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PROBABILISTIC NONLINEAR DYNAMIC ANALYSIS OF A PLANE FRAME WITH MATERIAL AND GEOMETRIC UCERTAINTIES

J. Alamatian[*a] and F. Shahabian[b]
[a]Department of Civil Engineering, Mashhad Branch, Islamic Azad University, 91735-413, Mashhad, Iran
[b]Department of Civil Engineering, Ferdowsi University of Mashhad, 91775-1111, Mashhad, Iran

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ABSTRACT

This paper focuses on probabilistic nonlinear dynamic analysis of a plane frame in which density, modulus of elasticity and section dimensions are random variables. The Monte Carlo method is used to simulate the random variables with different types of probability density functions and various coefficients of variation. The time-displacement responses of the frame show that randomness in the material and geometric properties can lead to significant uncertainty in the maximum response displacement. Also, the statistically dynamic responses obtained by the present approach will be useful for studying the structural safety and reliability.

Keywords: Probabilistic nonlinear dynamic analysis; structural safety; monte carlo simulation.

1. INTRODUCTION

Uncertainty on the various elements of the given structure may be arise from different sources such as geometry, material properties, boundary conditions, and so on. The need to incorporate uncertainties in an engineering design has long been recognized [1]. The traditional approach, the so called “deterministic analysis”, makes use of safety coefficients in order to prevent unpredicted failures due to the variability of the data. As a consequence, it is not possible to quantify the reliability of the structure, defined as the probability that the structure does not experience a failure [1]. On the other side, are liability new trend, named

* E-mail address of the corresponding author: alamatian@mshdiau.ac.ir (J. Alamatian)

"probabilistic analysis" [2,3,4], allowing the estimation of the reliability of the design, considers the stochastic variability of the data.

The probabilistic methods for treating uncertainty problems can be broadly classified into two categories: statistical approach and non-statistical approach. Direct Monte Carlo simulation technique is the most prevalent statistical approach. The Monte Carlo method, while requiring an effective and accurate tool for numerical generation of random fields, stands at the center of stochastic mechanics, providing a universal means of solving various complicated stochastic problems [5,6,7].

Some researchers have focused their interests on the non-statistical approach. Stochastic finite element method belongs to this category. This method in conjunction with Neumann expansion [8] or spectral approach [9] has been proposed to attain the statistics of response of stochastic structures.

The structures, however, usually unavoidably exhibit strong non-linearity during their service life. In spite of the paramount importance, it is so far difficult to use non-statistical approach to capture accurate probabilistic response of such problems.

On the other hand, the nonlinear stochastic dynamics [10], the probability density evolution methods [11] and the topic of variation of response due to the variation of parameters and excitations [12] have been developed in the past years.

The objective of this investigation is to study the probabilistic nonlinear dynamic analysis of a plane frame with randomness in material properties and section dimensions by using finite element method. In the probabilistic finite element program, all variables can be treated as random variables, in particular: material properties and geometrical properties. The Monte Carlo simulation method is used to generate the random variables. Each variable is simulated for any type of probability density function (PDF) including normal, lognormal and uniform distribution with various coefficients of variation (COVs) changing from 5% to 15%. The mean and the standard deviation of the top displacement of the plane frame are obtained for various "PDFs" and "COVs". These obtained statistical responses are very useful for estimating the structural safety and evaluating the sensitivity of dynamic responses to the type of "PDF" and values of "COV".

2. NONLINEAR DYNAMIC ANALYSIS

The dynamic analysis is usually performed by solving a set of time differential equations which contain nodal displacements, velocities and accelerations. In the other words, the dynamic equilibrium equation is a system of second order differential equations which can be formulated by the several well known approaches such as the Newton's second law of motion, principle of virtual work [13], Lagrange equation [13] or Hamilton's principle [14]. These procedures lead to the following system of dynamics equilibrium equations;

$$[M]^{n+1}\{\ddot{D}\}^{n+1} + [C]^{n+1}\{\dot{D}\}^{n+1} + \{f(D^{n+1})\} = \{P(t^{n+1})\} \tag{1}$$

Here, $[M]^{n+1}$, $[C]^{n+1}$, $\{f(D^{n+1})\}$ and $\{P(t^{n+1})\}$ are mass matrix, damping matrix, internal and external forces vectors, respectively. The structure has nonlinear behavior if inertia, damping or internal forces are nonlinear function of nodal displacement ($\{D\}^{n+1}$) at time t^{n+1}. Moreover, nodal velocity and acceleration vectors are presented by symbols $\{\dot{D}\}^{n+1}$ and $\{\ddot{D}\}^{n+1}$, respectively. The initial conditions of Eq. (1) can be written as follows;

$$\{D(0)\} = \{D_0\} \quad , \quad \{\dot{D}(0)\} = \{\dot{D}_0\} \tag{2}$$

where, $\{D_0\}$ and $\{\dot{D}_0\}$ are displacement and velocity at the beginning of analysis (t=0), respectively. It should be emphasized that there are some procedures which reduce the dynamic equation to a set of first order differential equations [15].

Since this paper deals with nonlinear dynamic analyses, the analytical methods such as modal analysis are not effective and applicable for solving Eq. (1). As a result, a numerical time integration approach is utilized here. Numerical schemes are known by step by step time integration. Generally, there are three groups of numerical algorithms: Explicit, Implicit and Predictor-Corrector.

Explicit integrations which run by vector operations are quite simple and rapid. The main defect of these procedures is numerical instabilities so that very small time steps should be utilized for increasing accuracy and stability. In these methods, displacement and velocity of the current increment are explicitly formulated using some available previous time steps data. As a result, displacement and velocity of the new time step are calculated by few vector operations. Then, these quantities are substituted into the dynamic equilibrium equation (Eq. 1), and acceleration of the current time step is obtained by solving a solving system of simultaneous linear equations, even for nonlinear analyses. Some of the well known explicit time integrations are as follows; Single step integration [15]; explicit integration with optimal dissipation [16]; Generalized weighted residual approach [17]; SSpj method [18]; β_m algorithm [19] and Hoff-Taylor approach [20] and etc.

In implicit methods, velocity and acceleration of the current time step are assumed to be function of current displacement and some available information of the previous steps. Utilizing these functions in dynamic equilibrium equation (Eq.1), an equivalent static system of simultaneous equations is obtained. For nonlinear analyses, this system will be nonlinear. Therefore, displacement vector of the new time step is calculated from the equivalent static system. These procedures are more stable and accurate than the explicit integrations so that greater time step can be used. Some of the well known implicit approaches are Newmark-β, Wilson-θ, Generalized-α method [21] and recently implicit higher order integrations (IHOA) [22].

The third group of time integration methods is Predictor-Corrector integrations which are formulated by combining implicit and explicit procedures. In such algorithms, the explicit and implicit integrations are utilized for estimating and correcting the answer, respectively [23]. The correction stage should be iterated successively to achieve more accurate response. In spite of implicit integrations, predictor-corrector methods can be performed by vector operations even in nonlinear analyses. In the other words, a linear system of simultaneous equations should be only solved in each correction of these methods (like explicit integrations). Such techniques present more stable and accurate integration compared with the explicit methods. Because of these specifications, higher order predictor-corrector integration called PC-m is used here for dynamic analysis of nonlinear portal frame [24]. This algorithm is formulated based on the implicit higher order integration (IHOA) by considering different weighted factors. In these higher order methods, accelerations of several previous time steps are used to predict and correct the dynamic response of structure. Therefore, displacement and velocity of the current time step are predicted as follows [24]:

$$\{D\}^{n+1} = \{D\}^n + \Delta t\{\dot{D}\}^n + \left(\frac{1}{2} - \sum_{i=0}^{\Gamma-1} \xi_i\right)\Delta t^2\{\ddot{D}\}^n + \Delta t^2 \sum_{i=1}^{\Gamma-1} \xi_i \{\ddot{D}\}^{n-i} \tag{3}$$

$$\{\dot{D}\}^{n+1} = \{\dot{D}\}^n + \left(1 - \sum_{i=0}^{\Gamma-1} \eta_i\right)\Delta t\{\ddot{D}\}^n + \Delta t \sum_{i=1}^{\Gamma-1} \eta_i \{\ddot{D}\}^{n-i} \tag{4}$$

where, $\{\ddot{D}\}^{n-i}$ and Γ are acceleration vector at time t^{n-i} and accuracy order of the method, respectively. Also, weighted factors ξ_i and η_i which control stability and accuracy of the integration can be found in reference [24]. Using Eqs. (3) and (4) an estimation of dynamic response of structure (displacement and velocity) is obtained. Then, the acceleration of the current time step ($\{\ddot{D}\}^{n+1}$) is calculated from the dynamic equilibrium equation (Eq. (1)). Now, the correction stage is started. For this purpose, the relationships of the IHOA integration is used; i.e. the displacement and velocity are corrected by the following implicit integration [22];

$$\{D\}^{n+1} = \{D\}^n + \Delta t\{\dot{D}\}^n + \left(\frac{1}{2} - \sum_{i=0}^{\Gamma-1} \alpha_i\right)\Delta t^2\{\ddot{D}\}^n + \alpha_0 \Delta t^2\{\ddot{D}\}^{n+1} + \Delta t^2 \sum_{i=1}^{\Gamma-1} \alpha_i \{\ddot{D}\}^{n-i} \tag{5}$$

$$\{\dot{D}\}^{n+1} = \{\dot{D}\}^n + \left(1 - \sum_{i=0}^{\Gamma-1} \beta_i\right)\Delta t\ \{\ddot{D}\}^n + \beta_0 \Delta t\{\ddot{D}\}^{n+1} + \Delta t \sum_{i=1}^{\Gamma-1} \beta_i \{\ddot{D}\}^{n-i} \tag{6}$$

Here, α_i and β_i are weighted factors of the IHOA method and can be found in reference [22]. In this paper the integration order is assumed to be 6 i.e. $\Gamma = 6$. In other words, the PC-6 algorithm is used for numerical dynamic analysis. More details of this algorithm can be found in [24].

To prove the accuracy of the PC-6 algorithm, an elastic pendulum shown in Figure 1 is analyzed [25]. This structure which has large deflection nonlinearity is modeled by a two nodes truss element [25]. Also, total LaGrange finite element approach is utilized to form the nonlinear equilibrium equations. The mass matrix is consistent [26] and the axial rigidity (AE) and material density per element length (ρA) are 10^4 N and 6.57 kg/m, respectively. Bathe has been analyzed this structure by using implicit integration and two time steps; i.e. 0.05 and 0.01 sec. Here, the PC-6 algorithm runs with time step 0.05 sec. Figure 2 shows the response of the horizontal displacement of the pendulum between times 45 and 50 seconds. It should be noted that the exact solution has been obtained by implicit integration and very small time step (0.0001 sec). It is clear that the PC-6 algorithm is more accurate than the implicit integration presented by Bathe [25] so that numerical results of the PC-6 algorithm have good consistency with the exact solution. Therefore, higher order predictor-corrector time integration can be utilized for any nonlinear dynamic finite element analysis.

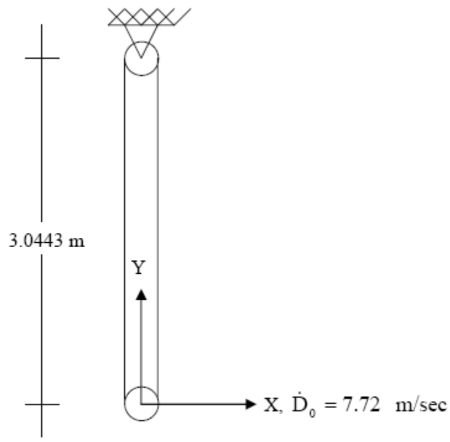

Figure 1. Elastic pendulum

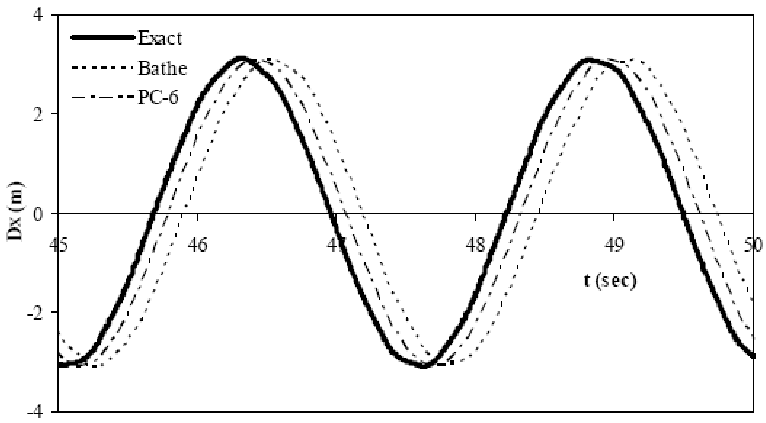

Figure 2. Response of horizontal displacement of the elastic pendulum for time step 0.05 sec

3. PROBABILISTIC DYNAMIC ANALYSIS

For the probabilistic dynamic analysis, a plane frame is analyzed with elastic geometrically nonlinear behavior [27]. Figure 3 shows this frame. For this purpose, the co-rotational finite element model is used [27]. The structure is subjected to earthquake base excitation, in the shape of El Centro acceleration record which is shown in Figure 4. For each simulation, time step is considered as 0.0005 second and top displacement of the frame is obtained using PC-6 method [24].

The modulus of elasticity, mass density and section dimensions are considered to be random variables. The type of probability density function, mean value and coefficient of variation for the variables are presented in Table 1.

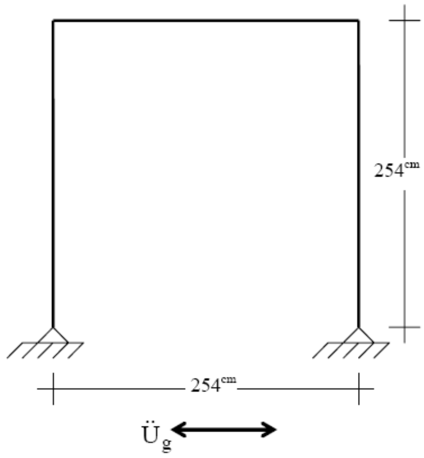

Figure 3. Plane frame for stochastic nonlinear dynamic analysis

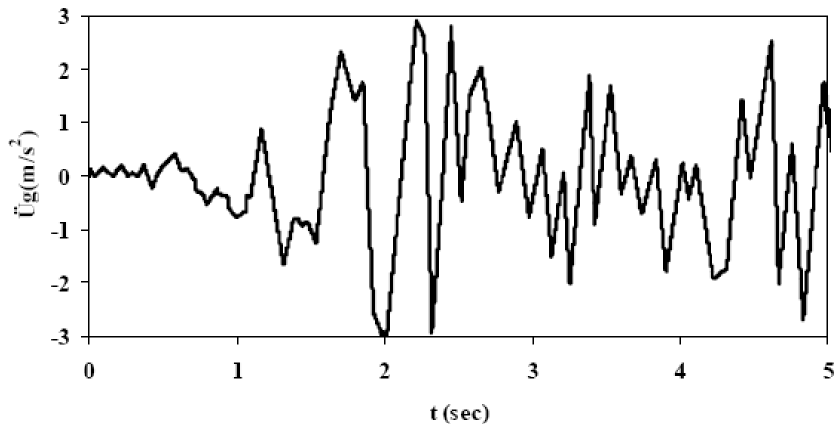

Figure 4. Earthquake excitation history for dynamic analysis

Table 1: Characteristics of random variables.

Stochastic variable	PDF	Mean Value	COV (%)
Modulus of elasticity	Normal Lognormal Uniform	2×10^{11} kg/m^2	5, 10, 15
Mass density	Normal Lognormal Uniform	39500 kg/m^3	5, 10, 15
Height of section dimension	Normal Lognormal Uniform	0.3 m	5, 10, 15

To study the probabilistic dynamic analysis of the plane frame, random variables i.e. density, modulus of elasticity and section dimensions are simulated by uniform, normal and lognormal probability density functions (PDF) and various coefficients of variation (COV) [28]. Herein, the Monte Carlo simulation is used in which one thousand data are generated based on the Sobol approach for each case [28]. For example, Figure 5 shows the histograph samples of the modulus of elasticity which has been plotted for some probability functions.

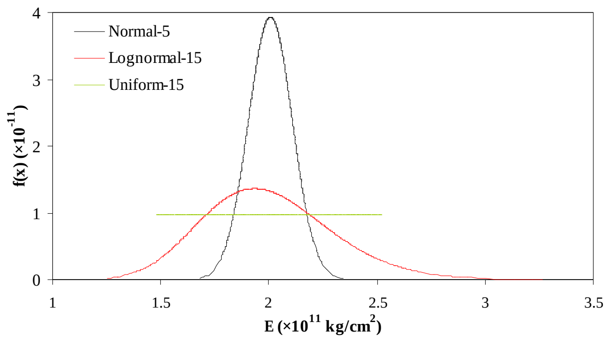

Figure 5. Simulated probability density function (PDF) for modulus of elasticity

Some important parameters in probabilistic analysis such as standard deviation and mean values of the dynamic response are obtained and discussed for various states as following. Figures 6, 7 and 8 present the results for the standard deviation of the top displacement of the frame, respectively, for normal, lognormal and uniform distributions, respectively. The time history of standard deviation can be used to predict the value of standard deviation for any type of probability density function. For this purpose, the standard deviation has been calculated in each time step of dynamic analysis. Figures 6, 7 and 8 show that the standard deviation of the structural response increases by increasing the coefficient of variation of the random variables. It can be concluded that when the coefficient of variation of variables increases 5%, the standard deviation of the structural response increases more than 100%.

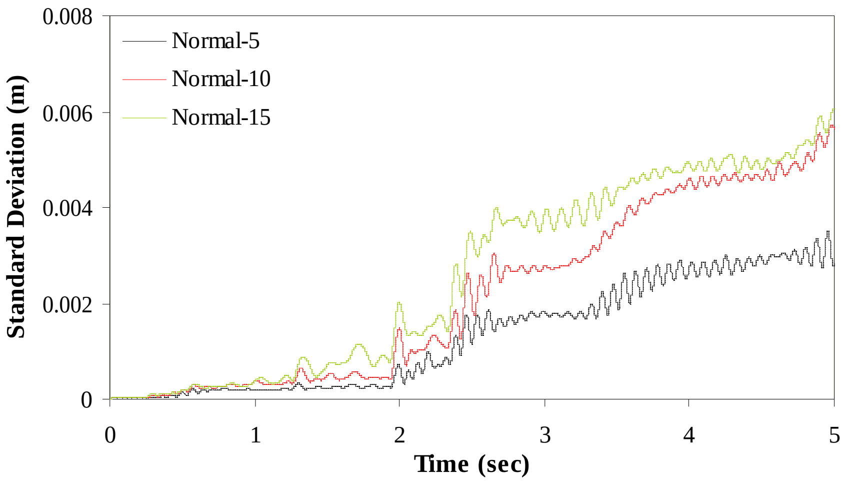

Figure 6. The standard deviation of the top displacement (Normal distribution)

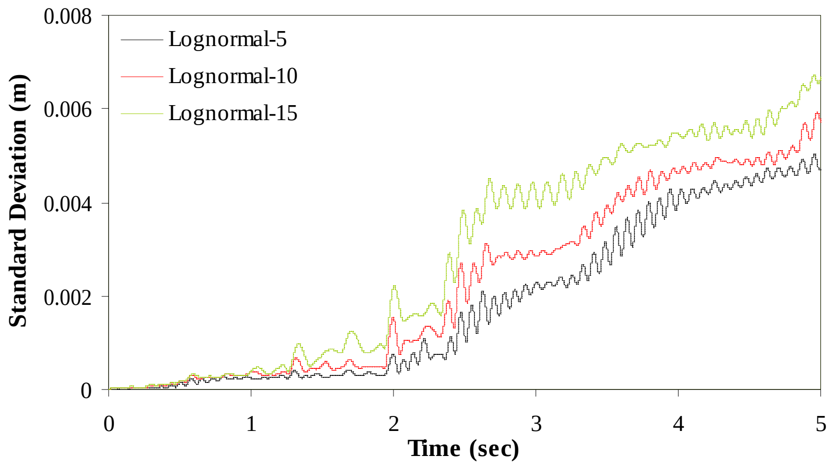

Figure 7. The standard deviation of the top displacement (Lognormal distribution)

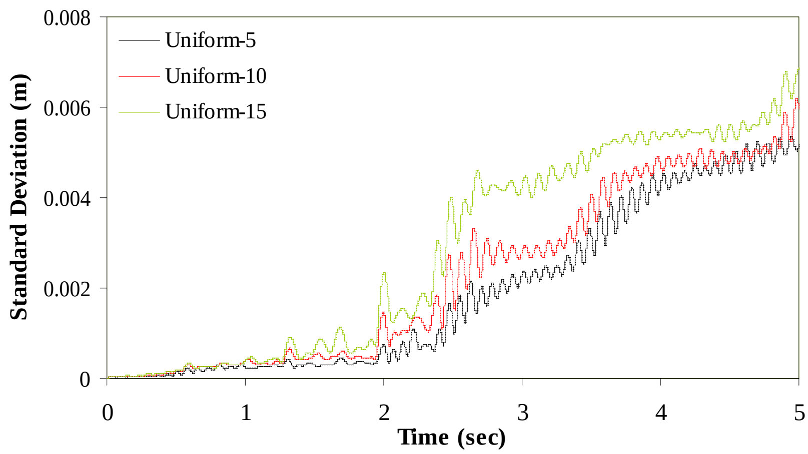

Figure 8. The standard deviation of the top displacement (Uniform distribution)

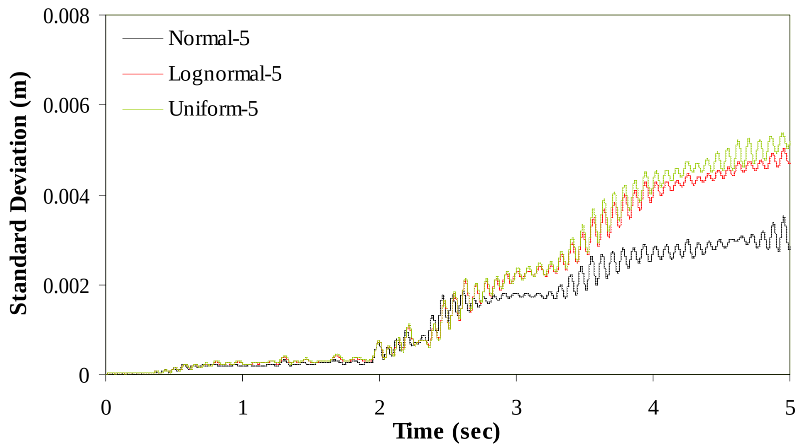

Figure 9. The standard deviation of the top displacement (COV=5%)

For comparison of three types of statistical distributions [28], standard deviation diagrams for different modes are drawn simultaneously. Figures 9, 10 and 11 show the standard deviation for three coefficients of variation i.e. 5%, 10%, and 15%, respectively. As it can be seen in Figures 9, 10 and 11, standard deviation of the structural response for uniform and lognormal distributions is more than normal one.

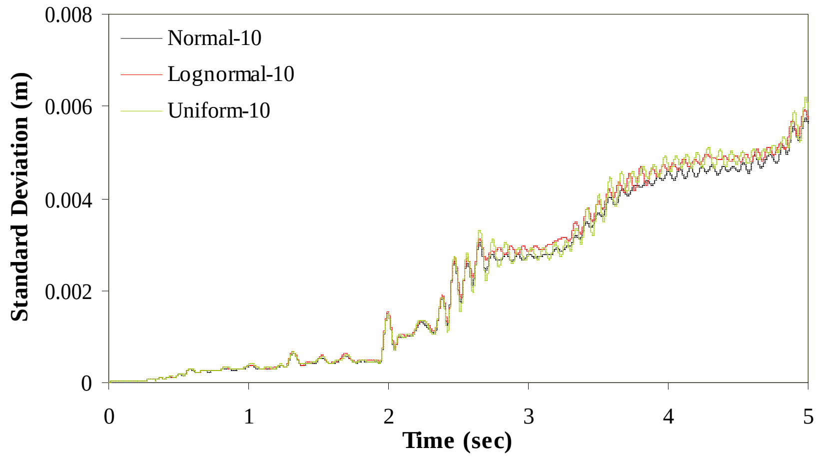

Figure 10. The standard deviation of the top displacement (COV=10%)

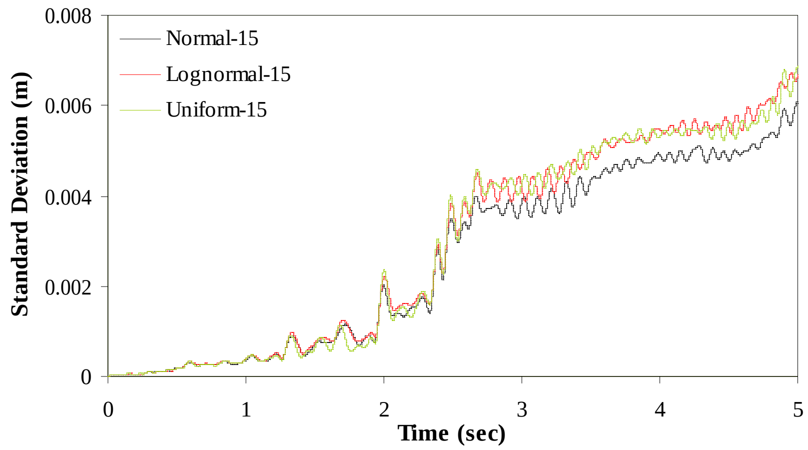

Figure 11. The standard deviation of the top displacement (COV=15%)

On the other hand, Figures 12, 13 and 14 show the time history of maximum values of the top displacement of the frame for various types of “PDFs” and various values of “COVs”, which are compared to the displacement time history with mean values of random variables which are regarded as deterministic values. As it can be seen in Figures 12, 13 and 14, the maximum values of the top displacement increase when the values of “COVs” increase. The maximum difference between the time histories of displacement with deterministic inputs and maximum values of displacement with various “PDFs” and “COVs” is about 200%. Therefore, the uncertainty and values of “COV” in mechanical properties and section dimensions affect on the dynamic response and especially on the maximum values of displacement.

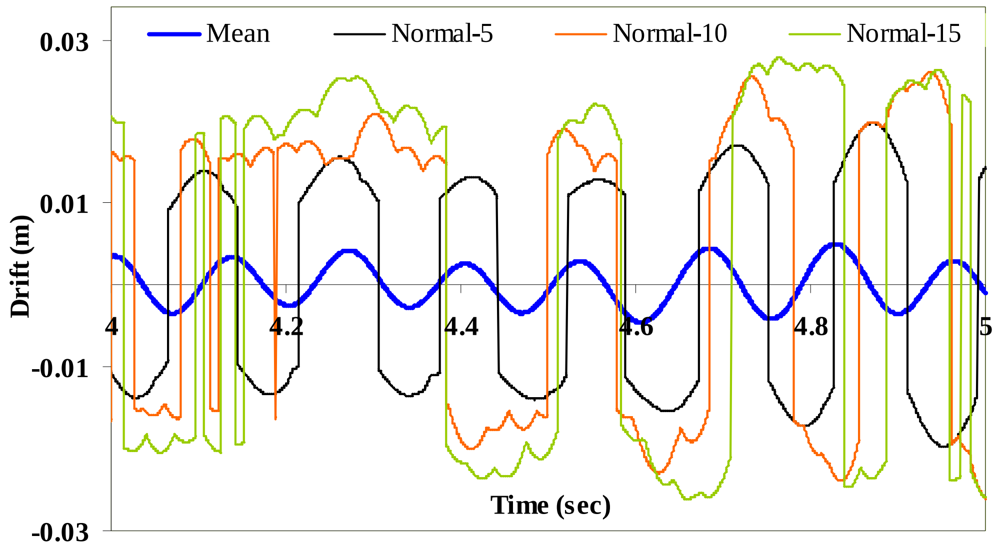

Figure 12. The mean and the maximum top displacement (Normal distribution)

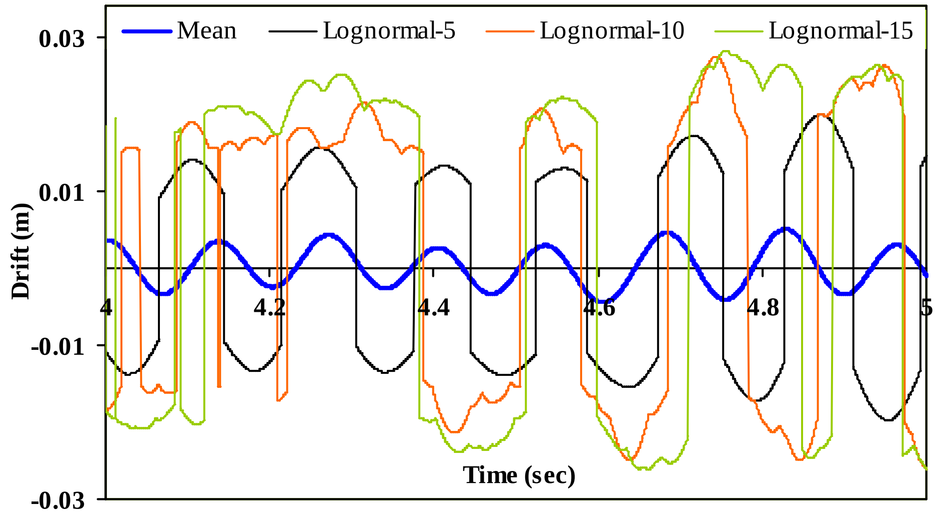

Figure 13. The mean and the maximum top displacement (Lognormal distribution)

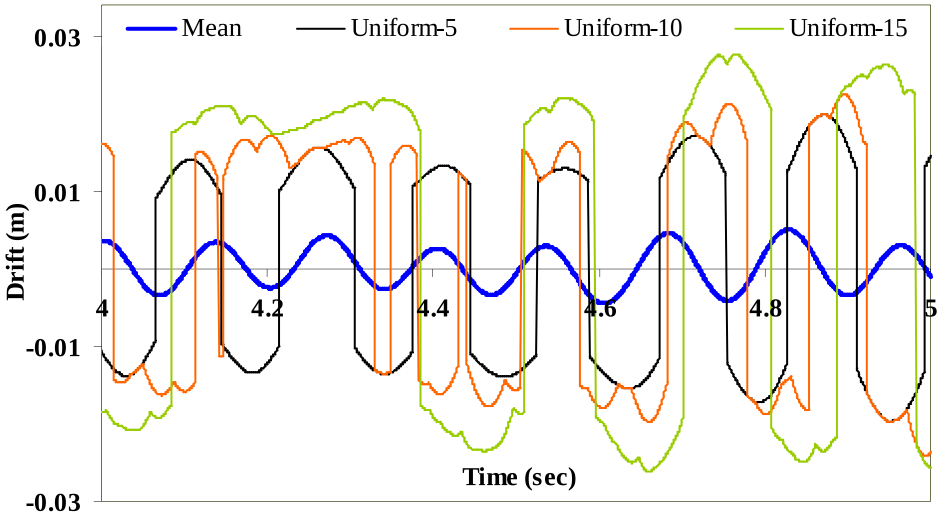

Figure 14. The mean and the maximum top displacement (Uniform distribution)

For comparison of the three types of statistical distributions, the time history of maximum values of the top displacement for different modes, are drawn simultaneously. Figures 15, 16 and 17 show the maximum top displacements for coefficients of variation 5%, 10%, and 15%, respectively. These Figures show that the dynamic response of the structure is not sensitive to the type of the probability density function.

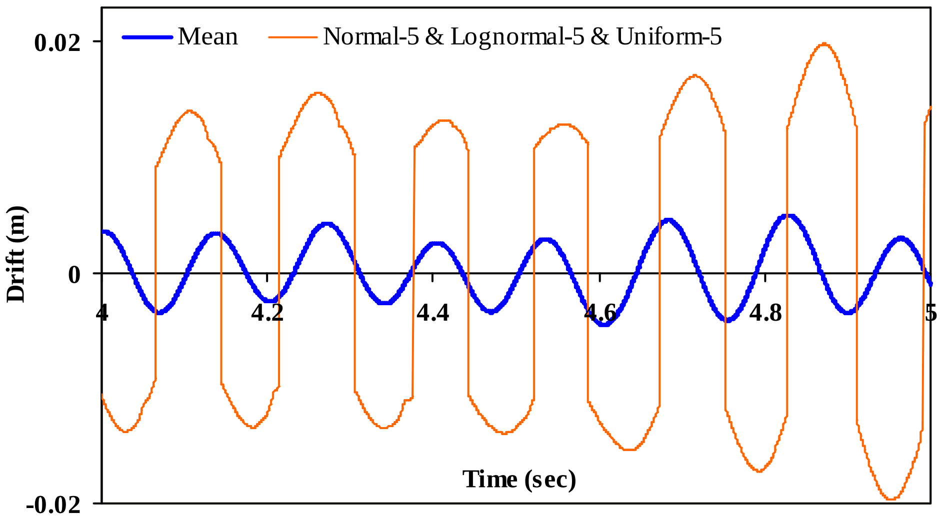

Figure 15. The mean and the maximum top displacement (COV=5%)

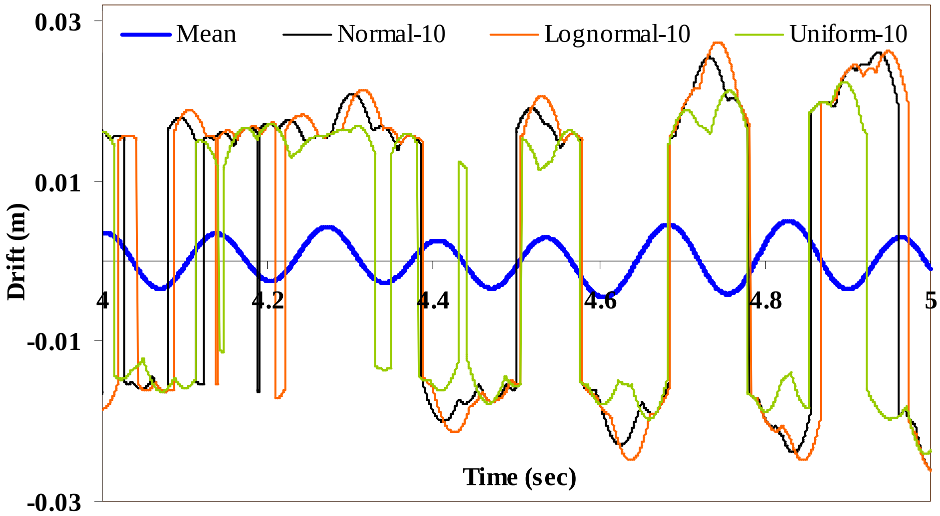

Figure 16. The mean and the maximum top displacement (COV= 10%)

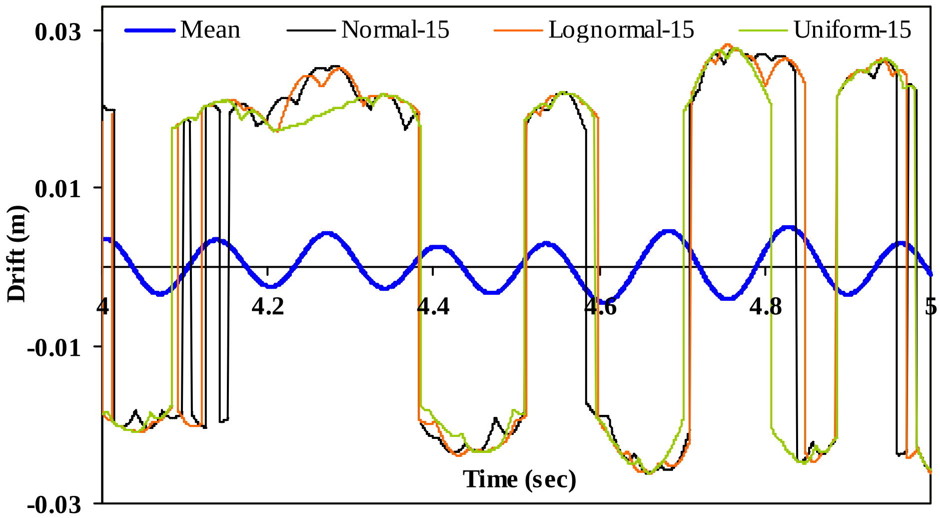

Figure 17. The mean and the maximum top displacement (COV= 15%)

4. STRUCTURAL FAILURE PROBABILITY

The failure probability of a structure is an important factor in the design procedure since it quantifies the probability that a structure will fulfill its design requirements. Probabilistic analysis is a tool that assists the design engineer to take into account all possible uncertainties during the design, construction and life of a structure in order to calculate its probability of failure P_f; i.e. to estimate the level of risk against a local or a global structural failure [6].

In probabilistic analysis the Monte Carlo method is often employed when the analytical solution is not attainable and the failure domain cannot be expressed or approximated by an analytical form [28]. The Monte Carlo estimation of P_f is given by

$$P_f = \frac{n}{N} \tag{7}$$

where N is the total number of simulations and n is the number of simulations which have greater values than the value for deterministic inputs (mean value of random variables).

The probabilities of failure are shown in Figures 18, 19, 20 and 21. As it can be seen in these figures, the failure probability increases when the coefficient of variation increases. Moreover, Figure 21 shows the failure probability is more sensitive to uniform and lognormal distributions than normal one.

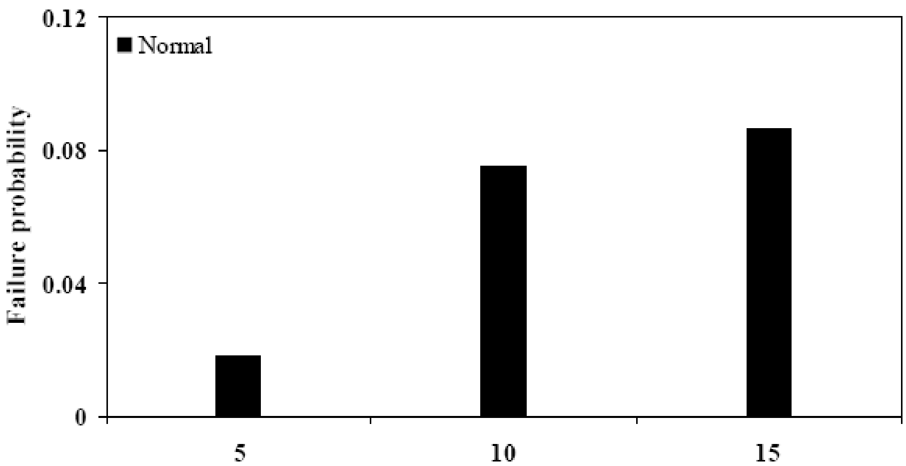

Figure 18. Probability of failure (Normal distribution)

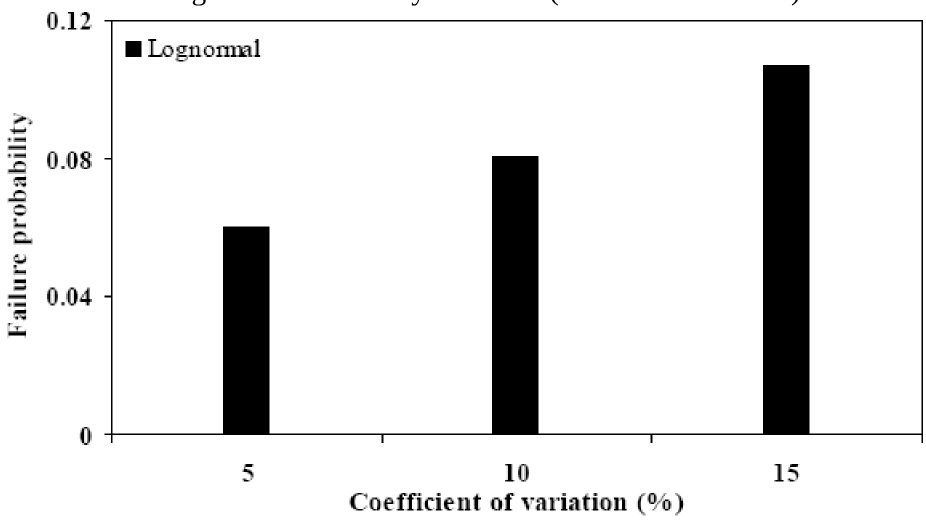

Figure 19. Probability of failure (Lognormal distribution)

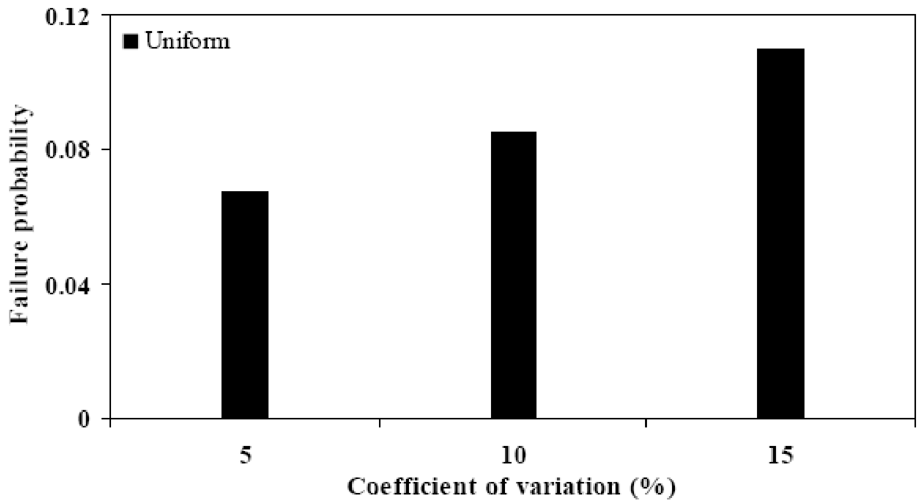

Figure 20. Probability of failure (Uniform distribution)

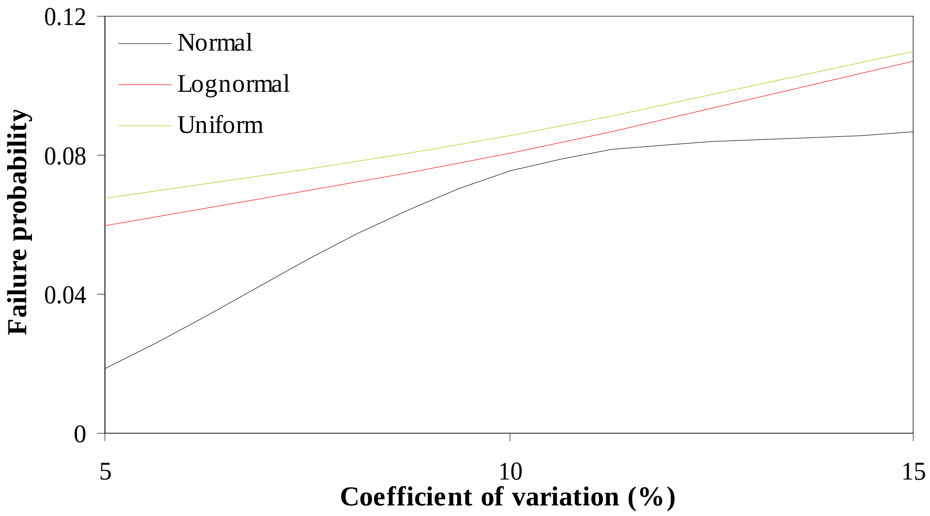

Figure 21. Comparison of the probabilities of failure

5. CONCLUSION

The probabilistic dynamic response has been investigated for a plane frame with random parameters under earthquake base excitation. The nonlinear finite element method with the Monte Carlo simulation have been employed to obtain the standard deviation, mean and maximum of displacement at the top of the plane frame.

The results have been obtained for random structural properties (mass density, modulus of elasticity and section dimensions) for different probability density functions with coefficient of variation changing from 5% to 15% and using the Monte Carlo simulation in which one thousand data have been generated for each case. It is found that the standard deviation of the structural response is increased when the coefficient of variation of the random variables is increased. Comparison of different types of statistical distributions shows that the standard deviation of the structural response is more sensitive to uniform and lognormal distribution than normal one. The time history of maximum values of the top displacement of the frame for various types of “PDFs” and various values of “COVs”, has been compared to the displacement time history with mean values of random variables which are regarded as deterministic values. It can be concluded that the maximum value of the top displacement is increased when the value of “COV” is increased. The maximum difference between the time histories of displacement with deterministic inputs and maximum values of displacement with various “PDFs” and “COVs” is about 200%. Comparison of the time history of maximum values of the top displacement for different types of statistical distributions shows that the dynamic response of the structure is not sensitive to the type of the probability density function. The results for the probabilities of failure show that the failure probability is increased when the coefficient of variation is increased. It is also shown that the failure probability is sensitive to the type of statistical distribution.

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