OPTIMAL DESIGN OF DOUBLE LAYER BARREL VAULTS USING IMPROVED MAGNETIC CHARGED SYSTEM SEARCH

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ABSTRACT

This paper presents an improved magnetic charged system search (IMCSS) and open application programming interface (OAPI) for optimization of double layer barrel vaults. In IMCSS algorithm, magnetic charged system search (MCSS) and an improved scheme of harmony search (IHS) are utilized and some of the most important parameters in the convergence rate of HS scheme are improved to achieve a good convergence and good solutions especially in final iterations. The OAPI is also utilized for the process of structural analysis, to link the analysis software with the IMCSS algorithm through the programming language. The results demonstrate the efficiency of OAPI as a powerful interface tool for analysis of large-scale structures such as double layer barrel vaults and also the robustness of the IMCSS as an optimization algorithm in fast convergence and achieving the optimal results.

1. INTRODUCTION

Barrel vault is one of the oldest type of space structure used since antiquity. This type of structures has lightweight and is cost effective structures which are used to cover large areas such as exhibition halls, stadium, markets, and concert halls. The earlier types of braced barrel vaults were constructed as single-layer structures. Nowadays, double-layer systems are utilized for covering large spans [1]. Double layer barrel vaults are generally indeterminate from static point of view. In such systems, due to the rigidity, the risk of

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instability can almost be eliminated. The use of this type of barrel vaults enhances the stiffness of the vault structure and provides structural systems of great potential, capable of having spans in excess of 100 m.

In the last decade, structural optimization has become one of the most interesting branches of structural engineering and many meta-heuristic algorithms have been developed and applied for optimization of truss structures. Every meta-heuristic method consists of a group of search agents that explore the feasible region based on both randomization and some specified rules. The rules are usually inspired by natural phenomena laws. Recently, a new meta-heuristic algorithm has been proposed, by Kaveh and Talatahari which is called Charged System Search (CSS) [2]. The CSS algorithm is based on the Coulomb and Gauss laws from physics and the governing laws of motion from the Newtonian mechanics. This algorithm can be considered as a multi-agent approach, where each agent is a Charged Particle (CP). Each CP is considered as a charged sphere with a specified radius, having a uniform volume charge density which can insert an electric force to the other CPs. After a while the CSS algorithm modified to Magnetic Charged System Search (MCSS) by Kaveh et al. [3]. This algorithm utilizes the governing laws for magnetic forces and includes magnetic forces in addition to electrical forces, so the movements of CPs due to the total force are determined using Newtonian mechanical laws.

Although many studies have been performed on optimization of truss structures using above algorithms, however, there are a few studies on optimization of double layer barrel vaults. For optimal design of double layer barrel vaults Kaveh and Eftekhar presented IBB-BC algorithm [4]. In another study optimal design of some single layer barrel vaults and a double arch barrel vault were presented by Kaveh et al. [5]. In several studies, a practical model of a braced barrel vault has been optimized by Hasançebi and Çarbaş using ant colony search method [6], Hasançebi et al. employed a reformulation of the Ant Colony [7] and Hasançebi and Kazemzadeh Azad utilized a reformulations of Big Bang-Big Crunch algorithm [8, 9].

In this paper, an improved magnetic charged system search (IMCSS) is proposed for optimal design of double layer barrel vaults. In this algorithm, in the process of position correction of CPs, an improved harmony search scheme is utilized and some of the most effective parameters in the convergence rate of the algorithm are improved to achieve better convergence and optimal results.

The present paper is organized as follows: in Section 2, the statement of the optimization design problem is presented and mathematically formulated. An introduction to CSS and MCSS algorithms are prepared in Section 3. In Section 4, IMCSS algorithm is introduced and also its discrete version is described. In Section 5, Open Application Programming Interface (OAPI) is presented as a tool for structural analysis. Section 6 contains two illustrative examples with continuous and discrete variables to confirm the capability of the new algorithm, and finally in Section 7, some concluding remarks are provided.

2. PROBLEM STATEMENT

The aim of the size optimization of trusses and space structures is to find the optimum

values for cross-sectional areas A_i of members, in order to minimize the structural weight W, and simultaneously satisfying the imposed constraints on the problem. Thus, the optimal design problem can be expressed as:

$$
\begin{aligned}
&\text{Find} \quad\quad X = [x_1, x_2, x_3, \ldots, x_n] \\
&\text{to minimize } \; Mer(X) = f_{penalty}(X) \times W(X)
\end{aligned}
$$

$$
\begin{aligned}
\text{subject to} \quad & \delta_{min} < \delta_i < \delta_{max} \quad & i = 1, 2, \ldots, nn \\
& \sigma_{min} < \sigma_i < \sigma_{max} \quad & i = 1, 2, \ldots, nm \\
& \sigma_i^b < \sigma_i < 0 \quad & i = 1, 2, \ldots, ns
\end{aligned}
\tag{1}
$$

where X is the vector containing the design variables; for the discrete optimum design problem, the variables x_i are selected from an allowable set of discrete values; n is the number of member groups; $Mer(X)$ is the merit function; $W(X)$ is the cost function, which is taken as the weight of the structure; $f_{penalty}(X)$ is the penalty function which results from the violations of the constraints corresponding to the response of the structure; nn is the number of nodes; nm is the number of members forming the structure; ns is the number of compression elements; σ_i and δ_i are the element stress and nodal displacements, respectively; min and max mean the lower and upper bounds of constraints, respectively. σ_i^b is the allowable buckling stress in member i when it is in compression.

The cost function can be expressed as:

$$
W(X) = \sum_{i=1}^{nm} \rho_i \cdot x_i \cdot L_i
\tag{2}
$$

where ρ_i is the material density of member i; L_i is the length of member i; and x_i is the cross-sectional area of member i as the design variable.

The penalty function can be defined as [10]:

$$
f_{penalty}(X) = \left(1 + \varepsilon_1 \cdot \sum_{i=1}^{np} \left(\phi_{\delta(i)}^k + \phi_{\sigma(i)}^k + \phi_{\sigma b(i)}^k\right)\right)^{\varepsilon_2},
\tag{3}
$$

where np is the number of multiple loadings. In this paper ε_1 is taken as unity and ε_2 is set to 1.5 in the early iterations of the search process, but gradually it is increased to 3 [10]. ϕ_δ^k is the summation of nodal displacement penalties, ϕ_σ^k is the summation of stress penalties and, $\phi_{\sigma b}^k$ is the summation of buckling stress penalties for kth charged particle which mathematically expressed as:

$$\phi_\delta = \sum_{i=1}^{nn} \max\left(\left| \frac{\delta_i}{\overline{\delta}_i} \right| - 1,0 \right),$$ (4)

$$\phi_\sigma = \sum_{i=1}^{nm} \max\left(\left| \frac{\sigma_i}{\overline{\sigma}_i} \right| - 1,0 \right),$$ (5)

$$\phi_{\sigma b} = \sum_{i=1}^{nm} \max\left(\frac{\lambda_i}{\overline{\lambda}_i} - 1,0 \right),$$ (6)

where δ_i, $\overline{\delta}_i$ are the displacement of the joints and the allowable displacement, respectively, σ_i and $\overline{\sigma}_i$ are the stress and allowable stress in member i, respectively, and λ_i, $\overline{\lambda}_i$ are the slenderness ratio and the allowable slenderness ratio of member i, respectively [11].

The allowable tensile and compressive stresses are used according to the AISC-ASD code [12], as follows:

$$\begin{cases} \sigma_i^+ = 0.6 F_y & for\ \sigma_i \geq 0 \\ \sigma_i^- & for\ \sigma_i < 0 \end{cases}$$ (7)

where σ_i^- is calculated according to the slenderness ratio

$$\sigma_i^- = \begin{cases} \left[\left(1 - \frac{\lambda_i^2}{2C_c^2}\right) F_y \right] \Big/ \left(\frac{5}{3} + \frac{3\lambda_i}{8C_c} - \frac{\lambda_i^3}{8C_c^3} \right) & for\ \lambda_i < C_c \\ \dfrac{12\pi^2 E}{23\lambda_i^2} & for\ \lambda_i \geq C_c \end{cases}$$ (8)

where E is the modulus of elasticity, F_y is the yield stress of steel, C_c is the slenderness ratio (λ_i) dividing the elastic and inelastic buckling regions ($C_c = \sqrt{2\pi^2 E / F_y}$), λ_i is the slenderness ratio ($\lambda_i = kL_i/r_i$), k is the effective length factor, L_i is the member length and r_i is the radius of gyration. The radius of gyration (r_i) can be expressed in terms of cross-sectional areas, i.e., $r_i = aA_i^b$ [13]. Here, a and b are the constants depending on the types of sections adopted for the members such as pipes, angles, and tees. In this paper, pipe sections (a = 0.4993 and b = 0.6777) were adopted for bars.

According to AISC-ASD the allowable slenderness ratio can be formulated as follows:

$$\overline{\lambda}_i = \begin{cases} \dfrac{k_i l_i}{r_i} \leq 300 & \text{for tension members} \\[3mm] \dfrac{k_i l_i}{r_i} \leq 200 & \text{for compression members} \end{cases} \tag{9}$$

where k_i is the effective length factor for the member i (k_i =1 for all truss members), r_i is the minimum radius of gyration for the member i and l_i is the length of member i.

3. INTRODUCTION TO CSS AND MCSS ALGORITHMS

The CSS algorithm was proposed by Kaveh and Talathari [2] for optimization. This meta-heuristic optimization algorithm takes its inspiration from the physic laws governing a group of CPs. These charge particles are sources of the electric fields, and each CP can exert electric force on other CPs. The movement of each CP due to the electric force can be determined using the Newtonian mechanic laws.

In physics, it has been shown that when a charged particle moves, produces a magnetic field. This magnetic field can exert a magnetic force on other CPs. Thus, for considering this force in addition to electric force, the CSS algorithm is modified to MCSS algorithm by Kaveh et al. [3] The MCSS algorithm can be summarized as follows:

Level 1. Initialization

Step 1: Initialization. Initialize CSS algorithm parameters; the initial positions of CPs are determined randomly in the search space.

$$x_{i,j}^{(0)} = x_{i,min} + \text{rand} \cdot (x_{i.max} - x_{i,min}), \ i = 1, 2, ..., n. \tag{10}$$

where $x_{i,j}^{(0)}$ determines the initial value of the ith variable for the jth CP; $x_{i,min}$ and $x_{i,max}$ are the minimum and the maximum allowable values for the ith variable; *rand* is a random number in the interval [0,1]; and n is the number of variables. The initial velocities of charged particles are zero

$$v_{i,j}^{(0)} = 0, \ i = 1, 2, ..., n. \tag{11}$$

The magnitude of the charge is defined as follows:

$$q_i = \frac{\text{fit}(i) - \text{fitworst}}{\text{fitbest} - \text{fitworst}}, \ i = 1, 2, ..., N. \tag{12}$$

where *fitbest* and *fitworst* are the best and the worst fitness of all particles; *fit(i)* represents

the fitness of the agent i; and N is the total number of CPs. The separation distance r_{ij} between two charged particles is defined as:

$$r_{ij} = \frac{\left\| X_i - X_j \right\|}{\left\| (X_i + X_j)/2 - X_{best} \right\| + \varepsilon}, \tag{13}$$

where X_i and X_j are the positions of the ith and jth CPs, X_{best} is the position of the best current CP, and ε is a small positive number to avoid singularities.

Step 2. CP ranking. Evaluate the values of the fitness function for the CPs, compare with each other and sort them in an increasing order.

Step 3. CM creation. Store CMS number of the first CPs and their related values of the objective function in the CM (based on CMS size).

Level 2: Search

Step 1: Force determination.

The probability of the attraction of the ith CP by the jth CP is expressed as:

$$p_{ij} = \begin{cases} 1 & \dfrac{\text{fit(i)} - \text{fitbest}}{\text{fit(j)} - \text{fit(i)}} > rand \ \ or \ \ \text{fit(j)} > \text{fit(i)}, \\ 0 & \text{else.} \end{cases} \tag{14}$$

where $rand$ is a random number uniformly distributed in the range of (0,1). The resultant electrical force $F_{E,j}$ acting on the jth CP can be calculated as follow:

$$F_{E,j} = q_j \cdot \sum_{i, i \neq j} \left(\frac{q_i}{a^3} r_{ij} \cdot i_1 + \frac{q_i}{r_{ij}^2} i_2 \right) \cdot p_{ij} (X_i - X_j), \begin{cases} i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a, \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a, \\ j = 1, 2, ..., N. \end{cases} \tag{15}$$

The probability of the magnetic influence (attracting or repelling) of the ith wire (CP) on the jth CP is expressed as:

$$pm_{ij} = \begin{cases} 1 & \text{fit(j)} > \text{fit(i)}, \\ 0 & \text{else.} \end{cases} \tag{16}$$

where $fit(i)$ and $fit(j)$ are the objective values of the ith and jth CP, respectively. This probability determines that only a good CP can affect a bad CP by the magnetic force.

The magnetic force $F_{B,ji}$ acting on the jth CP due to the magnetic field produced by the ith virtual wire (ith CP) can be expressed as:

$$F_{B,j} = q_j \cdot \sum_{i,i\neq j} \left(\frac{I_i}{R^2} r_{ij} \cdot z_1 + \frac{I_i}{r_{ij}} \cdot z_2 \right) \cdot pm_{ij} (X_i - X_j), \begin{cases} z_1 = 1, z_2 = 0 \Leftrightarrow r_{ij} < R, \\ z_1 = 0, z_2 = 1 \Leftrightarrow r_{ij} \geq R, \\ j = 1, 2, ..., N. \end{cases} \tag{17}$$

where q_i is the charge of the ith CP, R is the radius of the virtual wires, I_i is the average electric current in each wire, and pm_{ij} is the probability of the magnetic influence (attracting or repelling) of the ith wire (CP) on the jth CP.

The average electric current in each wire I_i can be expressed as:

$$(I_{avg})_{ik} = sign(df_{i,k}) \times \frac{|df_{i,k}| - df_{min,k}}{df_{max,k} - df_{min,k}}, \tag{18}$$

$$df_{i,k} = fit_k(i) - fit_{k-1}(i), \tag{19}$$

where $df_{i,k}$ is the variation of the objective function of the ith CP in the kth movement (iteration). Here, $fit_k(i)$ and $fit_{k-1}(i)$ are the values of the objective function of the ith CP at the start of the kth and k-1th iterations, respectively. By considering absolute values of $df_{i,k}$ for all of the current CPs, $df_{max,k}$ and $df_{min,k}$ will be the maximum and minimum values among these absolute values of df, respectively.

A modification can be considered to avoid trapping in part of search space (local optima) because of attractive electrical force in CSS algorithm.

$$F = p_r \times F_E + F_B, \tag{20}$$

where p_r is the probability that an electrical force is a repelling force which is defined as

$$p_r = \begin{cases} 1 & \text{r}and > 0.1 \cdot \left(1 - \frac{\text{iter}}{\text{iter}_{max}}\right), \\ -1 & \text{else.} \end{cases} \tag{21}$$

where $rand$ is a random number uniformly distributed in the range of $(0,1)$, $iter$ is the current number of iterations, and $iter_{max}$ is the maximum number of iterations.

Step 2: Solution construction. Move each CP to the new position and calculate the new velocity as follows:

$$X_{j,new} = rand_{j1} \cdot k_a \cdot \frac{F_j}{m_j} \cdot \Delta t^2 + rand_{j2} \cdot k_v \cdot V_{j,old} \cdot \Delta t + X_{j,old}, \tag{22}$$

$$V_{j,new} = \frac{X_{j,new} - X_{j,old}}{\Delta t}, \tag{23}$$

where $rand_{j1}$ and $rand_{j2}$ are two random numbers uniformly distributed in the range of (0,1). Here, m_j is the mass of the jth CP which is equal to q_j. Δt is the time step and is set to unity. k_a is the acceleration coefficient; k_v is the velocity coefficient to control the influence of the previous velocity. In this paper k_a and k_v are considered as

$$
\begin{aligned}
k_a &= c_1 \cdot \left(1 + \frac{iter}{iter_{max}}\right), \\
k_v &= c_2 \cdot \left(1 - \frac{iter}{iter_{max}}\right),
\end{aligned}
\tag{24}
$$

where c_1 and c_2 are two constants to control the exploitation and exploration of the algorithm, respectively.

Step 3. CP position correction. If a CP violates its allowable boundary, its position is corrected using harmony search-based approach. In this paper, the position correction has been improved and expressed in the next section.

Step 4: CP ranking. Evaluate and compare the values of the fitness function for the new CPs, and sort them in an increasing order.

Step 5: CM updating. If some new CP vectors are better than the worst ones in the CM (means better objective function), include the better vectors in the CM and exclude the worst ones from the CM.

Level 3: **Controlling the terminating criterion**.

Repeat the search level steps until a terminating criterion is satisfied. The terminating criterion is considered to be the number of iterations.


4. IMPROVED MAGNETIC CHARGED SYSTEM SEARCH

In the process of position correction of CPs using harmony search-based approach (Level 2 - Step 3), The *CMCR* and *PAR* parameters help the algorithm to find globally and locally improved solutions, respectively. *PAR* and *bw* in HS scheme are very important parameters in fine-tuning of optimized solution vectors, and can be potentially useful in adjusting convergence rate of algorithm to optimal solution [14]. The traditional HS scheme uses fixed value for both *PAR* and *bw*. Small *PAR* values with large *bw* values can led to poor performance of the algorithm and considerable increase in iterations needed to find optimum solution. Although small *bw* values in final generations increase the fine-tuning of solution vectors, but in early iterations *bw* must take a bigger value to enforce the algorithm to increase the diversity of solution vectors. Furthermore large *PAR* values with small *bw*

values usually led to the improvement of best solutions in final iterations which algorithm converged to optimal solution vector. To improve the performance of the HS scheme and eliminate the drawbacks lies with fixed values of *PAR* and *bw*, IMCSS algorithm uses IHS scheme with varied *PAR* and *bw* in correction step (Step 3). *PAR* and *bw* change dynamically with iteration number as shown in figure 1 and expressed as follow [14]:
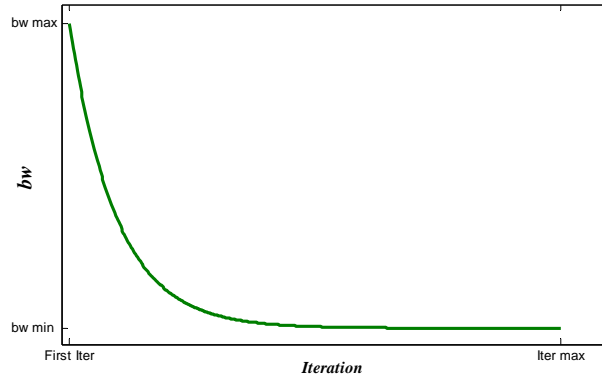
(a)
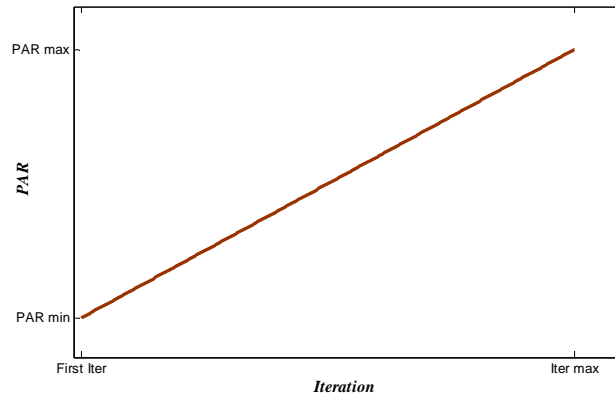
(b)

Figure 1. Variation of (a) *bw* and (b) *PAR* versus iteration number.

$$PAR(iter) = PAR_{min} + \frac{(PAR_{max} - PAR_{min})}{iter_{max}} \cdot iter \qquad (25)$$

And

$$bw(iter) = bw_{max} \exp(c \cdot iter), \qquad (26)$$

$$c = \frac{Ln\left(bw_{min}\big/bw_{max}\right)}{iter_{max}}, \tag{27}$$

where *bw(iter)* is the bandwidth for each iteration, bw_{min} and bw_{max} are the minimum and maximum bandwidth, respectively.

4.1 A discrete IMCSS

IMCSS algorithm is also applied to optimal design problem with discrete variables. One way to solve discrete problems using a continuous algorithm is to utilize a rounding function which changes the magnitude of a result to the nearest discrete value [15], as follows:

$$X_{j,new} = Fix\left(rand_{j1} \cdot k_a \cdot \frac{F_j}{m_j} \cdot \Delta t^2 + rand_{j2} \cdot k_v \cdot V_{j,old} \cdot \Delta t + X_{j,old}\right), \tag{28}$$

where *Fix(X)* is a function which rounds each elements of vector *X* to the nearest permissible discrete value. Using this position updating formula, the agents will be permitted to select discrete values.

5. OPEN APPLICATION PROGRAMMING INTERFACE

The Open Application Programming Interface (OAPI) is a powerful tool that allows users to automate many of the processes required to build, analyze and design models and to obtain customized analysis and design results. It also allows users to link SAP2000 with third-party software, providing a path for two-way exchange of model information with other programs. Most major programming languages can be used to access SAP2000 through the OAPI [16].

In this paper the language of technical computing MATLAB is used to access SAP2000 through the OAPI and MATLAB is also used for the process of optimization via IMCSS.

6. NUMERICAL EXAMPLES

In this section, two double layer barrel vaults are optimized via IMCSS algorithm to demonstrate the efficiency of this algorithm. For all of examples a population of 100 charged particles is used and the value of *CMCR* is set to 0.95. The values of PAR_{min} and PAR_{max} in IMCSS algorithm are set to 0.3 and 0.99, respectively.

The first Example is a 384-bar double layer barrel vault which is an optimization problem with continuous variables, and for this structure both process of optimization and analysis are performed in MATLAB.

The second problem is a 693-bar braced barrel which is a discrete optimum design problem, the variables are selected from an allowable set of steel pipe sections taken from AISC-LRFD code which is shown in Table 1 [17]. For analysis of this structure SAP2000 OAPI is used and the optimization process is performed in MATLAB.

Table 1: The allowable steel pipe sections taken from AISC-LRFD code [17]

	Type	Nominal diameter (in)	Weight per ft (lb)	Area (in2)	I (in4)	Gyration radius (in)	J (in4)
1	ST	1/2	0.85	0.25	0.017	0.261	0.082
2	EST	1/2	1.09	0.32	0.2	0.25	0.096
3	ST	3/4	1.13	0.333	0.037	0.334	0.142
4	EST	3/4	1.47	0.433	0.045	0.321	0.17
5	ST	1	1.68	0.494	0.087	0.421	0.266
6	EST	1	2.17	0.639	0.106	0.407	0.322
7	ST	1 1/4	2.27	0.669	0.195	0.54	0.47
8	ST	1 1/2	2.72	0.799	0.31	0.623	0.652
9	EST	1 1/4	3	0.881	0.242	0.524	0.582
10	EST	1 1/2	3.63	1.07	0.666	0.787	1.122
11	ST	2	2.65	1.07	0.391	0.605	0.824
12	EST	2	5.02	1.48	0.868	0.766	1.462
13	ST	2 1/2	5.79	1.7	1.53	0.947	2.12
14	ST	3	7.58	2.23	3.02	1.16	3.44
15	EST	2 1/2	7.66	2.25	1.92	0.924	2.68
16	DEST	2	9.03	2.66	1.31	0.703	2.2
17	ST	3 1/2	9.11	2.68	4.79	1.34	4.78
18	EST	3	10.25	3.02	3.89	1.14	4.46
19	ST	4	10.79	3.17	7.23	1.51	6.42
20	EST	3 1/2	12.5	3.68	6.28	1.31	6.28
21	DEST	2 1/2	13.69	4.03	2.87	0.844	4
22	EST	5	14.62	4.3	15.2	1.88	10.9
23	EST	4	14.98	4.41	9.61	1.48	8.54
24	DEST	3	18.58	5.47	5.99	1.05	6.84
25	ST	6	18.97	5.58	28.1	2.25	17
26	EST	5	20.78	6.11	20.7	1.84	14.86
27	DEST	4	27.54	8.1	15.3	1.37	13.58
28	ST	8	28.55	8.4	72.5	2.94	33.6
29	EST	6	28.57	8.4	40.5	2.19	24.4
30	DEST	5	38.59	11.3	33.6	1.72	24.2
31	ST	10	40.48	11.9	161	3.67	59.8
32	EST	8	43.39	12.8	106	2.88	49
33	ST	12	49.56	14.6	279	4.38	87.6
34	DEST	6	53.16	15.6	66.3	2.06	40
35	EST	10	54.74	16.1	212	3.63	78.8
36	EST	12	65.42	19.2	362	4.33	113.4
37	DEST	8	72.42	21.3	162	2.76	75.2
ST=standard weight, EST=extra strong, DEST=double-extra strong							

6.1 A 384-bar double layer barrel vault

The 384-bar double layer barrel vault shown in figure 2(a), was first optimized by Kaveh and Eftekhar using IBB-BC [4]. The material density is 0.288 lb/in^3 (7971.810 kg /m^3) and the modulus of elasticity is 30450 ksi（210000 MPa）. In this example, the yield stress of steel is taken as 58 ksi (400 MPa). The nodes are subjected to the displacement limits of ±0.1969 in (5 mm) in x, y and z directions.
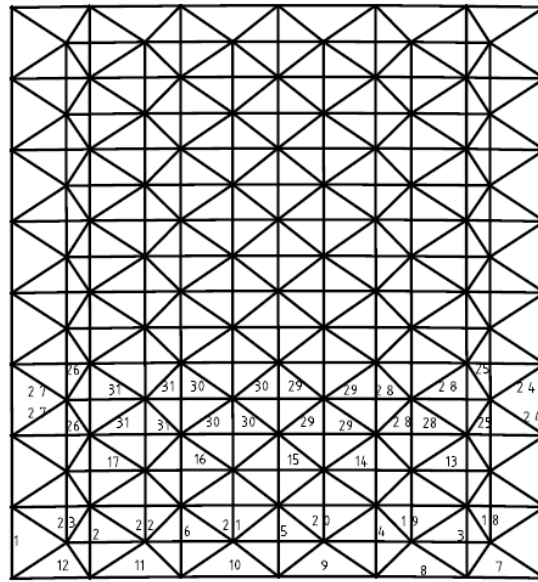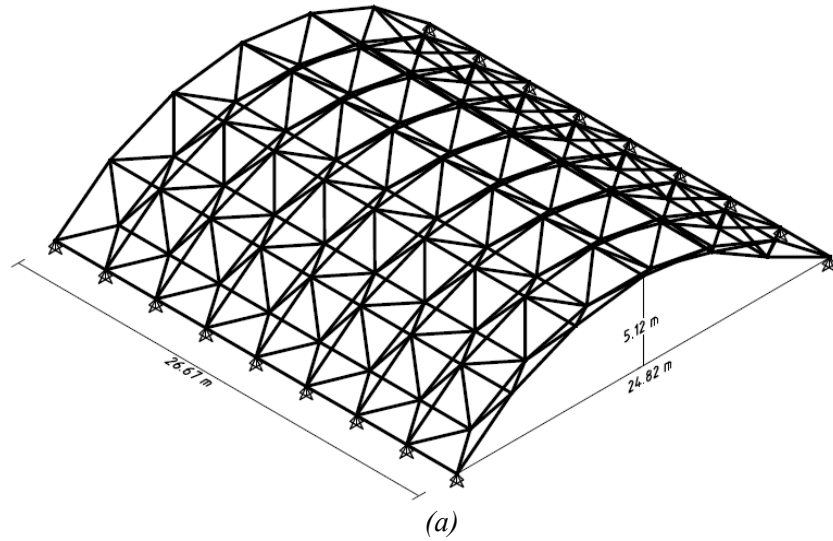
(a)

(b)

Figure 2. The 384-bar double layer barrel vault, (a) 3-dimensional view [1] and (b) Member groups in top view.

Table 2: Optimal design comparison for the 384-bar bar double layer barrel vault (in2) for Case 1

Element group	Kaveh and Eftekhar [4]			Present Work	
	HS	BB-BC	IBB-BC	MCSS	IMCSS
1	0.786	0.817	0.775	0.8316	0.7752
2	1.181	1.201	1.048	1.2055	1.2515
3	1.13	1.253	1.399	0.9916	0.7751
4	0.778	0.775	0.775	5.4264	5.2906

|					
5	6.28	5.753	6.523	0.8125	0.7751
6	0.776	0.776	0.775	1.2682	1.0878
7	14.917	14.392	13.288	13.1225	13.432
8	10.46	10.403	10.352	11.4493	11.2207
9	15.972	15.906	14.825	14.4281	16.2342
10	16.133	14.868	15.349	18.0413	16.2034
11	11.067	9.714	10.219	11.0402	10.687
12	12.681	14.54	13.747	13.5818	14.17
13	7.058	6.853	7.033	5.1206	6.4223
14	4.303	4.483	4.73	4.3391	4.3321
15	2.398	2.449	2.497	2.3865	2.3384
16	4.504	4.299	5.03	5.0637	4.3778
17	6.214	6.289	6.692	6.3834	6.6193
18	0.782	0.775	0.775	1.1101	0.775
19	0.789	0.79	0.775	1.0289	0.7767
20	0.776	0.789	0.775	1.3294	0.7785
21	0.829	0.793	0.775	0.9242	0.7751
22	0.787	0.775	0.775	0.8098	0.775
23	0.782	0.775	0.775	1.0791	0.7752
24	2.375	3.073	3.011	2.9766	2.436
25	1.458	1.502	1.811	1.6141	1.1545
26	1.722	1.617	1.732	1.7112	1.4576
27	2.368	2.757	2.824	3.0611	2.7649
28	1.242	1.278	1.217	1.2978	1.2236
29	1.269	1.419	1.279	1.4260	1.3542
30	1.327	1.252	1.255	1.6197	1.4034
31	1.227	1.229	1.231	1.3493	1.2101
Weight (lb)	62206	62096	61972	63760.08	62150.7
Penalty for Dispt. Constraint	0.3425	0.3449	0.3684	0	0
Penalty for Stress Constraint	1.1412	1.0551	1.1257	0	0
Penalty for Slenderness Constraint	0	0	0	0	0
No. of analyses	50,000	50,000	50,000	49,500	48,800

This spatial structure is subjected to two loading conditions:

In Case 1, which is a symmetric loading condition, the vertical concentrated loads of -20 ksi (-88.968 kN) are applied on free joints (non-support joints). In Case 2, which is asymmetric, the concentrated loads of -10 ksi (-44.484 kN) are applied at the right hand half of the structure and on non-supported joints. At the left hand half of the structure and on the non-supported joints the loads of -6 ksi (-29.69 kN) are applied.

This structure consists of two rectangular nets and for making it stable, angles of the bottom nets are put into the center of one of the above nets, and these are connected through diametrical elements as shown in figure 2. All members of this double layer barrel vault are categorized into 31 groups, as shown in figure 2(b). According to Ref. [1], the supports are considered at the two external edges of the top layer of the barrel vault.

figure 3 shows the convergence history for optimization of this structure for two cases using MCSS and IMCSS algorithms. Tables 2 and 3 are provided for comparison of the

optimal design results with those of the previous studies for both cases.
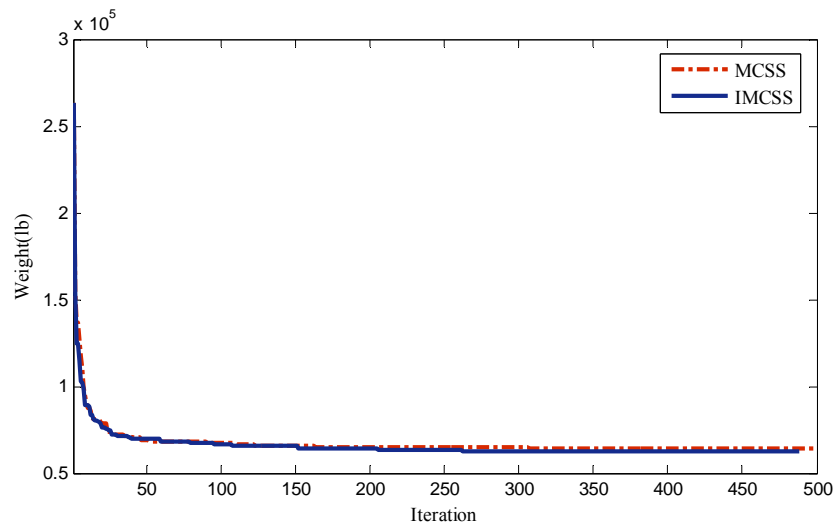
(a)
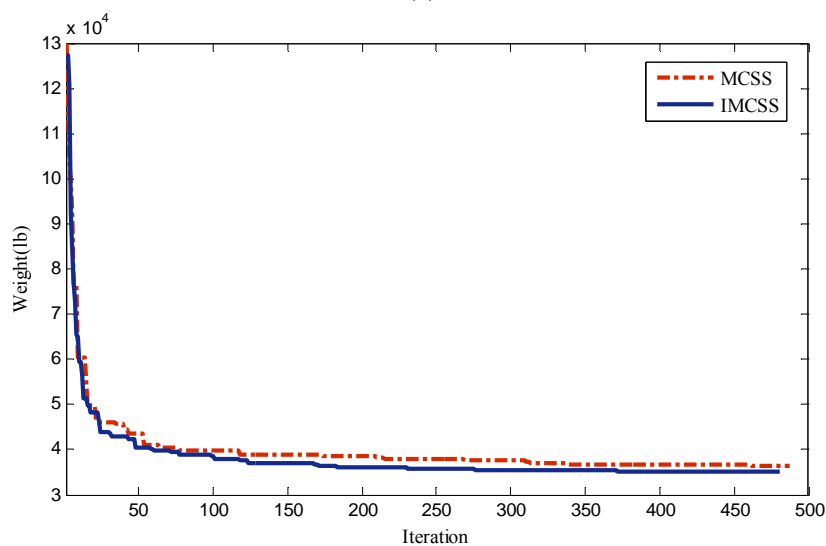
(b)

Figure 3. Comparison of the convergence rates between the MCSS and IMCSS algorithms for
the 384-bar bar double layer barrel vault, (a) Case 1 and (b) Case 2.

Table 3: Optimal design comparison for the 384-bar bar double layer barrel vault (in2) for Case 2

Element group	Kaveh and Eftekhar [4]			Present Work	
	HS	BB-BC	IBB-BC	MCSS	IMCSS
1	0.775	0.775	0.775	0.8251	0.775
2	0.823	0.868	0.775	0.9330	0.776
3	2.442	1.891	0.829	0.9315	0.775
4	1.78	2.077	2.151	1.8058	1.191

5	3.006	2.356	2.544	1.4406	1.042
6	0.918	0.775	0.775	0.9455	0.775
7	7.768	7.657	7.842	8.5558	8.949
8	8.002	7.564	7.717	8.0957	8.854
9	7.768	7.378	7.714	7.2711	8.112
10	4.833	5.704	5.128	4.4217	5.654
11	3.286	3.286	3.284	3.4532	3.361
12	5.024	4.774	4.957	5.8577	5.915
13	1.709	1.612	1.542	2.3414	1.507
14	1.435	1.426	1.397	1.5894	1.259
15	1.936	1.984	1.907	2.0375	1.806
16	5.371	5.332	5.053	4.0992	5.208
17	5.082	5.239	5.149	4.6918	4.340
18	0.775	0.775	0.775	1.3241	0.775
19	0.775	0.775	0.775	0.9577	0.776
20	1.537	1.147	0.775	0.8365	0.778
21	0.78	0.775	0.775	0.8032	0.775
22	0.775	0.775	0.775	0.9617	0.776
23	0.775	0.775	0.775	0.8154	0.779
24	0.799	0.868	0.784	1.2182	0.775
25	0.775	0.775	0.775	0.7758	0.775
26	1.373	1.24	1.283	1.3975	0.934
27	1.96	2.077	2.169	2.1500	1.992
28	0.775	0.775	0.775	1.0011	0.775
29	0.975	1.054	0.96	1.0844	0.905
30	1.621	1.798	1.774	2.2044	1.818
31	1.16	1.24	1.159	1.2054	1.162
Weight (lb)	35501	35372	34731	36313.81	35273.85
Penalty for Dispt. Constraint	0.0806	0.0228	0.0251	0	0
Penalty for Stress Constraint	0	0	0.0079	0	0
Penalty for Slenderness Constraint	0	0	0	0	0
No. of analyses	50,000	50,000	50,000	46,200	48,100

In Case 1, MCSS and IMCSS find the best solutions in 495 iterations (49,500 analyses) and 488 iterations (48,800 analyses), respectively, while for HS, BB-BC and IBB-BC algorithms, it takes 50,000 analyses to reach the best solutions. The best weights of MCSS and IMCSS are 63760.08 lb and 62150.7 lb, respectively, and for the HS, BB-BC and IBB-BC algorithms are 62206 lb, 62096 lb and 61972 lb, respectively.

In Case 2, the best weight of MCSS and IMCSS algorithms are 36313.81 lb and 35273.85 lb, but it is 35501 lb, 35372 lb and 34731 lb for the HS, BB-BC and IBB-BC algorithms, respectively. The MCSS and IMCSS algorithms get the best solution after 462 iterations (46,200 analyses) and 481 iterations (48,100 analyses), while for HS, BB-BC and IBB-BC algorithms 50,000 analyses is needed.

However, in both cases it can be observed that the best weights of MCSS and the IMCSS algorithms are not better than BB-BC and IBB-BC algorithms, but the MCSS and IMCSS

have no violation of displacement and stress constraints and no penalties for them, therefore these algorithms have better Merit function than BB-BC and IBB-BC algorithms which is the aim of this paper.

6.2 A 693-bar braced barrel vault

The optimal design of the 693-bar braced barrel vault was first presented by Hasançebi et al. [6-9]. This structure is taken from a braced barrel vault which was already built for roofing the platform shelters at the Thirumailai Railway Station in India [18]. The geometry of this structure is shown in figure 4(a) and the front and plan view are provided in figure 4(b) and figure 4(c), respectively. The material density is 0.283 lb /in^3 (7833.413 kg/m^3) and the modulus of elasticity is 29000 ksi $(203893.6$ MPa$)$. In this example, the yield stress *Fy* of steel is taken as 36 ksi (253.1 MPa). This structure consists of 259 joints and 693 members and as seen in figure 4(a) all members are grouped into 23 independent size variables groups considering the symmetry of the braced barrel vault about centerline.
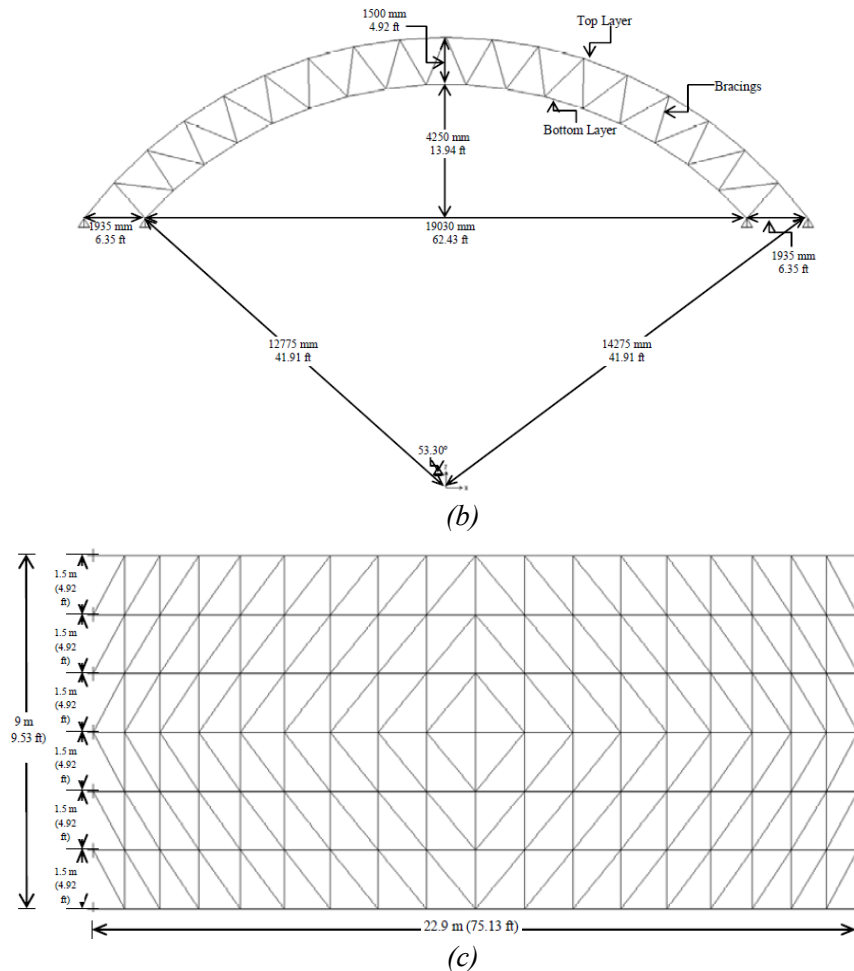

(a)

(b)

(c)
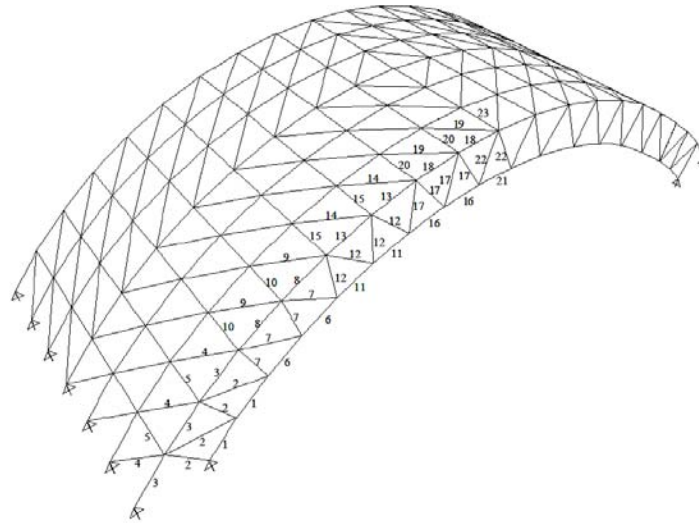
Figure 4. The 693-bar braced barrel vault, (a) 3-dimensional view, (b) Front view, (c) Plan view.

It is assumed that the barrel vault is subjected to a uniform dead load (DL) pressure of 35 kg/m^2, a positive wind load (WL) pressure of 160 kg/m^2, and a negative wind load (WL) pressure of 240 kg/m^2. For design purposes, these loads are combined under two separate load cases as follows:

Load case 1: 1.5(DL+WL) = 1.5(35 +160) = 292.5 kg/m^2 (2.87 kN/m^2)

Load case 2: 1.5(DL-WL) = 1.5(35–240) = –307.5 kg/m2 (–3.00 kN/m^2)

The nodes are subjected to the displacement limits of ±0.1 in (0.254 cm) in x, y and z directions. The strength and stability requirements of steel members are imposed according to AISC-ASD [12]. The structural members are selected from a list of 37 circular hollow sections taken from AISC-LRFD code [17].

Table 4 provides the comparison of the results of MCSS and IMCSS algorithms with those of the previous studies for this structure. The convergence history for MCSS and IMCSS algorithms are shown in figure 5.

Table 4: Optimal design comparison for the 693-bar braced barrel vault (in2)

Element group	Hasançebi and Çarbaş [6]	Hasançebi, Çarbas, and Saka [7]			Hasançebi and Kazemzadeh Azad [8,9]		Present Work	
	ACO2	sACO	HS	GA	BB-BC	MBB-BC	MCSS	IMCSS
1	3.17	4.03	3.68	3.02	4.03	3.68	3.02	3.68
2	0.494	0.494	0.433	0.669	0.494	0.494	0.494	0.494
3	0.669	0.494	0.494	0.639	0.333	0.333	0.433	0.639
4	0.881	0.494	0.494	0.494	0.669	0.494	0.32	0.333
5	0.333	0.494	0.433	0.333	0.333	0.333	0.32	0.494
6	4.3	0.333	3.17	4.41	3.68	3.68	3.02	2.66
7	0.669	0.639	0.669	0.639	0.494	0.494	0.881	0.494
8	0.881	0.333	0.333	0.333	0.494	0.494	0.494	0.669
9	3.68	2.68	2.68	2.66	0.494	0.494	0.333	0.32
10	0.669	4.03	0.494	0.639	0.333	0.333	0.32	0.25
11	0.433	0.494	0.669	0.669	2.25	2.23	2.25	2.23
12	0.799	0.639	0.881	0.799	0.799	0.799	1.07	0.881
13	0.799	0.881	1.07	1.07	1.07	1.07	1.7	1.48
14	0.669	0.639	0.881	0.799	0.494	0.494	0.333	0.25
15	0.433	0.333	0.333	0.494	0.333	0.333	0.333	0.333
16	0.799	0.639	0.881	0.669	1.07	0.881	0.669	0.881
17	1.48	0.881	0.881	1.07	0.669	0.669	0.799	0.799
18	0.669	0.494	0.669	0.799	1.7	2.23	2.23	2.23
19	0.669	0.669	0.639	0.669	0.669	0.494	0.433	0.333
20	0.333	0.333	0.333	0.333	0.333	0.333	0.25	0.333
21	2.25	1.7	1.7	2.23	0.494	0.494	0.669	0.494
22	0.799	0.669	0.494	0.669	0.494	0.333	0.433	0.639
23	0.333	0.494	0.639	0.433	0.333	0.333	0.333	0.433
W(lb)	**12133.47**	**10999.2**	**11232.1**	**12029.49**	**10859.42**	**10595.33**	**10812.39**	**10550.86**
W(kg)	5503.65	4989.15	5095.07	5456.48	4925.75	4805.96	4904.42	4785.81
NA	**50,000**	**-**	**-**	**-**	**27,150**	**27,150**	**14,300**	**9,200**

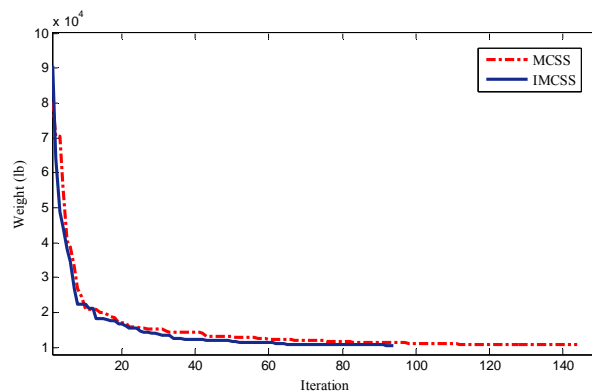NA: No. of analyses W: Weight

Figure 5. Convergence history for the 693-bar braced barrel vault using MCSS and IMCSS algorithms.

The MCSS and IMCSS algorithms find the best solutions in 143 iterations (14,300

analyses) and 92 iterations (9,200 analyses), respectively, while, it takes 50,000 analyses for the ACO2 and ACO1, and 27,150 analyses for the BB-BC and MBB-BC algorithms to reach the best solutions. The best weights of MCSS and IMCSS are 10812.39 lb (4904.42 kg) and 10550.9 lb (4785.81 kg), respectively, but it is 10595.33 lb, 10859.42 lb, 10999.20 lb for the MBB-BC, BB-BC and sACO algorithms, respectively.

As it can be seen in the results of 693-bar braced barrel vault, the performance of the IMCSS algorithm in fast convergence and finding the best solution in a lower number of analyses is better than all of the previous studies for this structure.

7. CONCLUDING REMARKS

In this study, an improved magnetic charged system search is proposed for optimization of double layer barrel vaults. In this algorithm, an improved harmony search scheme is utilized in the process of position correction and two of the most effective parameters (PAR and bw) in the convergence rate of algorithm are improved. In the process of structural analysis, the open application programming interface is utilized to link the analysis software with the programming language.

Optimal design via the proposed algorithm is performed for two double layer barrel vaults with continuous and discrete design variables. As it can be seen, the IMCSS algorithm in the first problem has not found better values for weight than the previous studies and the reason is that the aim of this algorithm is to find the best merit function and this algorithm has the best merit function without any violation of the constraints on the problem and therefore without any penalties for that violation than previous studies.

In the second problem as a more comprehensive and practical optimization problem, comparing the results of IMCSS algorithm with those of the previous studies, the robustness of the proposed algorithm in fast convergence and achieving the best optimal value for weight of the structure with a lower number of analyses, is demonstrated. It can also be concluded that the OAPI is a powerful tool for analysis of structural optimization problems, especially for practical large-scale structures such as double layer barrel vaults.

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