting of 25 particles and 25 ants are used for this algorithm; the value of constants  $c_1$  and  $c_2$  are set to 0.8 and  $c_3$  is taken as 0.6 as defined for HPSACO [5]. The value of inertia weight decreases linearly from 0.9 in the first iteration to 0.4 in the last iteration.

### 4.1 A welded beam design problem

The welded beam structure, shown in Figure 1, is a practical design problem that often has been used as a benchmark problem for testing different optimization methods. The objective is to find the minimum fabricating cost of the welded beam subject to constraints on shear stress ( $\tau$ ), bending stress ( $\sigma$ ), buckling load ( $P_c$ ), end deflection ( $\delta$ ), and side constraint.

There are four design variables, namely  $h(=x_1)$ ,  $l(=x_2)$ ,  $t(=x_3)$  and  $b(=x_4)$ . The detailed information about the constraints and objective function is presented in [5].

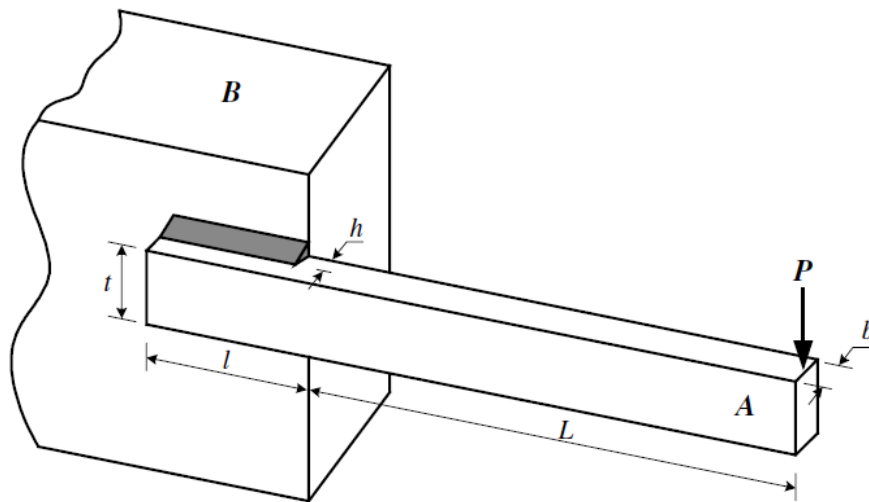


Figure 1. Welded beam structure

Coello and Montes [8] solved this problem using GA-based methods. He and Wang [9] using a PSO-based algorithm, and Montes and Coello [10] employing evolution strategies solved this problem. In addition, Kaveh and Talatahari utilized HPSACO [5], charged system search (CSS) [11], and imperialist competitive algorithm (ICA) [12] to solve this problem. Table 1 compares the result of the new algorithm with those obtained by other methods. This table also summarizes the statistical simulation results of these algorithms. Comparing

to other methods, it can be seen that, the standard deviation of the results by the new algorithm is very small and this means that the reliability of the proposed method is very high. In addition, the best result among these methods belongs to the new method.

Table 1: Optimum results for the welded beam design

Methods	$x_1 (h)$	$x_2 (l)$	$x_3 (t)$	$x_4 (b)$	Best result	Mean of results	Worst result	Std. Dev.
Coello & Montes [8]	0.205986	3.471328	9.020224	0.206480	1.728226	1.792654	1.993408	0.074713
He & Wang [9]	0.202369	3.544214	9.048210	0.205723	1.728024	1.748831	1.782143	0.012926
Montes & Coello [10]	0.199742	3.612060	9.037500	0.206082	1.737300	1.813290	1.994651	0.070500
Kaveh & Talatahari [12]	0.205703	3.47106	9.036654	0.205731	1.724906	1.742214	1.793435	0.017831
Kaveh & Talatahari [5]	0.205729	3.469875	9.036805	0.205765	1.724849	1.727564	1.759522	0.008254
Kaveh & Talatahari [11]	0.205820	3.468109	9.038024	0.205723	1.724866	1.739654	1.759479	0.008064
Present work	0.205803	3.468918	9.036615	0.205731	1.724762	1.726932	1.756281	0.006685

#### 4.2 Speed reducer design problem

The design of the speed reducer [13] shown in Figure 2 is considered with the face width  $x_1$ , module of teeth  $x_2$ , number of teeth on pinion  $x_3$ , length of the first shaft between bearings  $x_4$ , length of the second shaft between bearings  $x_5$ , diameter of the first shaft  $x_6$ , and diameter of the second shaft  $x_7$  (all variables are continuous except  $x_3$  that is integer). The weight of the speed reducer is to be minimized subject to constraints on bending stress of the gear teeth, surface stress, transverse deflections of the shafts and stresses in the shaft. The problem is defined with details in [12].

This example is solved by two constrained particle swarm optimizer algorithms (i.e. COPSO [14], SiC-PSO [15]) and by imperialist competitive algorithm [12]. A total of 30,000 and 24,000 objective function evaluations per run are considered for the COPSO and SiC-PSO algorithms, respectively. However, it is set to 5,000 for both ICA and the new algorithm. As shown in Table 2, even with this small number of function evaluations, the new algorithm as well as ICA could find the best result almost in all the runs. The results show that the new algorithm is a fast algorithm compared to PSO-based methods.

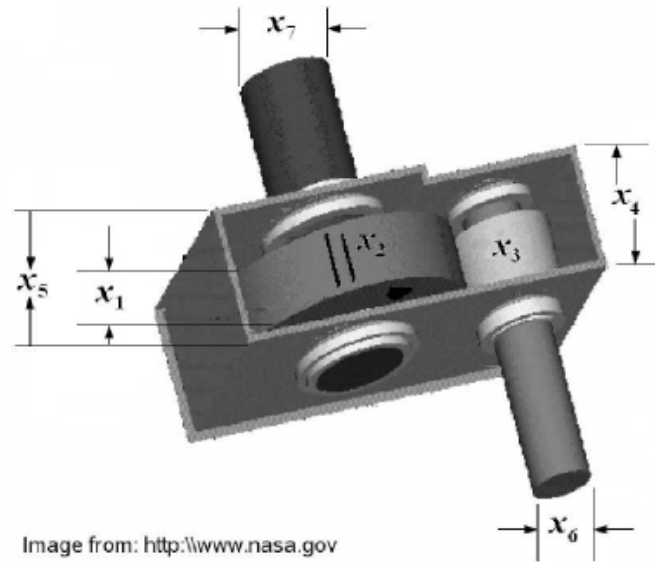


Figure 2. Speed reducer

Table 2: Optimum results for the speed reducer design

Methods	Best result	Mean of results	Std. Dev.
Hernandez et al. [14]	2,996.372	2,996.408	0.0286
Cagnina et al. [15]	2,996.348	2,996.348	0.0000
Kaveh & Talatahari [12]	2,996.348	2,996.348	0.0000
Present work	2,996.348	2,996.348	0.0000

#### 4.3 A four-storey, two-bay frame design problem

As the final example, a four-storey, two-bay frame is selected from [16]. The frame has 15 nodes and 20 elements constructed from I-beam sections and the elements are grouped into five different dimensional sets. The objective is to minimize the weight of the frame through finding the optimum cross-section dimensions  $b$ ,  $h$ ,  $t_w$  and  $t_f$  for each group of elements. Since there are four design variables for each group, 20 sizing variables are considered. The material density is  $7850 \text{ kg/m}^3$  and the elastic modulus is  $210 \text{ kN/mm}^2$ . Ref. [16] summarizes the detailed information of this example. The frame dimensions, configuration, loading, and grouping of the members are shown in Figure 3.

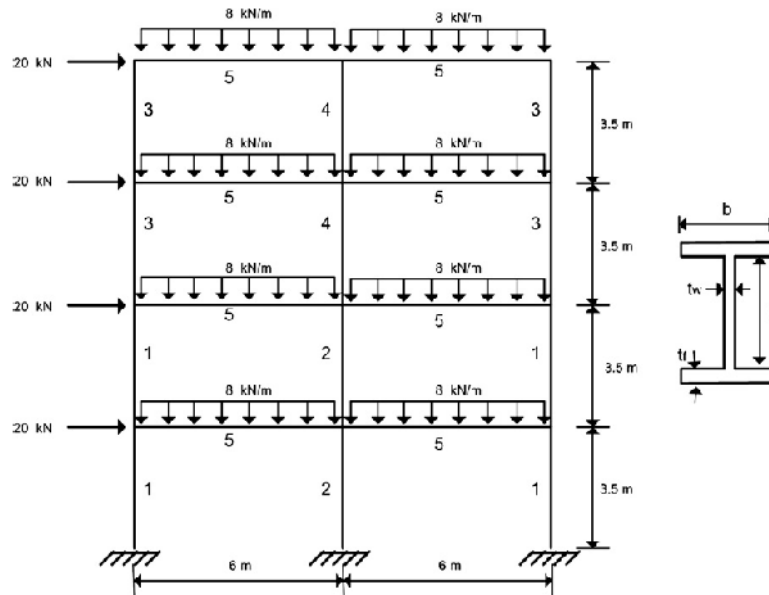


Figure 3. A four-storey, two-bay steel frame

Table 3 provides the best solution vectors and the corresponding weight using the proposed method, and compares the obtained results in this research with the outcomes of the harmony search (HS) [16], a hybrid harmony search algorithm (HSSA) [16], and HPSACO [5]. An optimal structural weight of 3,466.9 kg is achieved by the new algorithm while it was 10.9%, 7.7% and 2.8% lighter than for HS, HSSA and HPSACO, respectively. The optimum result is obtained after approximately 8,500 fitness function evaluations which are less than 41,000 and 10,500 function evaluations for the HSSA and HPSACO.

Table 3: Optimum results for the four-storey, two-bay steel frame

Group no.	1	2	3	4	5
$b$	9.543255	17.00268	10.34771	12.25897	12.08835
$t_f$	1.199895	2.153389	1.074574	1.118963	1.237647
$h$	32.43361	57.69905	35.3637	34.35955	41.18979
$t_w$	0.409722	0.922135	0.4000	0.4000	0.4000
Max. displacement (cm)				1.9998	
Max. stress (MPa)				68.32	
Weight for present work (kg)				3,466.92	
Weight for HPSACO (kg) [5]				3,564.25	
Weight for HS (kg) [16]				3,733.9	
Weight for HSSA (kg) [16]				3,845.2	

## 5. CONCLUDING REMARKS

Heuristic particle swarm-ant colony optimization is known as an advanced hybrid algorithm in the field of engineering design problems. HPSACO utilizes a PSOPC algorithm as a global search, and the idea of the ACO as a local search, and updating the positions of the particles is performed by a pheromone-guided mechanism. Here, an improved version of this algorithm is introduced. The new algorithm utilizes the BB–BC as a helping factor to improve the searching processes. First in the global searching level, the term of the center of the mass is added to PSO to increase the performance of the global algorithm and second in the local searching level, the center of mass point are also utilized in addition to global and local best points.

Comparing the results of the new algorithm with other heuristic algorithms and especially with the HPSACO, indicates that the new algorithm is more reliable and it can find better results with smaller computational costs. Three examples are considered in this paper. For the first one, it is shown that the new algorithm has the smallest standard deviation as well as the best result. This means that the new algorithm is more reliable than GA and PSO algorithms. From the second example, it can be concluded that the proposed algorithm performs faster than PSO-based methods. Comparison between the new algorithm with HS-based and HPSACO algorithms in third example also reveals the superiority of the new algorithm.

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