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OPTIMAL DESIGN OF LAMINATED COMPOSITE STRUCTURES VIA HYBRID CHARGED SYSTEM SEARCH AND PARTICLE SWARM OPTIMIZATION

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ABSTRACT

Composite laminates have many applications as advanced engineering materials, primarily as components in civil engineering structures, aircrafts, power plants, ships, cars, rail vehicles, robots, sports equipment, etc. Due to widespread use of these materials in various fields in this paper the minimum thickness design of laminated composite plates under in-plain loading is explored using a hybrid charged system search algorithm (CSS) and particle swarm optimization (PSO) where ply numbers and fiber orientations are considered as design variables. This optimization method is obtained by adding searching abilities of the PSO algorithm to those of the CSS approach. Static failure criteria are utilized to determine whether the load bearing capacity is exceeded for a configuration generated during the optimization process. In order to check the feasibility of solutions during an optimization procedure, both the Tsai–Wu and the maximum stress safety factors are employed. Numerical results are obtained and presented to evaluate the performance of the proposed algorithm for different loading cases. Compared to other approaches, the algorithm has proven to be quite reliable in performing these designs.

Keywords: Optimal design; laminated composite structures; hybrid algorithm; charged system search; particle swarm optimization

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1. INTRODUCTION

Design of a structural component using composites involves both material and structural design. Unlike conventional materials (e.g., steel), the properties of the composite material can be designed considering the structural aspects. Composite properties (e.g., stiffness, thermal expansion, etc.) can be varied continuously over a broad range of values under the control of the designer. Since 1960 composite materials have become common engineering materials and are designed and manufactured for various applications including automotive components, sporting goods, consumer goods, and have found applications in the marine and oil industries. The growth in composite usage also came about because of increased awareness regarding the product performance and increased competition in the global market for lightweight components [1]. Designing with laminated composites, on the other hand, has become a challenging problem for the designer because of a wide range of parameters that can be varied, and because the complex behavior and multiple failure modes of these structures require sophisticated analysis techniques. Finding an efficient composite structural design that meets requirements of a certain application can be achieved not only by sizing the cross-sectional areas and member thicknesses, but also by global or local tailoring of the material properties through selective use of orientation, number, and stacking sequence of laminae that make up the composite laminate. The possibility of achieving an efficient design that is safe against multiple failure mechanisms, coupled with the difficulty in selecting the values of a large set of design variables makes structural optimization an efficient tool for the design of laminated composite structures [2].

The mentioned potential capabilities of laminated composites have led many researchers to implement different algorithms to produce the most suitable structure for a typical application. Schmit and Farshi [3] first used linear programming (LP) to obtain minimum weight optimum design of composite plates subjected to multiple in-plane loading conditions. Mesquite and Kamat [4] employed nonlinear mixed integer programming (NLMIP) to maximize frequencies of stiffened laminated composite plates subject to frequency separation constraints and upper bound on weight. Hajela and Shih [5] utilized a methodology based on a piecewise linear representation of nonlinear problem to optimize a cantilever composite laminate beam for minimum weight and constraints on strength displacements, and natural frequencies.

These methods use gradient information to search the solution space near an initial starting point. In general, gradient-based methods converge faster and can obtain solutions with higher accuracy compared to stochastic approaches in fulfilling the local search task. However for effective implementation of these methods, the variables and cost function of the generators need to be continuous. Furthermore, a good starting point is vital for these methods to be executed successfully [6].

As an alternative to the conventional mathematical approaches, the meta-heuristic optimization techniques (genetic algorithm, ant colony optimization, particle swarm optimization, tabu search, etc) have been used to obtain global or near global optimum solutions. Due to their capability of exploring and finding promising regions in the search space in an affordable time, these methods are quite suitable for global searches and furthermore alleviate the need for continuous cost functions and variables used for mathematical optimization methods [6]. For the first time, Callahan and Weeks [7] applied genetic algorithm (GA) to demonstrate that GA can be a viable alternative to traditional search procedures in the design of composite laminates. Kogiso et al. [8] used GA with local improvement to optimize a laminated composite plate for buckling load maximization. Many others utilized this method or the modified type to optimize strength-to-weight ratio or other parameters [9–11]. Niranjan et al. [12] optimized stacking sequence of a laminate for buckling response, matrix cracking and strength that were conducted using a heuristic search technique known as tabu search (TS) and compared to the results obtained by GA. Aymerich and Serra [13] explored the potential of ant colony optimization (ACO) meta-heuristic for stacking sequence optimization of composite laminates. Again Aymerich and Serra [14] demonstrated application of the ant colony optimization (ACO) meta-heuristic to the lay-up design of laminated panels for maximization of buckling load with strength constraints. Omkar et al. [15] used VEPSO as a novel, co-evolutionary multi-objective variant of the popular particle swarm optimization algorithm (PSO) to achieve a specified strength with minimizing weight and total cost of the composite component. Erdal and Sonmez [16] maximized buckling load capacity using simulated annealing (SA). Also Akbulut and Sonmez [17] carried out direct simulated annealing (DSA) to minimize thickness of laminated composite plates subject to in-plane loading. Tabakov [18] showed the efficiency of big bang – big crunch optimization (BB-BC) method by an example of the lay-up optimization of multi-layered anisotropic cylinders based on a three-dimensional elasticity solution and proved to be more accurate than GA in such examples.

In this paper a new meta-heuristic optimization method based on charged system search (CSS) and PSO is utilized to achieve the minimum thickness by optimizing the number of laminates using Thai–Wu Failure and maximum principle stress criteria. The variables are ply orientation and ply numbers. The angles are supposed to be continuous and ply thickness is pre-assigned. The CSS utilizes a number of solution candidates which are called charged particles (CPs). Each CP is treated as a charged sphere and it can exert electrical forces on the other agents (CPs) according to the Coulomb and Gauss laws of electrostatics. The resultant force acts on each CP creating acceleration according to the Newton's second law. Finally, utilizing the Newtonian mechanics, the position of each CP is determined at any time based on its previous position, velocity and acceleration in the search space [6].

The remaining of this paper is organized as follows. Review of the hybrid CSS and PSO is briefly presented in Section 2. Structural optimization problem is formulated in Section 3. Design examples are studied in Section 4 and the results of the proposed method are presented. Finally, the conclusion is drawn in Section 5 based on the reported analyses.

2. HYBRID CHARGED SYSTEM SEARCH AND PARTICLE SWARM OPTIMIZATION

2.1 Standard CSS

The Charged System Search (CSS) algorithm is a meta-heuristic based on the Coulomb and Gauss laws from electrical physics and the governing laws of motion from the Newtonian mechanics [19]. This algorithm can be considered as a multi-agent approach, where each agent is a Charged Particle (CP). Each CP is considered as a charged sphere with radius *a*, having a uniform volume charge density and is equal to:

$$q_i = \frac{fit(i) - fitworst}{fitbest - fitworst} \quad i = 1,2,\ldots,N \tag{1}$$

where *fitbest* and *fitworst* are the best and the worst fitness of all the particles; *fit*(*i*) represents the fitness of the agent *i*, and *N* is the total number of CPs.

CPs can impose electric forces on the others, and its magnitude for the CP located inside the sphere is proportional to the separation distance between the CPs, and for a CP located outside the sphere is inversely proportional to the square of the separation distance between the particles. The kind of the forces can be attractive or repelling and it is determined by using ar_{ij}, the kind of force parameter, defined as

$$ar_{ij} = \begin{cases} +1 & \text{w.p. } k_t \\ -1 & \text{w.p. } 1-k_t \end{cases} \tag{2}$$

where ar_{ij} determines the type of the force, with +1 representing the attractive force and −1 denoting the repelling force, and k_t is a parameter to control the effect of the kind of force. Therefore, the resultant force is redefined as:

$$\mathbf{F}_j = q_j \sum_{i,i\neq j} \left(\frac{q_i}{a^3} r_{ij} \cdot i_1 + \frac{q_i}{r_{ij}^2} \cdot i_2 \right) ar_{ij} p_{ij} (\mathbf{X}_i - \mathbf{X}_j) \quad \left\langle \begin{array}{l} j = 1,2,\ldots,N \\ i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a \end{array} \right. \tag{3}$$

where $\mathbf{F}_j$ is the resultant force acting on the *j*th CP; r_{ij} is the separation distance between two charged particles defined as:

$$r_{ij} = \frac{\| \mathbf{X}_i - \mathbf{X}_j \|}{\| (\mathbf{X}_i + \mathbf{X}_j)/2 - \mathbf{X}_{best} \| + \varepsilon} \tag{4}$$

where $\mathbf{X}_i$ and $\mathbf{X}_j$ are the positions of the *i*th and *j*th CPs, respectively; $\mathbf{X}_{best}$ is the position of the best current CP, and ε is a small positive number to avoid singularity. P_{ij} determines the probability of moving each CP toward the others as:

$$p_{ij} = \begin{cases} 1 & \dfrac{fit(i) - fitbest}{fit(j) - fit(i)} > rand \vee fit(j) < fit(i) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

At the movement stage, each CP moves towards its new position under the action of the resultant forces and its previous velocity as

$$\mathbf{X}_{j,new} = rand_{j1} \cdot k_a \cdot \frac{\mathbf{F}_j}{m_j} \cdot \Delta t^2 + rand_{j2} \cdot k_v \cdot \mathbf{V}_{j,old} \cdot \Delta t + \mathbf{X}_{j,old} \tag{6}$$

$$\mathbf{V}_{j,new} = \frac{\mathbf{X}_{j,new} - \mathbf{X}_{j,old}}{\Delta t} \tag{7}$$

where k_a is the acceleration coefficient; k_v is the velocity coefficient to control the influence of the previous velocity; and $rand_{j1}$ and $rand_{j2}$ are two random numbers uniformly distributed in the range (0,1). If each CP moves out of the search space, its position is corrected using the harmony search-based handling approach as described in [20]. In addition, to save the best design, a memory (Charged Memory) is utilized.

2.2 Hybridization of PSO and CSS

A hybrid CSS and PSO algorithm is proposed by Kaveh and Talatahari [21,22]. This algorithm was applied to optimal design of some engineering problems. Also comparing the results with other meta-heuristic methods demonstrates that the proposed approach has a good capability of determining the approximate optimum solutions. The Particle Swarm Optimization (PSO) utilizes a velocity term which is a combination of the previous velocity, $\mathbf{V}^k_i$, the movement in the direction of the local best (i.e. the best visited position by the particle itself), $\mathbf{P}^k_i$, the movement in the direction of the global best (i.e. the best visited position of all the particles in its neighborhood), $\mathbf{P}^k_g$. In the present hybrid algorithm [21] the advantage of the PSO consisting of utilizing the local best and the global best is added to the CSS algorithm. The **CM** updating process is defined as follows:

$$\mathbf{CM}_{i,new} = \begin{cases} \mathbf{CM}_{i,old} & \text{fit}(\mathbf{X}_{i,new}) \geq \text{fit}(\mathbf{CM}_{i,old}) \\ \mathbf{X}_{i,new} & \text{fit}(\mathbf{X}_{i,new}) \leq \text{fit}(\mathbf{CM}_{i,old}) \end{cases} \tag{8}$$

Considering the above new **CM**, the electric forces generated by agents are modified as

$$\mathbf{F}_j = \sum_{i \in S_1} \left(\frac{q_i}{a^3} r_{ij} \cdot i_1 + \frac{q_i}{r_{ij}^2} \cdot i_2 \right) ar_{ij} p_{ij} (\mathbf{CM}_{i,old} - \mathbf{X}_j) + \sum_{i \in S_2} \left(\frac{q_i}{a^3} r_{ij} \cdot i_1 + \frac{q_i}{r_{ij}^2} \cdot i_2 \right) ar_{ij} p_{ij} (\mathbf{X}_i - \tag{9}$$

$$\begin{aligned} S_1 &= \{t_1, t_2, \ldots, t_n \mid q(t) > q(j), j = 1,2,\ldots,N, j \neq i, g\} \\ S_2 &= S - S_1 \end{aligned} \tag{10}$$

where subtitles S_1 and S_2 denote two sets of the numbers which determine the number of the agents utilized to calculate the resultant force by employing the agents sorted in the **CM** and the current agents positions, respectively. If the coefficient k_i is defined as

$$k_i = \left(\frac{q_i}{a^3} r_{ij} \cdot i_1 + \frac{q_i}{r_{ij}^2} \cdot i_2 \right) ar_{ij} p_{ij} \tag{11}$$

Then the resultant force formula can be simplified as

$$\mathbf{F}_j = k_1(\mathbf{CM}_{g,old} - \mathbf{X}_j) + k_2(\mathbf{CM}_{j,old} - \mathbf{X}_j) + \sum_{i \in S_1} k_i(\mathbf{CM}_{i,old} - \mathbf{X}_j) + \sum_{i \in S_2} k_i(\mathbf{X}_i - \mathbf{X}_j) \tag{12}$$

where the subtitle g denotes the number of the stored so far good position among all CPs. Therefore the first term directs the agents towards the global best position. When $i = j$, then the $\mathbf{CM}_{i,old}$ is treated similar to $\mathbf{P}^k_i$ in the PSO as considered in the second term of the above equation. This will direct the agents towards the local best [22].

3. PROBLEM STATEMENT

3.1. Laminate analysis

The structure is considered to include in-plain orthotropic plies with similar orientations in each lamina. The laminate itself consists of a number of laminae in the way that the whole structure becomes symmetric (Figure 1).

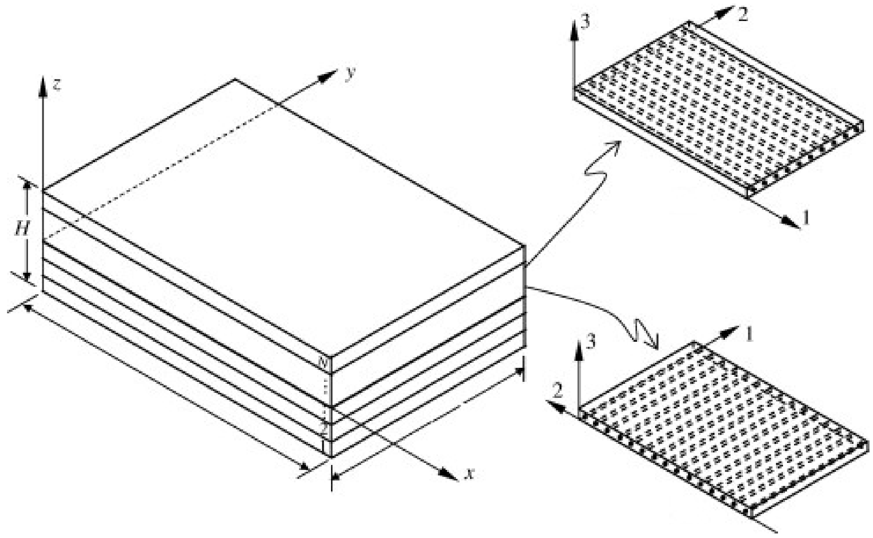

Figure 1. Schematic view of a plate composite laminate

The general relation between strain and curvature with force and moment components in composite materials is as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{13}$$

Where the blocks A and D are the extensional and flexural stiffness matrices, respectively. The blocks A relate the in-plane stress resultants to the mid-plane strains, and the blocks D relates the moment resultants to the curvatures. The blocks B, on the other hand, relates the in-plane stress resultants to the curvatures and moment resultants to the mid-plane strains, and hence is called the bending-extension coupling matrix. If it is undesirable, the blocks B can be avoided by a symmetric placement of the plies with different orientations with respect to the mid-plane of the laminate [2]. Figure 2 shows the resultant stress and moments in a laminate of composites.

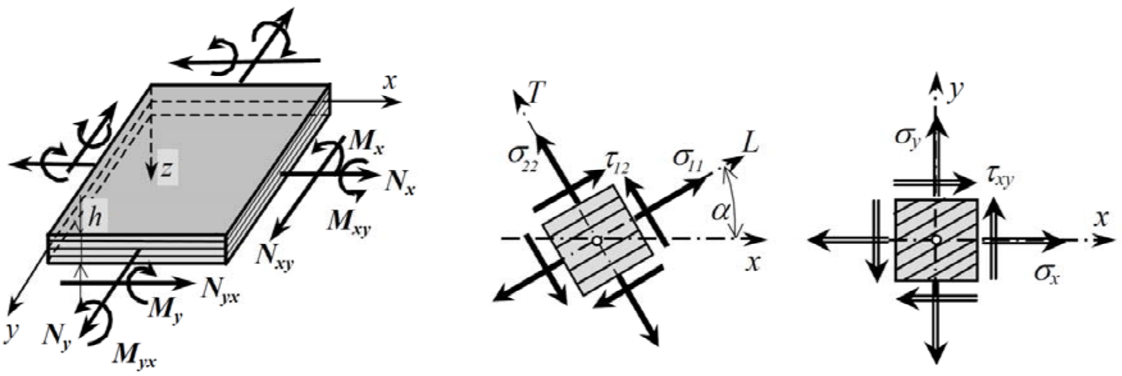

Figure 2. Stress and moment resultants for a plate

Due to symmetry, as mentioned above, B_{ij} blocks are equal to zero and since external loading is limited to in-plane forces, D_{ij} blocks are similarly zero. According to plane stress analysis and considering moment resultants to be zero, there is no out-of-plane deformation, so the curvature is dismissed and our analysis is independent of laminate height compared to mid-plane. Thus the general form of the relation above reduces to:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}_n = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}_n \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{14}$$

The index n is related to the number of the specified lamina for which the strain components will be calculated. Each lamina is comprised of plies with similar angles and a laminate consists of several laminas. Since the composite analysis in here is limited to in-plane loading and the geometry is symmetric, therefore strain components for each lamina is the same and independent of the index number. Accordingly, strain calculation is carried out for the entire structure in which extensional matric *A* components are calculated as:

$$A_{ij} = 2h\sum_{n=1}^{N/2} k_n (\bar{Q}_{ij})_n \tag{15}$$

where *h* is the thickness of each ply and has a constant value for all plies, since most commercially available composite materials come in fixed ply, and *k* is the number of plies in the specified lamina and *n* is the lamina number. The whole equation is multiplied by 2 because of symmetric geometry of structure. The mean reduced stiffness matric, $\bar{Q}_{ij}$, relates the stresses to strains as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_n = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_n \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{16}$$

The stiffness matric components are obtained using

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4 \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(n^4 + m^4) \\
\bar{Q}_{16} &= -mn^3Q_{22} + m^3nQ_{11} - mn(m^2 - n^2)(Q_{12} + 2Q_{66}) \\
\bar{Q}_{22} &= m^4Q_{22} + 2(2Q_{12} + 2Q_{66})m^2n^2 + Q_{11}n^4 \\
\bar{Q}_{26} &= mn^3Q_{11} - mn^3Q_{22} + mn(m^2 - n^2)(Q_{12} + 2Q_{66}) \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 4Q_{12})m^2n^2 + Q_{66}(m^2 - n^2)^2
\end{aligned} \tag{17}$$

where $m=cos(\theta)$ and $n=sin(\theta)$ and

$$\begin{cases} Q_{11} = \dfrac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \dfrac{E_2\nu_{12}}{1-\nu_{12}\nu_{21}} \\ Q_{22} = \dfrac{E_2}{1-\nu_{12}\nu_{21}}, \quad Q_{66} = G_{12} \\ \nu_{21} = \dfrac{E_2}{E_1}\nu_{12} \end{cases} \tag{18}$$

Using Eqs. (14), (15), (17), and (18), strain components for the composite are found. Thus the stress matric for lamina n is obtained and using transformation tensor principle, stress matric is calculated as:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_n = \begin{pmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{pmatrix}_n \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{19}$$

where the coefficient matric is named as tensor transformation.

3.2. Failure limits control

In order to be able to compare different structures with similar thickness, we use failure theories to compare the identical structures through the design factor. To control the failure limits, two failure criteria are used. The first major constraint is Thai-Wu criterion in which the function T should be less than 1.0 ($T \le 1$):

$$\begin{aligned} T &= \left(\frac{1}{S_{xt}} - \frac{1}{|S_{xc}|}\right)\sigma_{11} + \left(\frac{1}{S_{yt}} - \frac{1}{|S_{yc}|}\right)\sigma_{22} - \frac{1}{\sqrt{S_{xt}S_{xc}S_{yt}S_{yc}}}\sigma_{11}\sigma_{22} \\ &+ \frac{1}{S_{xt}|S_{xc}|}\sigma_{11}^2 + \frac{1}{S_{yt}|S_{yc}|}\sigma_{22}^2 + \frac{1}{S_{xy}^2}\tau_{12}^2 \end{aligned} \tag{20}$$

where S_{xt}, S_{xc} are tensile and compressive strengths along x direction, S_{yt}, S_{yc} are tensile and compressive strengths along y direction and S_{xy} is shear strength in the x–y plane.

The advantage of this theory is that there is an interaction between stress components and the theory does distinguish between the tensile and compressive strengths. This failure mode is conservative but in order to make sure all stress components do not exceed the limits we use another constraint known as the maximum principle stress theory as the second failure criteria to put more strict control on stress outputs.

In the maximum stress failure theory, failure of the lamina is assumed to occur whenever any normal or shear stress component equals or exceeds the corresponding strength. In this method the stress components are not related to each other and failure control is practiced over any of them individually. This theory is written mathematically as follows:

$$\begin{cases} \sigma_{11}^C < \sigma_{11} < \sigma_{11}^T \\ \sigma_{22}^C < \sigma_{22} < \sigma_{22}^T \\ |\tau_{12}| < \tau_{12}^F \end{cases} \tag{21}$$

where σ_{11} and σ_{22} are the maximum material stresses in the lamina, and τ_{12} is the maximum shear stress [23]. Using this criterion, we ensure the constraints over the composite failure is practiced strictly.

3.3. Optimization approach

The main goal of this article is to optimize the weight of structure by minimizing total thickness and maintaining in situ strength through failure control theories. The optimization process must continue in a way that the stress components do not go beyond the limits and no ply encounters failure. That is why the failure control procedure is practiced over the plies individually. In order to handle the constraints, a penalty function is utilized. If the constraints are between the allowable limits, the penalty is zero; otherwise the amount of penalty is obtained due to the violation of the maximum stress criterion and the Tsai–Wu criterion. Therefore the aim of the optimization is redefined by introducing the objective function (F) as

$$\text{Minimize } F = H + f_p = 2h\left(\sum_{m=1}^{n} k_m\right) + \left(\omega_1 P_{MS} + \omega_2 P_{TW}\right) \tag{22}$$

in which H is the total thickness of the composite, f_P is the penalty function, P_{ms} and P_{tw} are penalty values calculated based on the maximum stress and the Tsai–Wu criterion, ω_1 and ω_2 are the penalty factors, respectively. P_{MS} and P_{TW} are defined as:

$$\begin{aligned} P_{MS} &= \sum_{k=1}^{n}\left(P_x^k + P_y^k + P_{xy}^k\right) \\ P_{TW} &= \sum_{k=1}^{n}\left(P_{tw}^k\right) \end{aligned} \tag{23}$$

where

$$\begin{aligned} P_x^k &= \begin{cases} 0 & , FS_x^k \geq 1 \\ (1/FS_x^k) - 1 & , FS_x^k < 1 \end{cases} \\ P_y^k &= \begin{cases} 0 & , FS_y^k \geq 1 \\ (1/FS_y^k) - 1 & , FS_y^k < 1 \end{cases} \\ P_{xy}^k &= \begin{cases} 0 & , FS_{xy}^k \geq 1 \\ (1/FS_{xy}^k) - 1 & , FS_{xy}^k < 1 \end{cases} \\ P_{TW}^k &= \begin{cases} 0 & , FS_{TW}^k \geq 1 \\ (1/FS_{TW}^k) - 1 & , FS_{TW}^k < 1 \end{cases} \end{aligned} \tag{24}$$

where FS^k_x, FS^k_y and FS^k_{xy} are components of factors of safety for the maximum stress failure mode, and FS^k_{TW} is the factor of safety according to Tsai–Wu criterion for the kth lamina. The related safety factors are determined as

$$
\begin{aligned}
FS_x^k &= \begin{cases} S_{xt}/\sigma_{11} & ,\sigma_{11} > 0 \\ S_{xc}/\sigma_{11} & ,\sigma_{11} < 0 \end{cases} \\
FS_y^k &= \begin{cases} S_{yt}/\sigma_{22} & ,\sigma_{22} > 0 \\ S_{yc}/\sigma_{22} & ,\sigma_{22} < 0 \end{cases} \\
FS_{xy}^k &= S_{xy} / \left|\tau_{12}\right| \\
FS_{TW}^k &= \left| \frac{-b + \sqrt{b^2 + 4a}}{2a} \right|
\end{aligned}
\tag{25}
$$

where

$$
\begin{aligned}
a &= \frac{\left(\sigma_{11}^k\right)^2}{S_{xt}\left|S_{xc}\right|} + \frac{\left(\sigma_{22}^k\right)^2}{S_{yt}\left|S_{yc}\right|} - \frac{\left(\sigma_{11}^k\right)\left(\sigma_{22}^k\right)}{\left(S_{xt}S_{xc}S_{yt}S_{yc}\right)^{1/2}} \\
b &= \left(\frac{1}{S_{xt}} - \frac{1}{|S_{xc}|}\right)\sigma_{11}^k + \left(\frac{1}{S_{yt}} - \frac{1}{|S_{yc}|}\right)\sigma_{22}^k
\end{aligned}
\tag{26}
$$

In summary, the safety factor of the laminate according to the maximum stress criterion is calculated as follows: First, the principal stresses in each lamina are determined; then safety factors for each failure mode are calculated. In addition, the safety factor for the *k*th lamina according to the Tsai–Wu criterion is defined as the multiplier of the stress components at lamina *k*, which makes the right hand side of the Eq. (20) equal to 1.0.

4. NUMERICAL RESULTS AND DISCUSSION

In this section, some design examples are optimized in order to demonstrate the numerical efficiency and reliability of the proposed method. All programs are coded in MATLAB. These examples include

- Tension loading
- Compression loading
- Shear loading
- Reliability evaluation

For each example, 20 independent runs are carried out using the new hybrid algorithm. The number of 300 individuals for CPs is used and the value of constants k_v, k_a, ω_1 and ω_2 are set to 1.2, 0.9, 100 and 100, respectively. Also the maximum number of iteration is set to 120. The material properties of the composite used in this paper are the same as materials used in [17]. The materials include epoxy resin with long unidirectional graphite fibers with the properties as shown in Table 1.

Variables to consider here consist of the ply orientation and the number of plies in a lamina. In order to be able to obtain better results in the solution, angles are considered to be continuous in the range of [–90°, 90°] and the thickness is supposed to be constant equal to 0.00127 mm. Thus the ply angle and the ply number in a lamina are the design variables and total thickness minimization will occur through optimization of these two factors. Since different failure control methods influence the results, reliability factor for each one should be taken into consideration and logically the method with lower factor will be more conservative.

Table 1: Material properties of graphite-epoxy composite

Properties	Value	Unit
E_{11}	40.91	GPa
E_{22}	9.88	GPa
G_{12}	2.84	GPa
G_{21}	4.18	GPa
ν_{12}	779	MPa
S_{xt}	-1134	MPa
S_{xc}	19	MPa
S_{yt}	-131	MPa
S_{yc}	75	MPa
S_{xy}	779	MPa

4.1 Tension loading

In the first loading case according to Table 2, we increased the tension loading in biaxial state and considered no shear loading. As it is demonstrated, the laminate thickness is affected by changing tension loading and the general form of the stacking sequence is in the form of [θ_1, $-\theta_2$] and they are getting closer to unidirectional condition also orientations are becoming nearer to each other with rising of loading, the reason is the tendency of fibers to turn into the direction of the growing component to acquire the best resistance.

Table 2: The optimum layups using CSS+PSO under tensional loading

N_{yy}=5 *MN/m*, N_{xy}=0					
Load case	**N_{yy} (*MN/m*)**	**Stacking sequence**	**Number of plies**	**Failure mode**	
1	10	$[-35_{28}, 39_{26}]_s$	108	FS_{TW}	1.0043
				FS_{MS}	1.0249
2	20	$[-29_{25}, 34_{21}]_s$	92	FS_{TW}	1.0046
				FS_{MS}	1.1883
3	40	$[-26_{20}, 26_{20}]_s$	80	FS_{TW}	1.0061
				FS_{MS}	1.1885
4	80	$[-21_{26}, 20_{27}]_s$	106	FS_{TW}	1.0440
				FS_{MS}	1.2123
5	120	$[-18_{37}, 19_{34}]_s$	142	FS_{TW}	1.0044

				FS_{MS}	1.0955

Thickness decrease at the beginning of the loading can be according to no shear loading and increasing role of tension load that increases the uniaxial behavior and as we know fiber accesses the most strength in its direction. But in higher loads, in order to avoid the failure, thickness increases to improve its resistance to delamination.

4.2 Compression loading

According to material properties, compression strength in the direction of *y* axis is less than *x* axis. THUS the compression loading is increased in the direction of *y* to verify the failure limits (Table 3). In this case thickness increases with loading and fibers can not become closer and merge to decrease required thickness. The possible reason can originate from the fact that the strength limits for compression in *y* direction is low for this material, so fibers have to remain in the direction of loading to avoid failure. One of the orientations is forced to tend toward the loading angle in order to prevent structure from failure; on the other hand they are susceptible to failure in the vertical loading because they own weakest resistance in that direction so other fibers had to align in the nearly vertical direction to reinforce the structure and support the first oriented group of fibers.

Table 3: The optimum layups using CSS+PSO under compression loading

N_{xx}=10 *MN/m*, N_{xy}=0					
Load case	**N_{yy} (*MN/m*)**	**Stacking sequence**	**Number of plies**	**Failure mode**	
1	–10	$[73_7, 0_{14}]_s$	42	FS_{TW}	1.0332
				FS_{MS}	1.2147
2	–20	$[-4_{15}, -78_{15}]_s$	60	FS_{TW}	1.0022
				FS_{MS}	1.1432
3	–40	$[-4_{21}, 89_{24}]_s$	90	FS_{TW}	1.0072
				FS_{MS}	1.0991
4	–80	$[-8_{21}, -90_{51}]_s$	144	FS_{TW}	1.0054
				FS_{MS}	1.0912
5	–120	$[-8_{19}, 89_{79}]_s$	196	FS_{TW}	1.0086
				FS_{MS}	1.0982

4.3 Shear loading

As it is observed, shear load increase in Table 4 leads to drastic rise in composite thickness which shows the weakness of composite structures toward shear forces. That is because of low shear module of composites. Another reason can be that fibers can not bear shear load, so the load is not transferred from the matrix to fibers and the matrix itself has to stand it. Therefore the only way to increase the strength is the thickness improvement.

4.4 Reliability evaluation

In order to check reliability and superiority of the proposed optimization method, we used another similar problem based on [24] in which nonlinear programming (NLP) as a deterministic method was used and two different approaches were adopted, one of which took thickness as constant and ply angle as varying variable, and the other considered both as changing parameters. Since ply thickness is not categorized as our optimizing variable, we compared the results of example one. The objective function in this problem is maximization of safety factor leading to safer design.

Table 4: The optimum layups using CSS+PSO under shear loading

N_{xx}=10 *MN/m*, N_{yy}=10 *MN/m*					
Loade case	N_{xy} **(*MN/m***	**Stacking sequence**	**Number o plies**	**Failure mode**	
1	10	$[61_2, 40_9]_s$	22	FS_{TW}	1.1223
				FS_{MS}	1.0753
2	20	$[43_{30}, -76_{84}]$	68	FS_{TW}	1.0012
				FS_{MS}	1.0801
3	40	$[46_{59}, -22_{19}]$	156	FS_{TW}	1.0032
				FS_{MS}	1.1909
4	80	$[47_{107}, -67_{53}$	320	FS_{TW}	1.0018
				FS_{MS}	1.1932
5	120	$[68_{101}, -48_{146}$	494	FS_{TW}	1.0027
				FS_{MS}	1.2072

Different failure criterion was brought into the algorithm. Matrix tensile failure constraint similar to the one used in [25] was adopted where ply transverse tensile and shear strength were critical parameters specifying failure limits and modified expressions were included the same as [25] to have a fair judgment. Optimum results for 4 and 8 ply angles obtained in [24] were fetched into the algorithm and safety factors related to the mentioned failure method were extracted from [25] which was chosen as our comparing tool. Material properties and strength limits are illustrated in Table 5. Thickness of each lamina is 0.14 mm and the number of plies in each lamina is considered to be constant and equal to four.

Table 5: Material properties of graphite-epoxy composite for the reliability evaluation example

Properties	**Value**	**Unit**
E_{11}	138	*GPa*
E_{22}	11.7	*GPa*
G_{12}	4.56	*GPa*
ν_{12}	0.29	*MPa*
Y_t^0	44.5	*MPa*
S_t^0	48.2	*MPa*

As shown in Table 6, with the same stacking sequence, the present hybrid method displays comparable results with the larger safety factor indicating the advantage gained by global search algorithms. Optimum stacking sequence obtained for the maximum safety factor by DSA (direct simulated annealing) method proposed in [17] was also used which was carried out the method for the same problem. In order to compare the present hybrid approach with other meta-heuristic tool we added results accessed by the DSA. Again, the numbers extracted from the hybrid CSS and PSO is promising, showing its superiority for structural optimization problems.

Table 6: Comparison between different methods for a four ply angle laminate with constant thickness

	$[N_{xx}, N_{yy}, N_{xy}]$ (*kN/m*)	**Optimum layup**	**Optimization approach**	**Factor of safety**
(a)	[200, 200, 0]	$[62.68_4, -54.20_4, 81.23_4, -1.96_4]_s$	NLP	1.27
		$[50.80_4, -49.80_4, 26.59_4, -49.73_4]_s$	**DSA**	**1.51**
		$[52.76_4, -37.21_4, 33.14_4, -55.58_4]_s$	**CSS+PSO**	**1.58**
(b)	[200, 0, 200]	$[32.3_4, -56.61_4, -7.78_4, 33.87_4]_s$	**NLP**	1.57
		$[31.72_4, 31.72_4, 31.72_4, 31.72_4]_s$	**DSA**	**1.00**
		$[41.07_4, 26.19_4, -59.99_4, 31.72_4]_s$	**CSS+PSO**	**2.04**
(c)	[400, 200, 0]	$[-27.46_4, 57.58_4, -43.61_4, 20.11_4]_s$	**NLP**	1.04
		$[30.98_4, -36.57_4, 37.67_4, -37.20]_s$	**DSA**	**1.21**
		$[34.57_4, -37.66_4, 37.05_4, -34.85_4]_s$	**CSS+PSO**	**1.26**
(d)	[200, 200, 200]	$[45.45_4, 51.72_4, 43.38_4, 39.58_4]_s$	NLP	11.94
		$[45_4, 45_4, 45_4, 45_4]_s$	**DSA**	**7.6041×1014**
		$[45.39_4, 45.12_4, 44.81_4, 44.71_4]_s$	**CSS+PSO**	**7.6041×1014**

Case *d* shows a drastic size value for safety factor which shows the best optimal loading for in-plane combined loading with similar identical values is $[45_n]_s$ layup. To understand better, the safety factor for $[45_{16}]_s$ with the same loading as *d* is 7.6041×10^{14}. Similar alignment of fibers and resultant forces could be the main reason. Thickness of laminate in Table 7 is again constant and is proportionally increasing with load rising. Case *d* is similar to Table 6 and total loading is transferred to the fibers and resin has no roll in bearing the applied load. Therefore composite has the safest design in this case. Equations for safety factor calculations are available in [25].

Table 7: Comparison between different methods for an eight ply angle laminate with constant thickness

	$[N_{xx}, N_{yy}, N_{xy}]$ (*kN/m*)	**Optimum layup**	**Optimization approach**	**Factor of safety**
(a)	[400, 400, 0]	$[-3.08_4, -90.00_8, -28.51_4, 88.00_4, -$	NLP	2.51

		28.49_4 -27.59_4, $40.05_4]_s$		
		$[-73.68_4, 33.57_4, -50.69_4, 30.89_4, -51.55_4, -51.55_4, 30.75_4, 30.75_4]_s$	**DSA**	**2.61**
		$[-89.00_4, -1.00_4, -74.9_4, -12.04_4, 50.84_4, -33.37_4, 23.4_4, 85.77_4]_s$	**CSS+PSO**	**3.14**
(b)	[400, 0, 400]	$[36.66_4, -57.68_4, 36.47_4, 90.00_4, 37.31_4, 24.58_4, -57.77_4, 31.65_4]_s$	NLP	3.57
		$[31.72_8, 31.72_8, 31.72_8, 31.72_8]_s$	**DSA**	**1.00**
		$[28.97_4, -60.5_4, 30.56_4, 30.94_4, -57.78_4, 31.04_4, 64.88_4, 30.87_4]_s$	**CSS+PSO**	**4.01**
(c)	[800, 400, 0]	$[-26.94_4, 49.26_4, -37.54_4, 40.90_4, -42.78_4, 55.36_4, -24.16_4, 10.86_4]_s$	NLP	2.14
		$[-29.94_4, 35.60_4, -36.69_4, 36.31_4, 36.31_4, 36.31_4, -36.75_4, -36.75_4]_s$	**DSA**	**2.10**
		$[-43.00_4, 31.31_4, -31.15_4, 41.24_4, -35.07_4, 22.20_4, -38.00_4, 44.98_4]_s$	**CSS+PSO**	**2.23**
(d)	[400, 400, 400]	$[49.29_4, 49.27_4, 49.29_4, 26.60_4, 49.42_4, 11.91_4, 50.14_4, 49.33_4]_s$	NLP	5.32
		$[45_4, 45_4, 45_4, 45_4]_s$	**DSA**	**7.6041×10^{14}**
		$[43.91_4, 43.17_4, 46.60_4, 43.00_4, 50.00_4, 38.13_4, 48.89_4, 46.00_4]_s$	**CSS+PSO**	**7.6041×10^{14}**

5. CONCLUDING REMARKS

Since the early dawn of civilization, the strong and light material has always fascinated mankind for different applications. The idea of combining two or more different materials resulting in a new material with improved properties exists from ages. It was discovered long ago that composite materials have the combined advantages with superior performance compared to each of its constituting material. These materials are blended in special structures, one of which is stacking sequence. Stacking sequence is highly dependent on loading condition including sign of loading, magnitude of loading and material properties of the composite. The influenced variables are fiber orientation, and number of plies. According to previous studies, two orientation states often give the best answer with the two variables and composite withstand in-plane stress with the minimum thickness. Thus we used two angle–ply structures for the laminate layup. For some loading cases, many global or near-global optimum designs are found to exist. The proposed method proved to be quite reliable in locating these designs. In a single optimization run, the hybrid CSS and PSO algorithm could find one or more of them even with a large number of design variables. The resulting hybrid algorithm, tested over 460 runs on the different load cases, needs on average 30,000 analyses to locate a practical optimum with a high reliability.

In the case of shear loading, composite thickness makes strong change with increasing the load. This problem originates from nature of fiber interaction with the matrix and high shear loading may lead to delamination. In addition, in-plane loadings with equal values faces over design condition with nearly infinite safety factor which is because of complete transfer of loading to the fibers, removing the effect of resin in the composite. However the optimization procedure successfully managed to provide proper layup with the minimum possible thickness and weight in an acceptable criterion.

Safety factor values illustrated in the tables proved the feasibility of the design. In order to understand which constraint is more active similar penalty weighting coefficients ($\omega 1=\omega 2$) were allocated to the penalty function. The obtained safety factors from Thai-Wu method displayed lower values, showing more conservative behavior than maximum stress criterion, which means one can use different penalty weighting coefficients where $\omega 1 < \omega 2$. The future works should focus on the extension of the approach to a multi-objective based design of laminated composite structures for example simultaneous maximization of fundamental frequency and minimization of cost or maximization of the load carrying capacity and minimization of the mass of a graphite-epoxy laminate subjected to biaxial moments.

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