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Technical Note

DEFLECTIONS IN NON-PRISMATIC SIMPLY SUPPORTED PRESTRESSED CONCRETE BEAMS

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ABSTRACT

Pre-stressing is becoming a most common and indispensable technique in the recent past for beams and girders in buildings and bridges. This paper presents the application of Finite Difference Method (FDM) for finding deflections in Pre-Stressed Concrete (PSC) beams with non-prismatic sections. Equations are derived for dead and live load bending moment, eccentricity, and depth at any required section. Based on these equations, it is initially established that conventional techniques cannot be adopted for finding deflections in pre-stressed concrete beams with non-prismatic cross-sections and hence FDM was adopted as an alternative. The calculated deflections using FDM are compared with those obtained from equivalent STAAD.Pro. Model and they found to be in close agreement.

Keywords: Deflection; non-prismatic; serviceability; prestressing; FDM; double integration; simply supported.

1. INTRODUCTION

1.1 General

Although there are many techniques of prestressing structural elements, the most common method of applying prestressing force to a concrete member is by applying a predetermined force, which induces an axial compression that counteracts all, or part of, the tensile stresses set up in the member by applied loading. The serviceability limit state is extremely important in the design of PSC structures as it is one of the significant factors that affect their durability. Further, serviceability limit states are indirectly defined and controlled by providing maximum acceptable span to depth ratios. Depending on the type and

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functionality of the structure, deflections may have to be calculated and checked against permissible values.

1.2 Advantages of non-prismatic PSC beams

In case of prestressed prismatic continuous beams, the tendons are to be placed in a parabolic profile below centroid at center (to counter sagging moment) and at supports, the tendon is to be placed above the centroid (to counter hogging moment). In case of non-prismatic beams, the cable profile can be straight but can effectively be parabolic because, as the depth of the section varies, centroid shifts up and thus the cable can be positioned straight throughout the continuous/fixed span. (Fig. 1) The straight tendon profile in case of Non-prismatic PSC beam can be effectively used as the case of a parabolic profiled tendon in a rectangular PSC beam. For same load carrying capacity Non-Prismatic PSC beams are more efficient as the entire section is effectively used in the stress distribution when compared with the rectangular PSC beam.

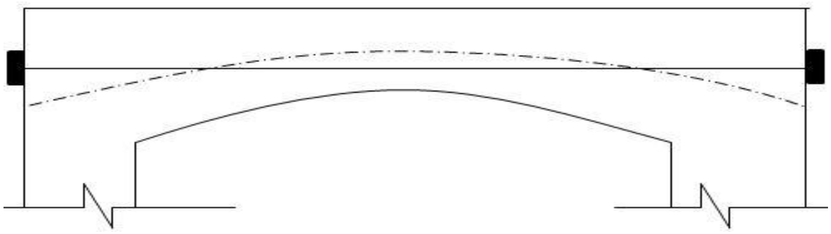


Figure 1.Straight cable profile and parabolic centroidal axis of non-prismatic beam.

1.3 Deflection of non-prismatic PSC beams

An economic beam design often demands different cross-sections along the span. Non prismatic or non-uniform simply supported beams have wide applications in long-span bridges and industrial structures with plate-girders and hinge connections to support heavy loads. There are several methods for solving the problem of deflections of prismatic beams whereas in case of non-prismatic beams the variations in the cross-sections are practically difficult and involves very long and tedious mathematical derivations. Hence this paper is an attempt to find an analytical solution using FDM for evaluating deflection of non-prismatic beams and validate them using a STAAD.Pro. model.

2. LITERATURE REVIEW

Many researchers have investigated the problem of deflection of non-prismatic beams subjected to different boundary conditions. A wide range of methods, including stiffness method, closed-form solutions, Equivalent system method and numerical techniques has been developed. Al-Gahtani and Khan [1] used the boundary integral method (BIM) to find the deflection, shear force and moment of non-prismatic beams with general boundary conditions at both ends. Bahadir Yuksel [2] proposed effective formulae and estimated dimensionless coefficients to predict the fixed end moments, forces, stiffness coefficients and carry-over factors with reasonable accuracy for the analysis and re-valuation of the non-prismatic beams having symmetrical parabolic haunches. He adopted a constant haunch length ratio of 0.5 using FEM. He also [3] proposed an approach for determining temperature effects on symmetrically haunched non-prismatic members. Eisenberger [4] developed an exact stiffness matrix for some particular cases of non-prismatic beam deflection analysis. Eisenberger and Reich [5] have studied static and vibrating behaviour of non-prismatic beams. Ganga Rao and Spyrakos [6] proposed a series solution for differential equations with variable properties over the domain. Karabalis and Beskas [7] developed a method based on exact stiffness and mass matrices for constant width linear height tapers. Miha Brojan [8] made a study on large deflections of slender, non-prismatic cantilever beams subjected to a combined loading of non-uniformly distributed continuous load over the span and a concentrated load at the free end of the beam. Markandeya Raju [9] developed a procedure for the analysis of non-prismatic parabolic profiled PSC concrete haunch beams and studied its performance. Raju Ponnada and Vipparthy [10] proposed a method for estimating deflection in prestressed steel I beams by mentioning the significance of deflection in prestressed elements

Based on the literature, it is clear that the non-prismatic beams are advantageous when compared to prismatic PSC beams. However they involve complexity in the analysis and design as the number of unknown variables increases compared to prismatic beams. Hence, the objective of this work is to formulate an expression for evaluating deflection by considering the combined effect of prestressing and total load on the deflection and to validate them using STAAD.Pro. model.

3. ESTIMATION OF DEFLECTION IN PRESTRESSED BEAMS

3.1 Expressions for estimation of deflection using double integration method

Consider a simply supported beam of span ‘l’, width ‘b’, Flexural rigidity EI_{xx} subjected to a total uniformly distributed load of w/unit length and prestressed by a force ‘P’ at an eccentricity ‘e_x’, ‘ρ’ being the density of the concrete and ‘h’ being the height of the parabola at center of the beam. (Fig. 2) Then the variation of the parabolic haunch (profile) is given by

$$y_x = \frac{4hx}{L} \times \left(1 - \frac{x}{L}\right) \tag{1}$$

The expressions for depth ‘dx’, moment of inertia ‘Ixx’ and area of cross-section ‘Ax’ at any section along the length of the beam are obtained from the basic principles [9]

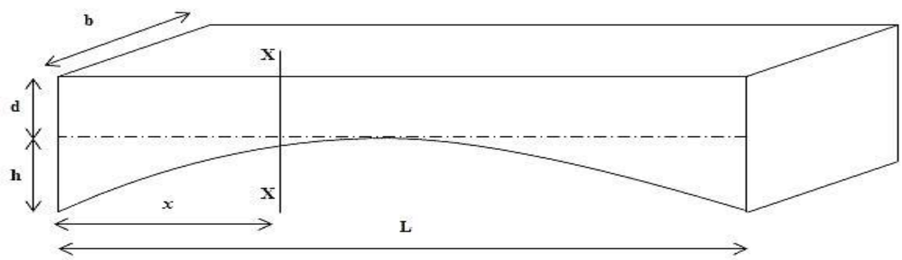


Figure 2. Details of non-prismatic beam

'M_x' at any section and are given as follows.

$$M_x = \left[\frac{b}{2}(d+h)x(L-x)\right]\rho - \left[\left(\frac{h}{3L^2}x^4 - \frac{2h}{3L}x^3 + \frac{hL}{3}x\right)\rho b\right] + 0.5W_x \times (L-x) \tag{2}$$

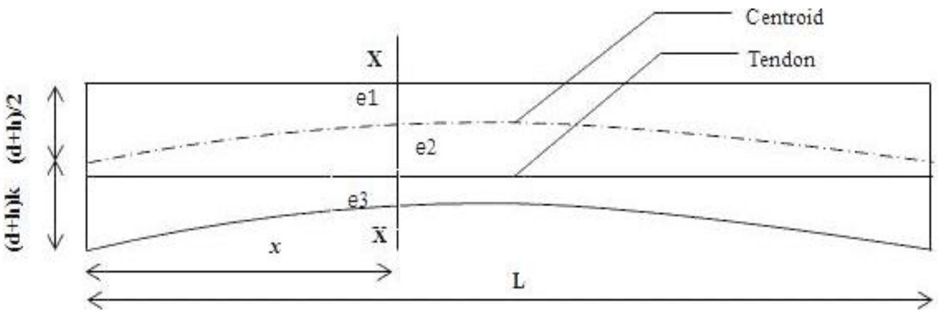


Figure 3. Location of tendon

There are two possibilities for the tendon location where the prestressing force 'P' acts, above the centroid and below the centroid. In the present study, it is limited to the latter case in which the tendon is located below the centroid for achieving the effective prestressing in the compression zone. (Fig. 3) For determining eccentricity, a non-dimensional parameter k is introduced which is defined as the ratio of the depth of the rectangular portion of the beam'd' to the total depth of beam (d + h). The eccentricity 'e_x' at any section is given as follows

$$e_x = \frac{d_x}{2} - (d+h)k + y_x \tag{3}$$

Hence from the theory of pure bending moment at transfer 'M_{xt}' and service 'M_{xs}' are as follows

$$M_{xt} = \left[\frac{b}{2}(d+h)x(L-x)\right]\rho - \left[\left(\frac{h}{3L^2}x^4 - \frac{2h}{3L}x^3 + \frac{hL}{3}x\right)\rho b\right] + Pe_x \tag{4}$$

$$M_{xs} = \left[\frac{b}{2}(d+h)x(L-x)\right]\rho - \left[\left(\frac{h}{3L^2}x^4 - \frac{2h}{3L}x^3 + \frac{hL}{3}x\right)\rho b\right] + \frac{W_x}{2} \times (L-x) + Pe_x \tag{5}$$

The deflection for non-prismatic PSC beams at transfer stage is obtained by solving the Eq. (4), which is given by Eq. (6). If Eq. (6) is simplified for any data, it results in a complex number and hence is futile in structural engineering point of view. Therefore double integration method is not suitable for finding deflections in non-prismatic PSC beams.

3.2 Expressions for estimation of deflection using finite difference method

Most of the problems of bending of beams and buckling of columns involve solution of ordinary differential equations for determining moments, shear forces and deflections.

Partial differential equations are invariably required in solving problems of two dimensional structural elements. Direct solutions are possible in cases where the load distribution, sectional properties and boundary conditions can easily be represented by mathematical expressions. In the current study, the beam has complex geometry with variable section properties. In the previous section it was established that exact integer solutions cannot be obtained by adopting conventional methods like double integration method. Forward difference method was adopted for evaluating deflections in which the following Eq. (7) is involved. In this work the FDM is adopted by dividing the beam into 2,4,6 and 8 parts and deflection is obtained at center at both transfer and service condition. Derivation for deflection of beam at center with 2 parts in service stage is as follows.

$$
\begin{aligned}
Y_t = & \left[\frac{18L^2(d+h)}{h} \left\{ \begin{bmatrix} \frac{L}{8d^2}\left[L-\frac{1}{2}\right]\log\left\{\frac{A}{B^2}\right\} \\ +\frac{\left[\frac{L}{4d}\left(\frac{1}{2}-L\right)-\frac{L^2}{8h}\right]x+\left[\frac{2L^3}{16h}-\frac{L^2}{16h}+\frac{L^2}{8d}\left(L-\frac{1}{2}\right)\right]}{A\times B^2} \\ +\left[\frac{L^2}{16hd}\log\left\{\frac{A}{B^2}\right\}\right] \end{bmatrix} \right\} \right] \\
& -\frac{3L^4}{2h^2}\left[\begin{matrix} -\frac{i}{L}\sqrt{\frac{h}{d}}\left[\frac{1}{2}(C)[\log(C)-1]-\frac{1}{2}(D)[\log(D)-1]\right]-\frac{(d+3h)}{2d}\left\{\log\frac{A}{B^2}\right\} \\ +\frac{(d+h)(d+5h)}{2}\left[\frac{1}{8d^2}\left\{\frac{A}{B^2}\right\}-\frac{\frac{1}{8d}}{a}+\frac{\frac{L}{8h}}{ab}\right] \end{matrix} \right] \\
& +\frac{3PL^2}{16bh}-\frac{3PL^2}{16bh}\left\{ \begin{bmatrix} (d+2h-2k(d+h))\log\left\{\frac{(d+h+A)}{B^2}\right\} \\ +\frac{2(d+h)(k-1)}{(d+h+a)}-\frac{1}{d}\log\left[\frac{A}{B^2}\right] \end{bmatrix} \right\}+c1x+c2
\end{aligned}
\tag{6}
$$

Where,

$$A=\frac{4h}{L^2}x^2-\frac{4h}{L}x+d+h,\ B=2x-L,\ C=B-iL\sqrt{\frac{d}{h}},\ D=B+iL\sqrt{\frac{d}{h}}$$

$$\frac{d^2y}{dx^2}=\left[\frac{y_{n+1}-2y_n+y_{n+1}}{h^2}\right] \tag{7}$$

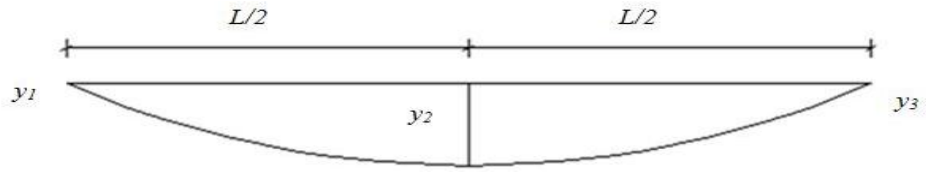


Figure 4. Deflection of beam with 2 parts

y_1, y_2, and y_3 are the deflections of the beam at their corresponding nodes. As the beam is divided into two parts interval for this case is h=L/2.

On substituting Eq. (5) with x= L/2 in the Eq. (7) with n=2 and applying boundary conditions for simply supported beam i.e.; y_1= y_3= 0, the deflection at L/2 is obtained. Similarly the expressions for central deflection at transfer and service stage that are obtained by diving the beam into 4, 6, 8 parts.

$$y_2 = \frac{3L^2\left[\frac{\rho bL^2}{8}\left(d+\frac{h}{6}\right)-P\left[(d+h)(0.5-k)+\frac{h}{2}\right]+\frac{WL^2}{8}\right]}{2Ebd^3} \tag{8}$$

4. VALIDATION

4.1 Need for validation and modelling procedure

To establish the accuracy of results obtained from FDM solutions a typical beam is modelled in STAAD.Pro. and the central deflections are compared. The beam of span ‘L’ is divided into ‘n’ (2, 4, 6, 8) parts. For each part the offsets in y-direction are calculated and applied such that the beam top for all parts is aligned. The width for all parts is constant and depths are calculated at each section and for each part the maximum depth at its ends are applied. After assigning the self-weight and live load, the prestressing force is applied such that the tendon is actually straight and at a constant distance from top level of the beam.

5. RESULTS AND DISCUSSION

Four non-prismatic PSC beams (with 2, 4, 6, 8 parts) are modelled in STAAD.Pro. (Fig. 5) for the following data (Table. 1) and the central deflections are obtained.

Table 1: Common data for case study considered for validation.

Span ‘L’ (mm)	Breadth ‘b’ (mm)	Live Load ‘W’ (kN/m)	Density ‘ρ’(N/mm3)
30000	600	30	2.4 x10-05
Prestressing Force ‘P’ (kN)	Young’s Modulus ‘E’ (N/mm2)	Depth of Haunch ‘h’ (mm)	
2000	33541	500	

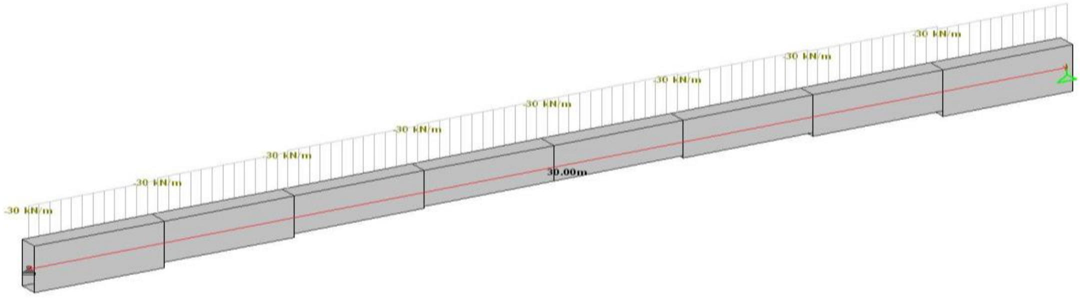


Figure 5. Typical non-prismatic beam with 8 parts modelled for data in Table 1

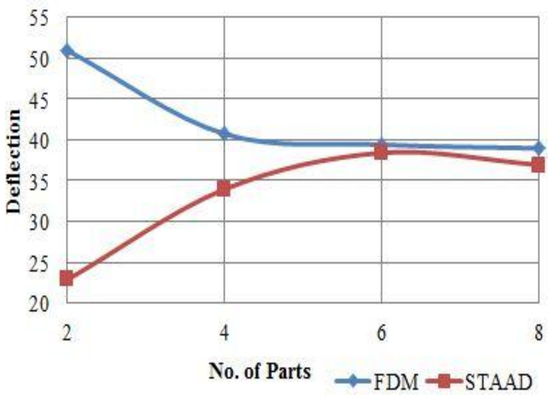
For the same data the results from FDM are obtained using Eq. (8) for 2 parts and similarly derived equations for 4, 6 and 8 parts. The results are presented as follows.

Table 2: Common data for case study considered for validation.

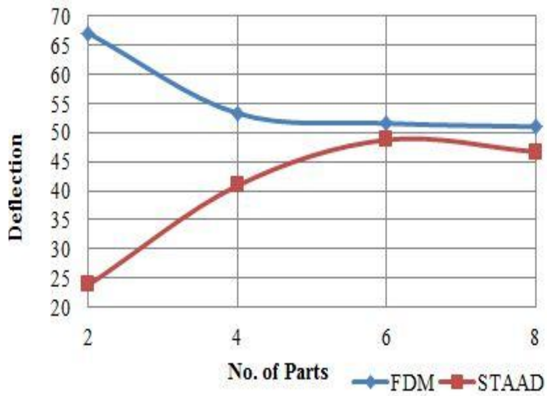
	Results from FDM				Results from STAAD. Pro. Model			
No. of parts Load Case	2	4	6	8	2	4	6	8
DL	50.98	40.73	39.42	38.98	22.86	33.96	38.42	36.94
LL	67.08	53.33	51.60	51.02	23.81	40.90	48.77	46.72
PT	-19.88	-17.75	-17.46	-17.37	-8.42	-13.87	-16.41	-15.87
DL+PT+LL	98.19	76.30	73.64	72.64	38.30	61.03	70.81	67.82

DL - Dead Load, LL – Live Load, PT – Prestressing

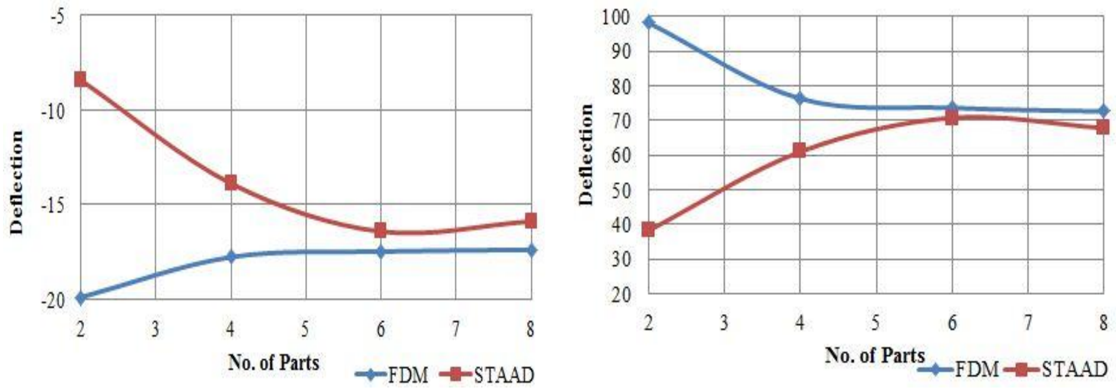
For the case study under consideration, the above results are represented in graphical format to ascertain the possibility of convergence of results.



(a) Dead Load



(b) Live Load



(c) Prestressing (d) Dead Load+LiveLoad+Prestressing

Figure 6. Convergence graphs for deflection.

From the above graphs (Fig. 6) it is observed that the deflection obtained from FDM and STAAD .Pro. are converging which indicates that the solution obtained from FDM is valid. Values obtained from FDM are marginally over estimated compared to values obtained from STAAD. Pro. Dividing the beam into more number of parts leads to accurate values. The variation in the results obtained from STAAD. Pro. and FDM is more in case of beam analysed with lesser number of parts and less in case of beams analysed with more number of parts.

5. CONCLUSION

From this work it can be concluded that conventional methods like double integration method are not suitable for finding deflections in non-prismatic PSC beams. As the results from FDM are converging with those obtained from STAAD.Pro. model the equations derived using FDM can be used safely for predicting the deflection of non-prismatic prestressed concrete beams.

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