ASIAN JOURNAL OF CIVIL ENGINEERING (BHRC) VOL. 15, NO. 1 (2014)
PAGES 107-122

FREE VIBRATION ANALYSIS OF TALL BULINDING WITH GEOMETRICAL DISCONTINUITES

A. Heidari[*1], R. Rahgozar[2] and R. Kamgar[2]
[1]Department of Civil Engineering, University of Shahrekord, Shahrekord, Iran.
[2]Department of Civil Engineering, University of Kerman, Kerman, Iran.

Received: 20 December 2012; **Accepted:** 5 April 2013

ABSTRACT

This paper deals with to determine the first natural frequency of tall buildings consists of framed tube, shear core, belt truss and outrigger system with multiple jumped discontinuities in the cross section of framed tube and shear core. In this paper, the continuous approach was accepted and by using energy method and Hamilton's variational principle, the governing equation for free vibration of tall building can be obtained. The entire length of tall building is partitioned into uniform segments between any two successive discontinuity points and therefore partial differential equation of motion, by applying the separation of variables method on time and space, is reduced to an ordinary differential equation with constant coefficients for each segment. Tall building characteristics matrix can be derived based on the boundary conditions and the continuity conditions applied at the partitioned points. This matrix is particularly used to find combined system natural frequencies and mode shapes. A numerical example has been solved to demonstrate the reliability of this method. The results of the proposed mathematical model give a good understanding of the structure's dynamic characteristics; it is easy to use, yet reasonably accurate and suitable for quick evaluations during the preliminary durations.

Keywords: Tall building; free vibration; geometrical discontinuity; Hamilton's variational principle; partial differential equation; ordinary differential equation.

1. INTRODUCTION

Free vibration analysis plays an important role in the structural design of tall buildings, especially in the first mode shape because it is the dominant shape in response to wind and earthquake induced vibrations in tall buildings. Therefore, it is important to investigate the

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[*] E-mail address of the corresponding author: heidari@eng.sku.ac.ir (A. Heidari)

calculating methods of natural frequencies and mode shapes for tall buildings. In this respect, numerous studies in structural engineering have devoted to obtain accurate theoretical results for the free vibration of tall buildings in the past decades. In Ref. [1] an analytical solution for the dynamic response of a free-free discontinuous beam with a single step change and an aligned neutral axis presented. They considered the case of free-free boundary conditions to obtain direct frequency response functions due to harmonic force or couple excitation at both ends location. In Ref. [2] an analytical method to calculate the frequencies of beams with up to three step changes in cross section presented. They considered the combinations of the classical clamped, pinned, sliding, free, general and degenerate types of elastic end supports. The governing equations of wall-frame structures with outriggers through the continuum approach and the whole structure was idealized as a shear-flexural cantilever with rotational springs [3]. The effect of shear deformation and flexural deformation of the wall-frame and outrigger trusses were considered and incorporated in the formulation of the governing equations. In Ref. [4] Smith and Salim, presented formulae that were developed for estimating the optimum levels of outriggers to minimize the drift in outrigger braced buildings. They presented the analyses and formulae for outrigger structures in which the core and columns were uniform through their height and the outriggers had the same flexural stiffness. Then the results of an investigation on drift reduction in uniform and non-uniform belted structures with rigid outriggers under several lateral load distributions, which were likely to be encountered in practice [5]. Design aids in the form of graphical presentations of the somewhat complex solutions were provided. Stewart and Andrew [6], presented an approximation dynamics method by using Hamilton’s principle about including applications to non-conservative and conservative systems. They had shown that this method was suitable for both multi degree of freedom and constrained systems. In Ref. [7], free vibration analysis of asymmetric plan frame structures demonstrated. They emphasized on analysis of lateral-torsional vibration of the structures, where lateral shear vibrations in two orthogonal directions were coupled with St. Venant torsion vibration. The governing equation of coupled vibration of the problem was derived by them; also the corresponding eigenvalue equation was derived. A theoretical method of solution was proposed to solve the eigenvalue problem and a general solution was given to determine the natural frequencies and associated mode shapes of the structure. A simplified analytical method for outrigger structure has been presented in Refs. [8-11]. Geourgoussis [12], presented a simple mathematical model for assessing periods of vibration and mode shapes of common cantilever bents used in concrete structures, such as shear walls, coupled walls, rigid frames and wall-frame assemblies. He used Dunkerley’s formula for calculating natural frequencies of mentioned structures and considered the effect of column axial shortenings in the analysis of structural bents. In Ref. [13] the Timoshenko beam model and obtained partial differential equation (PDE) of framed tube structure is used. This PDE was reduced to an ordinary fourth-order differential equation with constant coefficients and was solved over a specified interval by applying appropriate boundary equations. In many real applications, the investigation of non-uniform cross-section tall building may provide a realistic distribution of mass and stiffness desired for accurate structural analysis.

In this paper, a simple mathematical model is presented to calculate the first natural frequency of combined system including framed tube, shear core, outrigger and belt truss system. Framed tube system consists of closely spaced exterior columns along the periphery interconnected by deep spandrel beams at each floor. This produces a system of rigidly connected jointed orthogonal frame panels forming a rectangular tube, which acts as a cantilevered hollow box [14-16]. The effect of belt truss and outrigger system is modeled as a concentrated rotational spring located at the belt truss and outrigger system location. Here by adopting the Hamilton's variational principle (HVP); the following items are derived: PDE of the structural vibration, boundary displacements (kinematic boundary conditions), boundary forces (natural boundary conditions) and eigenvalue solution form. By selecting partitioned method along the height of the structure and by using the assumption of the harmonic motion, the PDE is reduced to ordinary differential equation (ODE) with constant coefficients. By applying boundary conditions and continuity conditions, the eigenvalue problem for finding the first natural frequency of tall building is obtained. In order to illustrate the efficiency and accuracy of the proposed method a numerical example has been carried out by the proposed method and SAP 2000 software. The result is shown in very good agreement.

2. FORMULATION AND SOLUTION

In this section by adopting the following assumptions and using HVP, the vibration PDE of the combined system including framed tube, shear core, belt truss and outrigger system is derived. These assumptions are as follows:

1. The floor slabs of the system are not deformable in their planes and have no motion perpendicular to their planes.
2. The effect of belt truss and outrigger system is considered as a rotational spring with constant rotational stiffness, which acts on the position of belt truss and outrigger system.
3. Spacing of columns and beams are constant throughout the building's height and the dimensions of all beams and columns are the same in each storey and segment.
4. Shear core and columns are fully fixed at the base.
5. Member's connections of the outrigger system are assumed to be rigid and connections of members of the belt truss are assumed to be pinned.
6. The material of structure is linearly elastic, homogeneous and obedient to the Hook's law.
7. The structure is assumed symmetric in plan of all stories and therefore cannot twist.
8. The thickness of the shear core and the dimensions of the columns and beams of framed tube structure change in a stepwise in the height of the structure.
9. The dimension of members of the belt truss and outrigger system are constant and do not vary in the height of the structure.
10. The mass of the outrigger and belt truss system is supposed that distributed uniformly in the height of the structure in that segment which this system has been located.

With above assumption, the structure can be modeled as a beam with a box variable cross section in height (Figure 1) and by using HVP, the differential equation of vibration of combined system can be obtained. HVP, considering fundamental law of dynamic, encompasses Newton's equations of motion, Lagrange's equations for structure dynamics, D'Alembert's principle and the principle of virtual work. Vibration of structures can be conveniently formulated in terms of HVP. HVP is energy functional based, the diverse areas of structure dynamics, numerical solutions of PDE, finite element methods, and functional analysis can all be linked in a single development. Structure energy is expressed in terms of the functional Lagrangian energy, and HVP requires that this functional energy have a stationary value. The term functional is used to denote a general expression for a continuous function of the domain V of the structure in space and time [17].

Consider a combined system of framed tube, shear core, belt truss and outrigger system as a continuous beam with flexural stiffness $EI(x)$ shear stiffness $AG(x)$ mass per unit height $m(x)$ dynamic displacement $w(x,t)$ and total height L (figure 1). The effect of belt truss and outrigger system is considered as a rotational spring at the location of the belt truss and outrigger system. Structure is defined over the closed domains $0 \le x \le L$, where x is the spatial position of any material point of the system and t is the time of vibration of any material point of the system. In figure 1, dashed line shows the variation of thickness along the height of structure, and E, G and t_{tte} are modulus of elasticity, shear modulus and total equal thickness of shear core and framed tube in both of flange and web panel. The dynamic system under considering has the kinetic energy as follows:

$$T(x,t) = \frac{1}{2} \int_0^L m(x)\, [\dot{w}(x,t)]^2 \, dx \tag{1}$$

and the potential energy is:

$$V(x,t) = \frac{1}{2} \int_0^L \left(EI(x)\, [w''(x,t)]^2 + AG(x)\, [w'(x,t)]^2\right) dx + \frac{1}{2} K_r\, [w'(a,t)]^2 \tag{2}$$

Which K_r is the equivalent stiffness of the rotational spring including the effect of the belt truss and outrigger system on the framed tube acting at $x = a$ and primes and dots on w denote partial derivatives with respect to x and t, respectively. By considering Eqs. (1)-(2), the total energy of the structure is defined as follows:

$$B(x,t) = T(x,t) - V(x,t) \tag{3}$$

The action or principle function of dynamics A, can be expressed as the time integral of B between two times t_1 and t_2 [17].

$$A = \int_{t_1}^{t_2} B(x,t)\, dt = \int_{t_1}^{t_2} [T(x,t) - V(x,t)]\, dt \tag{4}$$

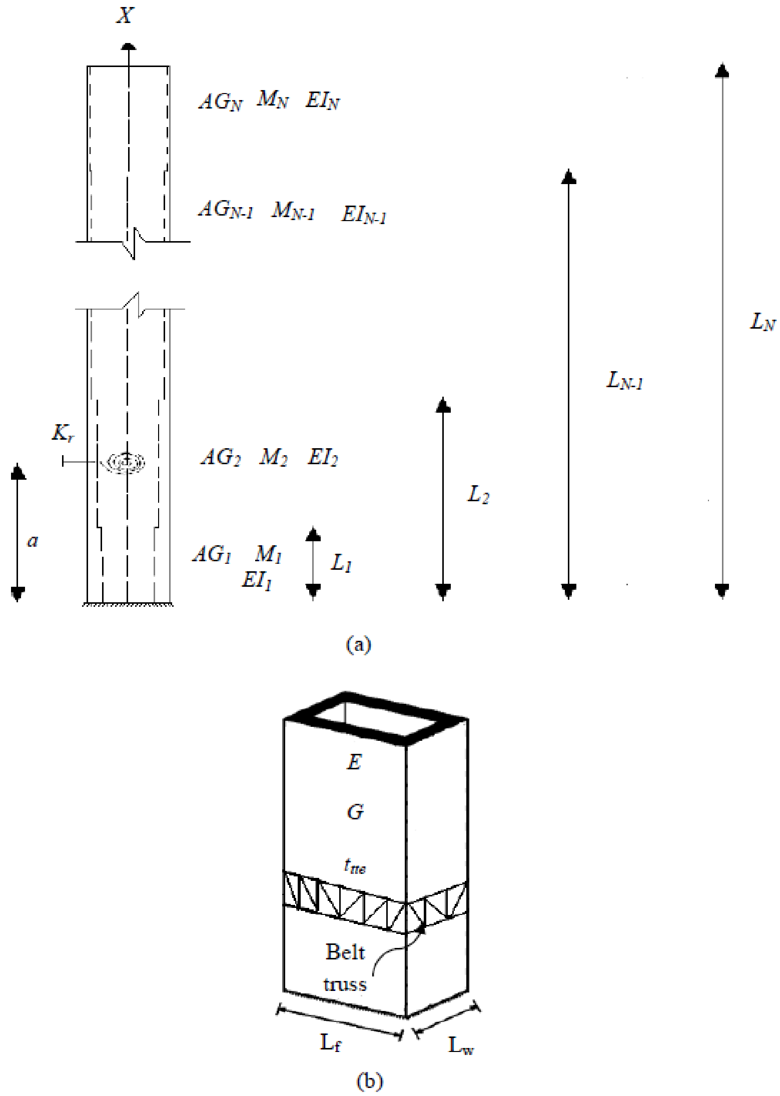

Figure 1. Approximate model of tall buildings
(a) Elevation of combined system (b) 3D model of combined system

HVP states that A has a stationary value expressed as $\delta A = 0$ where δ is termed as the variation operator. The δ operator over a structure domain V implies that complete description of the structure shape requires an infinite number of degree of freedom- one degree of freedom for each shape. HVP requires that the action defined in Eq. (4) has a minimum or stationary value over all possible structure variations, that is:

$$\delta A = \delta \int_{t_1}^{t_2} B(x,t)\, dt = \int_{t_1}^{t_2} \delta B(x,t)\, dt = \int_{t_1}^{t_2} \delta [T(x,t) - V(x,t)]\, dt = 0 \qquad (5)$$

This expression of HVP in terms of δA utilizing the δ operator properties and integration by parts provides

1. The differential equation of motion termed the Lagrange's equation
2. The boundary displacements (kinematic boundary conditions)
3. The boundary forces (natural boundary conditions)
4. The eigenvalue solution form

By substituting Eqs. (1) and (2) into Eq. (5), the following equation is obtained:

$$\delta A = \int_{t_1}^{t_2}\int_0^L [m(x)\,\dot{w}(x,t)\,\delta\dot{w} - EI(x)\,w''(x,t)\,\delta w'' - AG(x)\,w'(x,t)\,\delta w']\,dx\,dt \\ -\int_{t_1}^{t_2} k_r\,w'(a,t)\,\delta w'\,dt \tag{6}$$

Integration by parts from Eq. (6) with respect to time and space, gives:

$$\delta A = -\int_{t_1}^{t_2}\int_0^L [m(x)\,\ddot{w}(x,t) + \frac{\partial^2}{\partial x^2}(EI(x)\,w''(x,t)) - \frac{\partial}{\partial x}(AG(x)\,w'(x,t))]\,\delta w\,dx\,dt \\ -\int_{t_1}^{t_2} [\,k_r\,w'(a,t) + EI(x)\,w''(x,t)]\,\delta w'\Big|_0^L dt + \int_{t_1}^{t_2} (\frac{\partial}{\partial x}[EI(x)\,w''(x,t)] - \\ AG(x)\,w'(x,t))\,\delta w\Big|_0^L dt \tag{7}$$

The equation of vibration and related boundary conditions can be obtained as follows:

$$m(x)\,\ddot{w}(x,t) + \frac{\partial^2}{\partial x^2}(EI(x)\,w''(x,t)) - \frac{\partial}{\partial x}(AG(x)\,w'(x,t)) = 0 \quad \text{for } 0 \le x \le L, \;\; 0 \le t \tag{8}$$

and the boundary conditions are:

$$\begin{aligned} &\frac{\partial}{\partial x}[EI(x)\,w''(x,t)] - AG(x)\,w'(x,t) = 0 && at \quad x = L \\ &k_r\,w'(a,t) + EI(x)\,w''(x,t) = 0 && at \quad x = L \\ &w(x,t) = 0 && at \quad x = 0 \\ &w'(x,t) = 0 && at \quad x = 0 \end{aligned} \tag{9}$$

where the first two of Eq. (9) are boundary forces and the last two of Eq. (9) are boundary displacements. Using the method of variables separation, we assume:

$$w(x,t) = \varphi(x)\,sin(\omega t) \tag{10}$$

where ω is the natural frequency and $\varphi(x)$ is the mode shape function of the system. By substituting Eq. (10) into Eq. (8) and dividing by $sin(\omega * t)$ yields:

$$\frac{d^2}{dx^2}[EI(x)\ \frac{d^2\varphi(x)}{dx^2}] - \frac{d}{dx}[AG(x)\frac{d\varphi(x)}{dx}] = m(x)\,\omega^2\ \varphi(x) \tag{11}$$

For a tall building with parametric discontinuities (e.g., jump in the flexural stiffness, shear stiffness and mass distribution) Eq. (11) cannot be solved using conventional approaches. An alternative method is to partition the tall building into uniform segments between any two successive stepped points and apply the continuity conditions at these points. figure 1, illustrates a tall building with N jumped discontinuities in its spatial height. As we known, the tall building with different kinds distribution of stiffness along the height can be considered as a cantilever beam that has a uniform cross section at each segment. Hence, Eq. (11) can be divided into n uniform equation expressed as:

$$\begin{aligned}&(EI)_n\frac{d^4\varphi_n(x)}{dx^4} - (AG)_n\frac{d^2\varphi_n(x)}{dx^2} = m_n\ \omega^2\ \varphi_n(x) \qquad l_{n-1}\langle x\langle l_n\\ &n = 1,2,3,\ldots,N \qquad\qquad l_0 = 0\end{aligned} \tag{12}$$

Where $\varphi_n(x)$, $(EI)_n$, $(AG)_n$ and m_n are mode shape, flexural stiffness, shear stiffness and mass per unit height of tall building for the n^{th} segment, respectively. Let,

$$\alpha_n = \frac{(AG)_n}{(EI)_n} \qquad\qquad \beta_n = \frac{\omega^2 m_n}{(EI)_n} \tag{13}$$

So far, Eq. (12) can be rewritten in a more recognizable form as follows:

$$\frac{d^4\varphi_n(x)}{dx^4} - \alpha_n\frac{d^2\varphi_n(x)}{dx^2} - \beta_n\ \varphi_n(x) = 0 \tag{14}$$

The general solution of Eq. (14) is

$$\varphi_n(x) = A_n\ cos(\sqrt{-\lambda_{2,n}}x) + B_n\ sin(\sqrt{-\lambda_{2,n}}x) + C_n\ cosh(\sqrt{\lambda_{1,n}}x) + D_n\ sinh(\sqrt{\lambda_{1,n}}x) \tag{15}$$

where parameters λ_1 and λ_2 are defined as follows:

$$\lambda_{1,n} = \frac{\alpha_n}{2} + \sqrt{\frac{\alpha_n^{\ 2}}{4} + \beta_n} \qquad\qquad \lambda_{2,n} = \frac{\alpha_n}{2} - \sqrt{\frac{\alpha_n^{\ 2}}{4} + \beta_n} \tag{16}$$

where A_n, B_n, C_n and D_n are the constants of integration which can be determined by applying the boundary and continuity conditions. The continuity conditions for displacement, slope, bending moment, and shear force at point of discontinuity are given by:

$$\begin{aligned}
&\varphi_n(l_n) = \varphi_{n+1}(l_n) \\
&\frac{d\varphi_n(l_n)}{dx} = \frac{d\varphi_{n+1}(l_n)}{dx} \\
&(EI)_n \frac{d^2\varphi_n(l_n)}{dx^2} = (EI)_{n+1} \frac{d^2\varphi_{n+1}(l_n)}{dx^2} \\
&(EI)_n \frac{d^3\varphi_n(l_n)}{dx^3} = (EI)_{n+1} \frac{d^3\varphi_{n+1}(l_n)}{dx^3}
\end{aligned} \tag{17}$$

and the boundary conditions of Eq. (9) can be rewritten as follows:

$$\begin{aligned}
&\varphi_1(0) = 0 \\
&\frac{d\varphi_1(0)}{dx} = 0 \\
&(EI)_N \frac{d^2\varphi_N(l_N)}{dx^2} + k_r \frac{d\varphi_j(a)}{dx} = 0 \\
&\frac{d^3\varphi_N(l_N)}{dx^3} - \alpha_N \frac{d\varphi_N(l_N)}{dx} = 0
\end{aligned} \tag{18}$$

The characteristics matrix of the system can be formed by applying Eqs. (17) and (18) into Eq. (15) at each point of discontinuity as well as at the boundaries. It is remarked that β_n is function of tall building natural frequency with an explicit expression given in Eq. (13). Therefore, the characteristics matrix becomes only function of a single parameter β. The characteristics equation is then given by:

$$\boldsymbol{H}_{4N\times 4N}\boldsymbol{C}_{4N\times 1} = 0 \tag{19}$$

where $\boldsymbol{H}=\boldsymbol{H}(\beta)$ is the characteristics matrix and $\boldsymbol{C}$ is the characteristics vector of the system:

$$\boldsymbol{C} = [A_1B_1C_1D_1A_2B_2C_2D_2 \ldots A_NB_NC_ND_N]^T_{1\times 4N} \tag{20}$$

In order to obtain a non-trivial solution for Eq. (19) to find the first natural frequency and the associated mode shape, the determinant of matrix $\boldsymbol{H}$ must be zero

$$det[\boldsymbol{H}(\beta)] = 0 \tag{21}$$

Since this matrix is a function of only parameter $\beta \in (0,\infty)$, its determinant can be numerically evaluated for its zero values. The values of β, which satisfy Eq. (21), lead to the calculation of the natural frequencies of tall buildings.

Control of top drift and base overturning moment in the core of a tall structure subjected to lateral loads is the main concern in design of tall buildings. There are many structural forms such as rigid frame, braced frame and shear-walled frame, frame-tube, braced-tube, bundled-tube and outrigger systems that can be used to enhance the lateral resistance in tall buildings [18]. Belt truss system restrains the bending of the core by introducing a point of inflection in its deflection curve. This reversal in curvature reduces the bending movement above the belt truss and outrigger system. The belt truss functions as horizontal fascia stiffeners and engages the exterior columns, which are not directly connected to the outrigger trusses. A general important or up to 25 to 30 percent in stiffness can be realized in contrast to the same system without such trusses because instead of individual columns acting as tiedowns, all the façade columns participate in resisting the lateral load. Placement of a rigid truss at the top of the building eliminates differential movement between interior and exterior columns by providing compressive restraint for exterior columns in expansion and tension restraint.

In order to determine the stiffness of belt truss and outrigger system, the work that are done in Refs. [3,8] can be utilized. It has pointed out that magnitude of the reductions depends on the flexural rigidities of the core, the outriggers, and the column acting axially around the core's centroid. The reductions depend also on the locations of the outriggers up the height of the core [4]. Equivalent stiffness of rotational linear spring K_r can be given:

$$K_r = 1/\theta \tag{22}$$

where θ denotes the total rotation in the outrigger and belt truss system due to the restraining moment, and can be obtained by splitting up the rotation as:

$$\theta = \theta_a + \theta_b + \theta_s \tag{23}$$

First, the restraining forces in the exterior columns will cause rotation of the outrigger resulting from the axial lengthening and shortening of the columns. The outrigger rotation θ_a due to the resulting restraining moment can then be defined as the column change in length divided by the length of the outrigger (d):

$$\theta_a = (2\,a)/(d^2 AE) \tag{24}$$

where d and AE are the distance between center to center of exterior columns and the axial stiffness of the exterior columns, respectively. The flexural deformation of outrigger due to the action of the column force will cause additional drifts between adjacent floors. The resulting rotation θ_b is given by:

$$\theta_b = (d)/(12\ EI_{oe}) \tag{25}$$

where EI_{oe} is the effective flexural stiffness of the outrigger, modeled as though its length extended from the column to the centroid of the core. EI_{oe} can be obtained from the outrigger's actual flexural rigidity EI_{ou} by converting the flexural rigidity of a wide-column beam, figure (2a), to that of an equivalent full-spam beam, figure 2b as follows [4]:

$$EI_{oe} = EI_{ou}(1+((b_c/2)/((d-b_c)/2)))^3 \tag{26}$$

where b_c is the length of the shear core and EI_{ou} can be calculated by using theory of parallel axes.

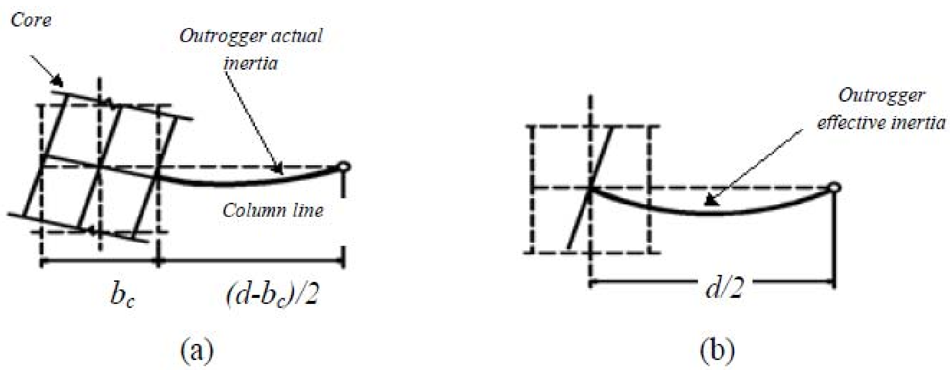

Figure 2. Wide-column effects of core and outrigger
(a) Outrigger actual inertia (b) Outrigger effective inertia

The rotation caused by the shear force in the outrigger and belt truss system θ_s results from strain in diagonals, and can be expressed as:

$$\theta_s = \left[1/(h\,AG_{ou})\right] \tag{27}$$

where h is the height of the outrigger, and AG_{ou} is racking shear stiffness of the outrigger and belt truss system. This racking shear stiffness can be calculated for specific outrigger truss type. The racking shear stiffness is a property for which the method of determination is most particular to the type of belt. It depends on the deformation of the web members as the structure racks under the shearing action. It should be noted that the vertical members do not have any influence on the racking shear stiffness of the segment. For various type of belts, the value of AG_{ou} have been given [8]. The value of K_r which corresponds to stiffness of the spring at $x=a$, can be derived as follows:

$$K_r = [(2a)/(d^2 AE) + (d)/(12\,EI_{oe}) + (1/h\,AG_{ou})]^{-1} \tag{28}$$

where A is the area of the exterior columns in position of belt truss and outrigger system that are perpendicular to direction of vibration of structure.

3. ACCURATE OF THE RESULTS

To illustrate the accuracy of the proposed approximate method, a numerical example is given to demonstrate the ease of application. A high-rise 40-storey reinforced concrete consisting of framed tube, shear core and belt truss, as shown in Figs. 3 and 4, is analyzed. The sizes of all beams, columns that change in height of the structure have been listed in Table 1. The height of each storey is 3m and the center-to-center spacing of the columns is 2.5m. The outrigger and belt truss system has been located in 30m from the base. The all dimensions of outrigger and belt truss system are equal to $0.8\text{m}\times 0.8\text{m}$. The spacing of members of outrigger and belt truss system as shown in figure 4, are $S_v = S_h = 5\,m$ and $S_{oh} = S_{ov} = S_b = 2.5\,m$. The Young's and shear modulus of the material which have been used in structural elements such as beams, columns, slabs, shear core, belt truss and outrigger system are $E = 2\times 10^9\,kg.m^{-2}$ and $G = 8\times 10^8\,kg.m^{-2}$, respectively. Other specifications that are used in numerical example are as follows:

$$L_f = 35m,\ \ L_w = 30m,\ \ s = 2.5m,\ \ \rho = 2400kg.m^{-3},\ \ t_{slab} = 0.3m,\ \ L = 120m,\ \ y = 3m$$

where y, t_{slab} and ρ are height of each storey, thickness of the floor slab and mass per unit volume of materials of the system, respectively. Dimensions of the core are $5m\times 5m$, thickness of shear core panels (t_{sc}) that changes in height of the structure have been listed in Table 2. The Poisson ratio assumed to be 0.25.

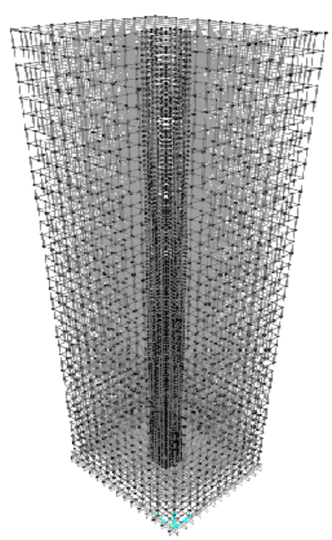

Figure 3. Model of numerical example.

Table 1: The properties of combined system framed tube, shear core, belt truss and outrigger system

No. Storey	Height from the base (m)	Dimensions of beams and columns (cm)	Thickness of shear core (cm)	$AG\,(kg)$	$EI\,(kg.m^2)$	$M\,(kg)$	$G_{eft}\,(kg.m^{-2}$
21	63	80	25	$44.8021*10^{8}$	$1.0548*10^{13}$	25575578.38	$1.4596*10^{8}$
40	120	60	20	$23.7687*10^{8}$	$5.9091*10^{12}$	18816566.36	$7.4387*10^{7}$

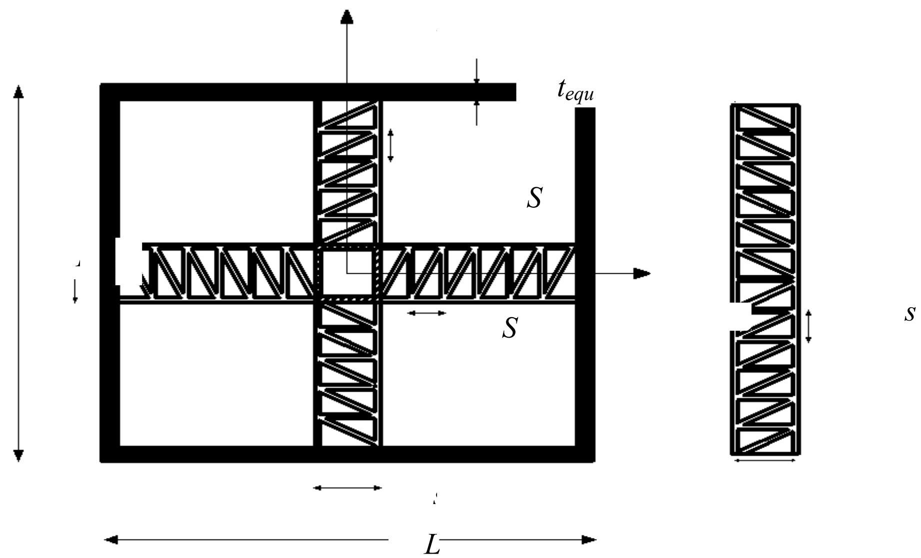

Figure 4 Plane of outrigger and belt truss system.

Table 2: The thickness of shear core panels

No. Step	$t_{equ} = A_c / s\ (m)$	$A_{fts} = 2t_{equ} * (L_w + 2t_{equ})\quad (m^2)$	$t_{tts} = t_{equ} + t_{sc}\quad (m)$
1	0.256	15.6221	0.506
2	0.144	8.7229	0.344

The moment of inertia of the shear core (I_{sc}) and framed tube (I_{ft}) about the Y-axis that changes in height of the structure can be calculated simply and therefore, the total flexural of framed tube and shear core (EI_t) is shown in Table 3. Also, the effective section area of shear core (A_{sc}) and framed tube (A_{fts}) for computing the shear rigidity that changes in height of the structure are shown in Table 3.

Table 3: The total flexural and effective section area

No. Step	$EI_t = E(I_{sc} + I_{ft})$ $(kg.m^2)$	$A_{sc} = (a + t_{sc})*2t_{sc}$ (m^2)	$A_{fts} = (L_w + 2t_{equ})*2t_{equ}$ (m^2)
1	$1.0548*10^{13}$	2.6250	15.6221
2	$5.9091*10^{12}$	2.0800	8.7229

The equivalent elastic parameters for the analogous orthotropic membrane tube (G_{eft}), as evaluated by Kwan [19] which change in every step of the height of the structure, have been listed in Table 4. In calculation of the effective shear area of beams and columns, we use the Hutchinson's *k*. In rectangular cross section Hutchinson's *k* is defined as follows:

$$k = -\frac{2(1+v)}{[\frac{9}{4a_1^5 b_1} C_4 + v(1 - \frac{b_1^2}{a_1^2})]}$$

$$C_4 = \frac{4}{45} a_1^3 b_1(-12a_1^2 - 15va_1^2 + 5vb_1^2) + \sum_{n=1}^{\infty} \frac{16v^2 b_1^5 (n\pi a_1 - b_1 \tanh(\frac{n\pi a_1}{b_1}))}{(n\pi)^5 (1+v)}$$

where the depth of the column or beam (y-direction) is $2a_1$ and the width of the column or beam (z-direction) is $2b_1$ (figure 5).

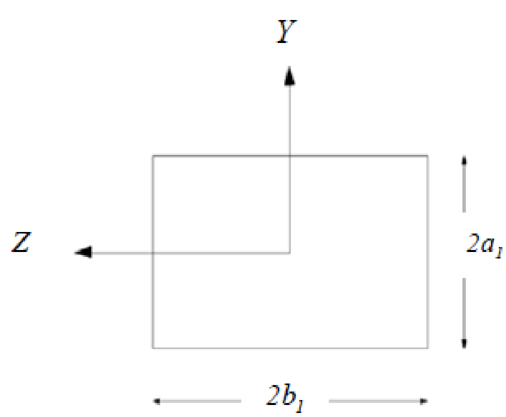

Figure 5. Plane of a column.

The total shear stiffness (AG_t) of the structure which changes in every step of the height of the structure and the Hutchinson's *k* are shown in Table 3. By using Eq. (28), the value of K_r is calculated as follows:

$$K_r = 5.0115 \times 10^9 \ kg.m, \quad d = 30m, \quad AE = 1.92 \times 10^{10} \ kg, \quad EI_{oe} = 1.7591 \times 10^{11}, \quad AG_{ou} = 916467325.4$$

The structure has been analyzed and the result of the first natural frequency which has been obtained by the proposed approximate method is compared with the result of the finite element method which has been obtained from SAP 2000 software.

Table 4: the equivalent elastic parameters

No. Step	$A_{Gt} = (A_{sc} * G) + (A_{fts} * G_{eft})\ \ (kg)$	k
1	$44.8021*10^8$	0.86623
2	$23.7687*10^8$	0.86623

In this example, the outrigger and belt truss system locate in stories 9-11 or in height 27-33 (m) from the base of the structure. By inserting these data into Eq. (21) and solving Eq. (21) through an iterative numerical process, we obtain $\omega_1 = 1.9616 (rad/s)$. The value of ω_1 which can be obtained by using the finite element method is $2.0197 (rad/s)$.The proposed approximate method underestimate the natural frequency by 2.96%. Therefore, the proposed method shows a good understanding of structural behavior, easy to use, yet reasonably accurate and suitable for quick evaluations during the preliminary design stage, which requires less time. The main sources of error between the proposed approximate method and the finite element method are as follows:

1. Modeling the frame panels as equivalent orthotropic membranes (framed tube), so it can be analyzed as a continuous structure.
2. The equivalent elastic properties are derived for the frame tube, shear core, and belt truss.
3. The equivalent stiffness of the rotational spring used to model the effect of the belt truss and outrigger system on frame tube.
4. The approximate values of EI_t and AG_t are derived for distribution of them in height of tall building.
5. The effect of shear lag has been neglected in approximate method.

4. CONCLUSIONS

In this paper, a simple approximate method has been developed to determine the first natural frequency of combined system consists framed tube, shear core, belt truss and outrigger system. By using the HVP, the governing equation of vibration of mentioned structure obtained and by using partitioned method, the governing equation was reduced to an ODE with constant coefficients. The accuracy of the proposed method is verified by a numerical example. The numerical example shows that the approximate value of natural frequency obtained by the proposed method appears to be 2.96% smaller than the more accurate finite element method result. From the point of view of a structural engineer, this error within the acceptable range of engineering practice and therefore the proposed method may be used to estimate the natural frequency at the preliminary stage in the structural’s design, which requires less time.

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