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STATIC AND DYNAMIC BEHAVIOR OF LATERALLY LOADED SINGLE PILES AND DETERMINATION OF P-Y CURVES

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ABSTRACT

To improve the understanding of soil-pile interaction under horizontal dynamic loads and seismic events, a parametric centrifugal study was undertaken. Flexible piles with pile caps of different masses and instrumented with 20 strain gauges on the length of the pile were used for this purpose. The piles were impacted and the resulting displacement and acceleration for different levels of force were measured. The equation of the movement of a beam equivalent to the pile under dynamic loading has been established and all the terms of this equation was determined using the experimental results. The term of inertia was divided into two parts, one related to the mass of the pile and the other related to the mass of the associated soil. The contribution of each term to the equation at different period (or time of) of vibration was illustrated. Distribution versus time of the displacements and the reactions of the soil at any depth were deduced from the profiles of the bending moments by a double integration and a double derivation respectively. Then the dynamic P-y curves or loops were constructed based on these results. The procedures of experimental tests and P-y curves construction are explained and a comparison between static and dynamic P-y curves is also indicated.

Keywords: Soil-pile interaction; p-y curves; piles; centrifuge; damping; dynamical pile testing.

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1. INTRODUCTION

Lateral loads on piles are developed both by the superstructure and by the wave propagation through the soil. The dynamic loads due to the horizontal movement of the superstructures are mainly generated by wind effects, machine vibrations, impact of vehicles or boats; the loads due to the wave propagation is primarily because of earthquakes. Therefore, the total forces are the result of two types of interaction: an inertial one from the movement of the superstructure and a kinematical one from the soil motion (see Figure 1).

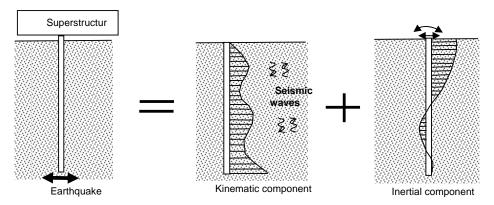


Figure 1. Modeling of the soil-pile-structure interaction under a seismic event

The high degree of the coupling between the modes and the components of the interaction explains the complexity of the dynamic soil-pile-structure interaction. Common methods to solve the problem of this type of interaction consider independently for the two aspects. The kinematic effect of ground and the inertial interaction effect are evaluated separately, and are combined by superimposition to obtain the solution. A fundamental step towards understanding the behavior of pile-soil-structure interaction is believed to be the study of the free vibration response of piles.

Apart from the FEM and BEM methods which often require extensive computing resources, the use of the Winkler model for nonlinear support of a beam is considered a practical method for the design of laterally loaded piles. From the early developments, the modeling has consisted of taking into account the different aspects of pile-soil interaction such as the nonlinear behavior of the soil, the strain rate effects, the phenomena of compaction, damping, gapping and slippage. All these features have been accounted by P-y relationships between the pile and the soil. The consideration of the dynamic aspects in the Winkler model has been made in complementing the P-y curves by rheological elements such as masses and dashpots. The results given by the recent literature clearly show the difficulty in extending the static P-y curves to dynamic P-y curves (loops). The major objective of this research work is to study the inertial interaction of soil-pile systems and to derive static and dynamic P-y curves from experimental data.

Considering the state of research on the topic, experimental approaches are considered most suitable for developing adequate the rheological elements for the soil-pile

interaction response. Some investigators have used impact and ring-down full-scale pile tests to determine the dynamic and static parameters of soil-pile interaction [1-3]. These full-scale tests are very expensive and limited to the specific field conditions. The horizontal Statnamic test is one of the more recently developed full-scale experimental approaches for studying the behavior of piles. From these types of tests, parameters are used for the modeling of dynamic loads caused by earthquakes, ship impacts and wind effects on structures and foundations. In any case, full-scale testing methods do not extensively respond to the lack of knowledge of the physical effects involved in soil-pile behavior. Centrifuge modeling is a less expensive tool and a more flexible alternative experimental technique to field tests. With the similitude laws, the behavior of piles and soils can be correctly modeled in the centrifuge and parametric studies can be performed. The modeling of models approach has proven that the scale effects are negligible and the recorded pile responses are reproducible, Hajialilue-Bonabet al. [4].

2. EXPERIMENTAL TEST SET-UP

Fontainebleau sand which is a uniform silica sand that consists of fine and rounded particles was used in the experiments. Some physical properties of this sand are shown in Table 1.

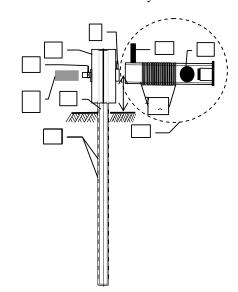
Table 1: Some physical properties of the Fontainebleau sand

D ₅₀ (mm)	$\frac{\gamma_s}{(kN/m^3)}$	$\gamma_{d \text{ min}} (kN/m^3)$	$\gamma_{d \text{ max}} (kN/m^3)$	e _{min.}	e _{max.}	φ	ν
0.2	26.44	13.64	16.83	0.616	0.940	35°	0.2

The ratio between the diameter of the model pile (18mm) and the grain size is 90, which is higher than the value of 30 above which there is no important scale effect on the bearing capacities of foundations [5]. Monotonic and cyclic behavior of this sand is presented by Georgiannou et al. [6]. The method applied for the sand reconstitution was sand pluviation. The average density achieved in this way was 16.05 kN/m³ which gives a relative density of 85 %.

The pile model is 1/40 of prototype and all the tests were carried out in 40g acceleration. The model piles are made with AU4G aluminium hollow pipes. The outside diameter is 18 mm (720 mm in prototype scale) and the thickness is 1.5 mm. They have a total length of 380 mm (15.2 m in prototype scale) for an embedded length of 300 mm *i.e.* 12 m in prototype scale. The stiffness EI converted to prototype scale is equal to 505.4 N.m². The free height H above the soil surface where the impact was applied is equal to 2.2 m, see the Figure 2.

- 1: 20 Strain gauges
- 2: Model pile
- 3: Laser displacement sensor
- 4: Acceleration transducer
- 5: Pile cap
- 6: Force transducer
- 7: Electromagnetic impact device
- 8: Power supply to the ball gun.
- 9: Steel ball
- 10: Diode for evaluation of ball velocity



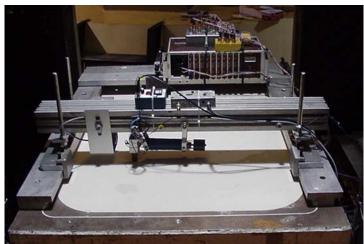


Figure 2. Experimental set-up for dynamic test

A method to characterize the relative stiffness of the pile is to calculate its transfer length l_0 , namely:

$$l_0 = \sqrt[4]{\frac{4 \cdot E_p I_p}{E_S}} \tag{1}$$

 $E_p I_p$ is the stiffness of pile in bending and E_s , is the module of reaction of the soil. The pile is regarded as flexible if $L/l_0 > 3$ (L is the embedded length of the pile) and as rigid if $L/l_0 < 1$ according to Frank [7]. The difficulty of evaluation of l_0 lies in the choice of modulus of soil reaction which varies with the depth and the magnitude of loading.

Remaud [8] conducted a study by interpolation and correlations from a cone penetrometer test in centrifuge (40g), allowing to estimate the module of reaction E_S . He estimated the modulus of reaction equal to 30 MPa at the depth of 4m (the level of the first upper third of the pile). This level corresponds to an area where stress and strains are the most significant. The length of transfer I_0 is therefore of 2.8 m and the ratio L/I_0 is equal to 4.3. The modeled pile is therefore considered as flexible.

In this study the inertial effect on top of the pile was reproduced by adding different pile caps. The pile caps were designed with a small thickness in the direction of the impact in order to decrease the effect of the rocking mode. Four pile caps were manufactured for the parametric studies relating to the mass of the superstructure as shown in Figure 3.

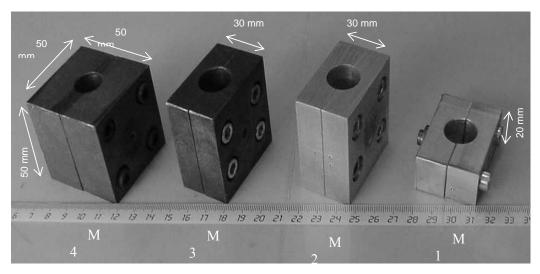


Figure 3. Geometry and dimensions of the pile cap models

The total masses of the pile caps including the instrumentation i.e. force sensor and accelerometer, are given in Table 2.

Table 2: Masses of the pile caps

	M_1	M_2	M_3	M_4
Prototype (kg)	6554	13360	32019	54502

To measure the pile deformations and to compute the bending moments from them, the model piles were instrumented with 20 strain gauges assembled in half-bridge. The gauges were pasted on the outer surface of the piles. The strain gauges were protected by a layer of special material. Without protection, they are likely to deteriorate; particularly, when the pile is driven. The drawback is that the nominal diameter of pile, *i.e.* 18 mm, had increased. The increase in diameter can reach 1.5 mm. This protection did not have any significant effect on pile flexural stiffness because the coating material is very soft and its Young modulus is

very low compared to that of aluminum. Another effect would be on friction difference between aluminum with soil and coating material with soil. Two static tests were performed using two piles, one with coating and another without coating and strain guages. The head force-displacement curve did not show any noticeable differences [9].

In the static tests, 20 pairs (or couples) of gauges were used, whereas in the dynamic tests only 7 pairs of theses gauges were utilized because of channel limitation for transferring and data acquisition. To evaluate the applied force, a piezoelectric sensor was mounted on the pile cap. To determine the displacement, two methods were employed: a direct measurement by an analog laser sensor with integrated amplifier, and a piezoelectric accelerometer fixed on the pile cap which allows determination of the displacement by integration.

The impact force was generated by an innovative device shooting a steel ball against the pile. This device consists of a tubular guide wound by a coil. The coil is fed by a short high driving current pulse from the in-flight capacitors and triggered by a signal generator from the command room. The exit is just near the impact point, less than 1 cm, so that the ball bounces back in the tube. A light slope in the tubular guide enables the gravity to draw the ball back to its initial position, and ready for the next impact. This system is able to repeat impacts in flight either to reproduce the same pulses one by one or to cumulate impacts for other applications. By changing the capacitor tension and the current pulse duration in flight one can vary the amplitude of the impact force. The time duration of the impact is typically t = 0.01 s (prototype scale) which corresponds to a Dirac-type impulse [10].

In the static tests, the pile was the same but the lateral loading of the pile was applied by a hydraulic servo-actuator as shown in the Figure 5a. Instead of the laser displacement transducer, two LVDTs placed at 20 mm and 65 mm above the soil surface (model scale dimensions) measured the pile head deflections and the resulting rotation.

The load is applied in 20 steps of 80 N each.

Pile installation

The piles were driven in 1g with a manual hammering system. A simple device designed at LCPC allows a good repeatability of the pile driving, regardless of the operator (Figure 4). During the pile driving operation, the number of hammer blows per unit length of penetration was recorded in order to check the sand homogeneity. At the end of the driving, a little conical depression around the pile was observed. It was approximately 10mm deep, indicating a densification of the soil around pile. This system could not be performed in 40g. The reason is the strain gauges and coating material would be deteriorated because of high friction effect in 40g. Two other type of installation at 1g which are *board pile* (the pile was fixed in container before sand raining) and *jacked pile* (an hydraulic jack is used to drive the pile in the sand at 1g level) were also performed for comparing the effects. The pile responses for these three methods were discussed by Hajialilue-Bonab et al. [10]. The resonance frequencies were slightly different for three cases. The resonance frequency for driven pile was higher than two other cases and the resonance frequency of jacked pile was higher than board pile. The difference did not exceed 5% in any case.

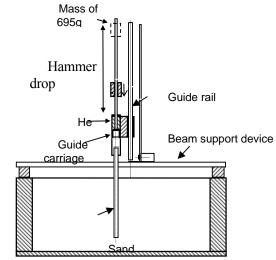
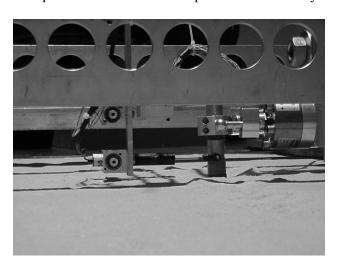


Figure 4. Pile driving by hammering at 1g

3. STATIC P-Y CURVES

A static test was performed in order to obtain P-y curves. It was a displacement control test using a hydraulic actuator. Two LVDTs were measured the pile head displacement at two different point (Figure 5a). A very good quality load-cell enabled to measure the lateral load continually during the test. The strain gauges fixed on the piles give the bending moment profiles during static lateral loading. The study was based on the analysis of the following experimental data. For a given lateral load, the measured distribution of bending moment M, with depth z, was used to derive the soil reaction P, by double derivation, and the displacement profile y, by another double integration. Static P-y curves are illustrated at the Figure 5b. This test was performed in order to compare the static and dynamic P-y curves.



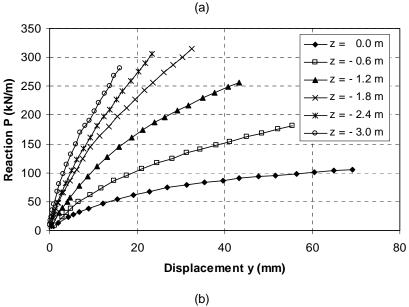


Figure 5. (a) Experimental set-up for static test (b) Static P-y curves

4. EFFECT OF PILE CAP MASS ON THE MAXIMAL RESPONSE

Two types of pile responses can be observed under a lateral impact. For design purposes, the maximum displacement and energies transmitted to the pile are the main parameters. It should be noted that the damping has a very low influence on this transient response. For seismic purposes; however, the vibration behavior of the piles and the damping are important issues to study.

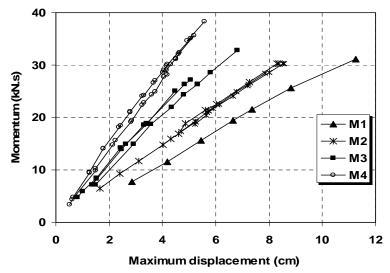


Figure 6. Effect of pile cap mass on maximum response

Experiments and analyses were performed on the transient responses of the pile by applying different amplitudes of impact. Tests were run and repeated for the pile with four different pile caps. The main results are shown in Figure 6. A quasi-linear relationship between the momentum and the maximum displacement has been observed. An important observation is that the slope of the lines depends on the mass of the pile caps.

5. STUDY OF THE PILE VIBRATION

For an overall assessment of the test data an equivalent single-degree-of-freedom (SDOF) system was used to evaluate the dynamic response of the pile foundation system to lateral impact. The dynamic response of a SDOF system is governed by the following differential equation:

$$M \overset{\bullet}{u} + C \overset{\bullet}{u} + K u = F(t) \tag{2}$$

where u, u, and u are the acceleration, velocity, and displacement of the system, respectively. This equation defines the physical state of the system at any instant of time (t). F(t) is the forcing function vector, and M, C, and K are the generalized mass, damping, and stiffness parameters of the SDOF system. The mass and the damping parameters are usually considered constant during a single loading event. The effective mass M is composed of the total mass of the pile cap and a part of the pile mass. In this simplified approach, this part can be limited to the length of the pile above the soil. Equation 1 may also be expressed as follows:

$$F_{inertial} + F_{damping} + F_{static} = F_{impact}$$
 (3)

where:

F inertial = inertial resistance from the effective mass of the pile foundation,

F damping = effective viscous damping resistance,

F static = effective static soil resistance,

 F_{impact} = force measured on pile cap.

Once the damping ratio (ξ) is evaluated, one will be able to evaluate the global stiffness of system (**K**) and damping coefficient(C) for an equivalent SDOF system. The average damping ratio for a response signal was measured from the half power method as noted above in this paper. The stiffness and damping coefficients can be calculated as follows:

$$K = m\omega^2$$
 , $C = 2m\omega\xi$

Where K is the equivalent stiffness of the system, m is the mass and C is the equivalent dashpot constant.

A number of parametric studies were performed on the pile and the results are discussed in this section. One of the issues of interest was the effect of the impact repetition on the pile response. To investigate this, a series of constant impacts were applied and the variations of the equivalent stiffness and damping were computed. The results are displayed in Figure 7 which reveals that the stiffness increases with impact repetition due to soil densification. However, both the stiffness (Figure 7a) and the damping coefficient (Figure 7b) tend to constant values after a number of impacts. The number of impacts where the two coefficients become constant is 25 which could constitute a characteristic number (depending on the amplitude and duration of the impact, i.e. the transmitted energy).

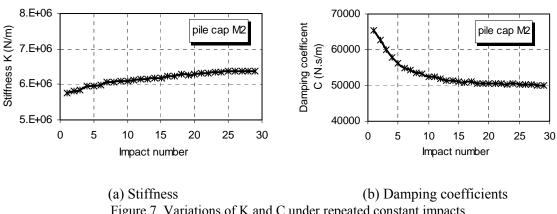


Figure 7. Variations of K and C under repeated constant impacts

6. DYNAMIC P-Y CURVES

The equation of the movement of a beam on visco-elastic supports under dynamic loading can be expressed as:

$$E_{p}I_{p}\frac{\partial^{4}y}{\partial z^{4}} + c_{p}I_{p}\frac{\partial^{5}y}{\partial z^{4}\partial t} + m\frac{\partial^{2}y}{\partial t^{2}} + c\frac{\partial y}{\partial t} - p = 0$$

$$\tag{4}$$

where the first term corresponds to the force resulting from the deformation of the pile, the second term is the internal force of damping of the pile which depends on the frequency, the third term is the inertia produced by the vibration of the pile, m is the mass of the soilpile system which includes the mass of the pile and the so-called added mass of the soil, and the fourth term corresponds to an equivalent force of viscous damping in the soil. The last term lumps the various forms of damping in the soil including the radiation damping of the soil-pile system. There is no analytical solution for this differential equation; however, the experimental approach makes it possible to simplify this equation. Using the strain gauges i.e. deformations along the piles, one can determine the bending moment profile of the pile at each time step. The relation between displacement and the bending moment is expressed as follows:

$$y(z,t) = \int \left(\int \frac{M(z,t)}{E_p I_p} dz \right) dz + C_1 z + C_2$$
 (5)

This equation, which is valid whether the stiffness is variable or constant, can be used to calculate the displacement profile of the pile at each time step of time.

Equation (4) can then be rewritten as follows:

$$\frac{\partial^2 M(z,t)}{\partial z^2} + \frac{c_P}{E_P} \frac{\partial}{\partial t} \left(\frac{\partial^2 M(z,t)}{\partial z^2} \right) + m \frac{\partial^2 y(z,t)}{\partial t^2} = p - c \frac{\partial y(z,t)}{\partial t}$$
 (6)

The left side of this equation represents the force exerted by the pile and the inertia force due to its mass and added soil mass , and the right side is the reaction of the soil which has a component related to the pressure exerted by the deformation of the ground and a component which is produced by the damping of the soil. In this equation, the bending moment profile in each time step is known. Then by double integration of the bending moment one can derive the displacement time histories y(z, t), and by and double differentiation one can compute the first and second terms in the above equation. The remaining terms in this equation can be determined by differentiation of the displacements with respect to time. The procedures of double integration and differentiation with respect to depth in each time step are explained in the following section.

6.1 Procedure of double integration

For each time step of, a polynomial function was fitted to the variation of the experimental bending moments using the method of least squares. It was observed that polynomials of odd degree usually gave more satisfactory results than those of even degrees. For the constants of integration, the following two conditions were imposed:

- 1. The displacement at the point of impact (z = 2.2 m) was obtained by the laser incremental position sensor. It was found out that the displacement of the pile must be measured as close as possible to the surface of the soil for better accuracy.
 - 2. The displacement at pile tip was considered zero.

6.2 Procedure of double differentiation

The profiles of the experimental bending moments were analyzed with the software SLIVALIC5 [11] to compute the profiles of lateral pressure. Initially, the software smoothes the gathered data by quintic splines, and then, it carries out two successive derivations. A problem is encountered on the pile head and on pile tip, *i.e.* at the boundaries of the interval of smoothing, because for these two end points there is not sufficient information. The calculated spline is third degree in these points, and double differentiation has to be carried out on a line at the ends of the profiles. To overcome this problem, the necessary data were added at the head of the pile. In the case of static tests, three points were added which correspond to the bending moments at z = +0.6m, z = +1.2m and z = +1.8m, calculated from

the lateral force applied to z = +2.2m multiply by the distance from the load. In the case of dynamic tests where only 7 gauges were used, two points were added corresponding to the bending moments at z = +0.6m, z = +1.8m, calculated from the estimated force at z = +2.2m. Under free vibrations, there is no external force and the pile vibrates under the inertial effect. It can be assumed that this force is concentrated at the centre of gravity of the part of the pile above the soil. It is possible to calculate this force, either by multiplication of the mass and acceleration or by the division of the bending moment obtained by the first gauge of deformation (located at z = -0.6 m) by the distance between this gauge and the center of gravity (L= 2.8 m). The inertia forces at various times of vibration were evaluated by the two methods; a very good agreement was observed between the two results.

6.3 Evaluation of pile internal damping

To evaluate the internal damping in the pile, the pile was fixed on a support and set into free vibration, and the damping ratio (ξ) was calculated by the logarithmic decrement method. The average value obtained for damping ratio (ξ) was equal to 0.9%.

6.4 Identification of the terms of the equation of motion

The intention of this section is to evaluate the contribution of each parameter in the equation of motion (3). For a given test and at a depth z = -1.8 m, which is close to the maximum reaction, different terms of the equation were calculated. Figure 8 shows the time variations of the various terms for a pile with pile cap M3 and with natural frequency of 2.3 Hz ($\omega = 14.45 \text{ rad/s}$). The other parameters in the calculation are as follows:

 $E_P = 7.8 \times 10^{10} \text{ Pa (N/m}^2)$

 $M_p = 530 \text{ kg/m}$ (pile including wire of connections and coating)

 $C_P = 2.M_P.\omega.\xi = 2 \times 530 \times 14.45 \times 0.009 = 138 (N.s/m)/m$

The second term in the equation which corresponds to internal damping of the pile is given by:

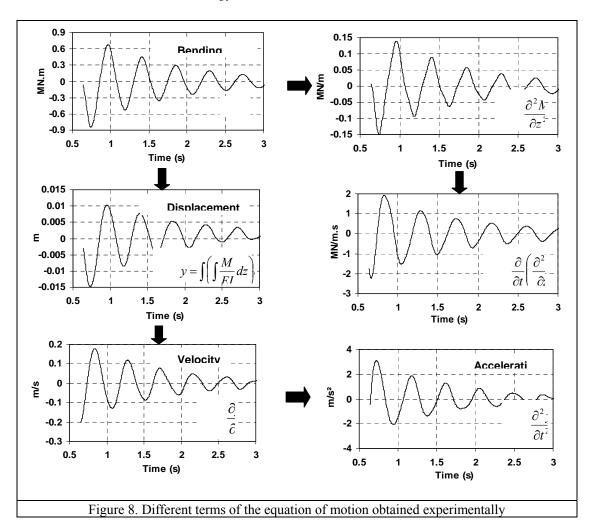
$$\frac{138}{7.8 \times 10^{10}} \frac{\partial}{\partial t} \left(\frac{d^2 M}{dz^2} \right) \quad \text{or} \quad 1.77 \times 10^{-6} \frac{\partial}{\partial t} \left(\frac{d^2 M}{dz^2} \right)$$

The maximum value of this term is equal to 3.5×10^{-4} N/m. It can be observed that the value of internal damping compared to other terms in the equation is negligible. The third term in the equation which corresponds to the inertia force can be divided into two parts, one related to the mass of the pile and the other related to the added soil mass. As the term of inertia is in phase with that obtained by the double differentiation, the importance of the effect of pile mass can be compared with that obtained by double differentiation in each time step. For example, at the time of the first maximum of the signal (T= 0.75 s), the value given by the double differentiation (first term) is equal to:

$$\frac{\partial^2 M}{\partial z^2}_{(t=0.75s)} = 1.4 \times 10^5 \, \text{N/m},$$

However, the term of the inertial force of the pile mass in the same time step is:

$$m_p \frac{\partial^2 y}{\partial t^2}_{(t=0.75s)} = 530 \text{ x } 3 = 1590 \text{ N/m},$$



The ratio between these terms is equal to $1590/1.4 \times 10^5 = 1.1\%$. The inertial force of the pile is just 1% of the force represented by the term resulting from the double differentiation, which is practically negligible. Therefore, the equation of motion (3) can be simplified as follows:

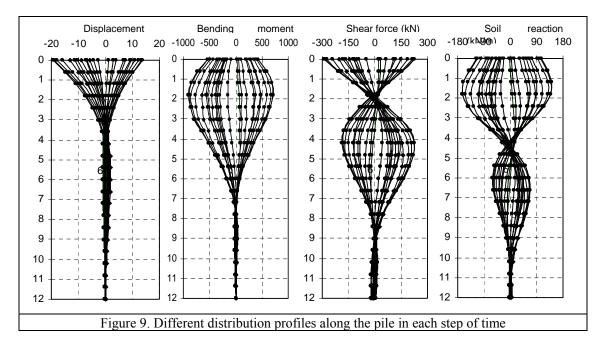
$$\frac{\partial^2 M(z,t)}{\partial z^2} = p - c_S \frac{\partial y(z,t)}{\partial t} - m_S \frac{\partial^2 y(z,t)}{\partial t^2}$$
 (7)

In which c_S is the equivalent damping ratio of the soil and m_s is the equivalent added soil

mass. It thus appears that the double differentiation of the profile of the bending moments gives the dynamic reaction of the ground in which, there is the term of the reaction due to the deformation of soil, the effect of damping and the effect of inertial force due to the soil.

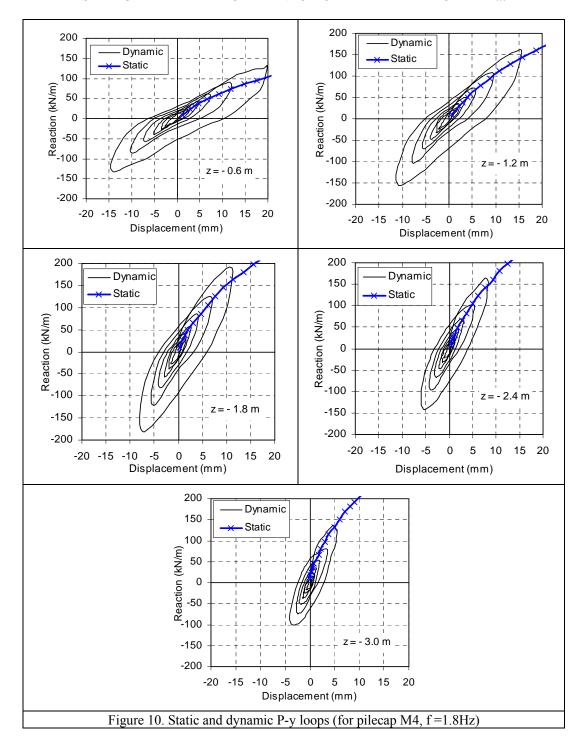
6.5 Dynamic p-y curves

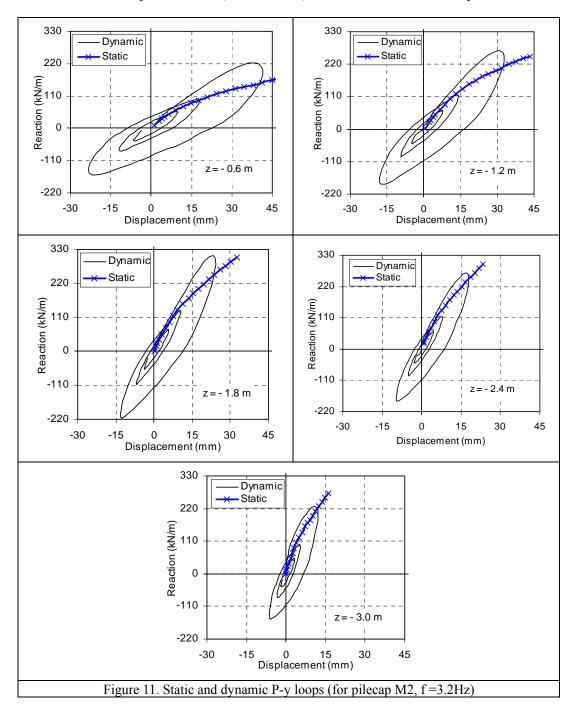
With the procedure of double integration and double differentiation, the displacement and the dynamic reaction of the pile, at various depths and different time steps can be calculated. In static tests, the relations between the reaction of the soil and displacement are called "P-y curves". In dynamic tests, on the other hands, these relations are presented in the form of series of hysteretic loops. The profiles of the displacements, the experimental bending moments, the shear force and the dynamic reactions of the soil are represented in Figure 9.



The obtained dynamic P-y loops for tests with pilecap M4 (frequency = 1.8Hz) and with pilecap M2 (frequence = 3.2 Hz) are illustrated in Figure 10 and 11 respectively at various depths until z = -3.0 m and beyond this depth the displacement of the pile is almost negligible and P-y curves can be assumed linear for small displacement.

The following are a number of observations from these results. First, we observe the increase of the stiffness with depth which is reflected by an increase in the slope of the P-y curves. This was also observed in the static tests. It is also observed that as the amplitude of the deformation is reduced, the nonlinear behavior of the soil is reduced correspondingly. The same general observations were obtained for the two other pile caps. An important result of these computations is the form of the P-y loops. These curves can be used in calibrating/validating simple models of dynamic springs commonly used in the numerical analysis based on the Winkler assumption.





7 CONCLUSION

This paper presented the results of a series of centrifuge tests for the study of the static and dynamic behavior of piles under horizontal impact loads. The paper also presented the details of the instrumentation and the numerical methods used for interpreting the collected data

A SDOF model was used for a basic interpretation of the free vibration response. The study showed that the cumulative constant impacts increase the stiffness and decrease the damping coefficient due to the soil densification. This variation becomes negligible after a characteristic number of impacts.

Increasing the amplitude of the impacts results in a reduction of the stiffness of the pile-soil system due to the nonlinear behavior of the soil.

The analysis of the displacement and strain gauge signals has provided an insight into the variation of the damping of the free vibration response; namely, it increases strongly with the maximum pile head displacement.

Moreover, a simplified differential equation of motion of a pile in vibration was established. Different terms of this equation were evaluated using the experimental data. It was observed that the value of the pile internal damping compared to the other terms in the equation was negligible. Similarly, the effect of the pile inertial force was found negligible compared to the other terms.

Special schemes for double integration and double differentiation were used to derive the variation with depth of the pile displacement and soil pressure at all time steps using the measured bending moment profiles. The results were then used to establish the dynamic P-y curves or hysteresis loops for different depths. These curves highlight the hysteretic characteristics of the soil and are believed to be very useful for the characterization of the nonlinear springs and dashpots in numerical analyses of Winkler type.

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