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# NEW MATHEMATICAL MODELING OF STEEL PANEL ZONE WITH THIN TO THICK COLUMN FLANGES

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## ABSTRACT

The response of a steel moment resisting frame (MRF) depends on the specifications of its main components, namely the columns, beams and connections. One important connection element which can significantly affect frame behavior is the panel zone (PZ). The PZ is described to be an element mainly subjected to shear stresses and its failure mode is often governed by shear yielding. Several analytical models for PZ behavior exist, in terms of shear force-shear distortion relationships. Among these models, the Krawinkler PZ model is the most popular one which is used in codes. Some studies have pointed out that Krawinkler's model gives good results for the range of thin to medium column flanges thickness. The model presented here is applicable to both thin and thick column flange. More than four-hundred beam-column connections are included in the parametric study, with varied parameters being: beam depth, column flange thickness, column web thickness, and beam flange thickness. The elastic stiffness, shear yield strength and ultimate shear strength of the PZ obtained from FE analysis, are compared with those obtained from available mathematical models to show differences, especially in the case of thick column flanges. In the paper a simple mathematical model for estimating the stiffness and shear strength in the PZ is introduced. In this model both shear and bending deformations are considered. A comparison between the results of proposed method herein with FE analyses shows the average error is significantly reduced which demonstrates the accuracy, efficiency, and simplicity of the proposed model.

**Keywords:** Panel zone; shear strength; beam-column connection; mathematical model; FEM analysis.

## 1. INTRODUCTION

In linear analysis, the beam-column joint is treated as a point at the intersection of the beam

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and the column and generally the joint stiffness is formulated based on the clear span of the member. To further the calculation of the joint deformations in the structure without using additional elements, a reduced rigid end offset as a fraction of the actual joint size at both ends of the beams and columns can be defined [1]. Using this approximate approach it is difficult to determine the appropriate fraction of actual joint sizes for the member end offsets to account for the flexibilities of these joints. In addition, in nonlinear analysis, the beam-column joint can yield in shear due to the large moment transferred through the joint. The hinge formation pattern of the structure will be erroneous without considering the relative flexibilities of joints with respect to other elements; therefore, a separate element that realistically specifies the behavior of the beam-column joint is needed. For this aim, a panel zone (PZ) joint element for modeling steel beam-column joints for nonlinear analyses of moment resisting frames was developed.

In the 1994 Northridge earthquake, brittle fractures in beam-column connection areas occurred causing considerable damage. Following this event, various related American institutions have been conducting experimental researches on the behavior of steel moment connection and developing analytical modeling techniques. In a modified pre-Northridge connection, now called the post-Northridge connection, there are only two sources to dissipate seismic energy, the beam-end and the PZ [2-4].

Depending on the basis for computing the required shear strength, PZs can behave quite differently. If the PZ is weak relative to the girder flexural strength, most of inelastic behavior may take place within the connection, while stronger PZs will allow shared energy dissipation between the joint and the connected girders. Specifically, a weak PZ will put relatively high stress and strain concentrations at the location of the kink in the column flange adjacent to the critical girder flange-to-column welds. This may increase the potential for low-cycle fatigue and brittle fracture. A strong PZ may increase the stress and strain concentrations in the girder, on the other side of the critical girder flange-to-column welds and at the critical weld access hole area. It is presently not clear whether a weak or strong PZ is best for the overall resistance of the connection [5].

The PZ, which is the region in the column web defined by the extension of the beam flange lines into the column (Fig. 1), is known to have stable hysteretic and ductile properties [6,7]. These features make the PZ an attractive component for energy dissipation in steel MRFs under seismic loading.

At the beginning of the 1970's, studies were conducted to understand the inelastic behavior of joints in moment-resisting frames [7,8]. In order to figure out different loading regimes, several loading conditions were simulated on the tests, whereby gravity and cyclic seismic loads were applied to different sub-assemblages. Several years later, a number of test were performed by Popov et al. [9] in order to verify the extreme loading conditions on joints and to study the cyclic behavior of large beam assemblies. Tsai et al. [10] carried out further testing on similar joint subassemblies. The main purpose was to study the performance of seismic steel beam-column moment joints. The research concluded that the PZ has a significant effect on joint behavior and that the inelastic deformation capacity of the joint can be increased if the PZ is correctly proportioned. Hence, among the parameters studied in both studies was the design criteria used for the PZ.

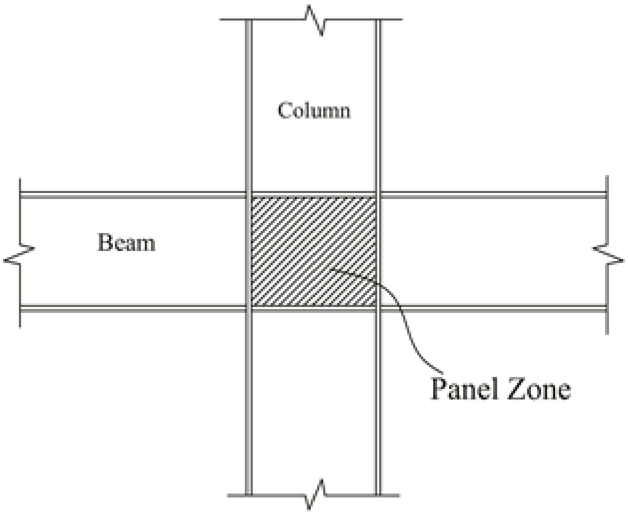



Figure 1. Joint panel of an interior beam-to-column connection

It has been demonstrated analytically and experimentally that high shear forces are often developed in a joint panel, and the PZ shear and deformation effect will have a pronounced influence on frame behavior [11].

Among several analytical models, the Krawinkler joint model is used in codes and guidelines (e.g. [12,13]); however, this model gives good results for joints with thin to medium column flange thicknesses [11]. In this paper, using an extensive nonlinear finite element analysis, a new sample mathematical model is proposed to cover the range of thin to thick column flanges. In the recent years several studies have been done to evaluate the behavior of panel zones [14-17].

## 2. EXİSTİNG MATHEMATİCAL MODELS

Mathematical models for the behavior of the steel PZ in terms of shear force-shear distortion $(V-\gamma)$ relationships have been suggested by many researchers (e.g., [18-20] based on either experimental observations or modifications to pre-existing models. This representation can be carried out by means of different relationships and levels of precision. Techniques for modeling the beam-column connections in steel structures are classified in the following sections [21,22].

*2.1 Linear centerline model*

To design structures or evaluate the performance of existing buildings, two criteria are needed, the strength of members and the stiffness of system. The linear elastic model using the central line model is suitable for designing a steel moment resisting frame. Although the model shows appropriate results for design, it cannot accurately forecast the distribution of the inelastic member forces created by the dynamic load.

*2.2 Elastic model with panel zone*

Fig. 2 shows the scissors model, which includes a PZ. In this model, beams and columns are connected via rigid links in a PZ, and the crossroad hinge is connected via a spring with the stiffness of the PZ. Since this model contains dimension and stiffness of the PZ, it forecasts more accurately the distribution of shear forces, flexural moments, and axial forces than the above model.

*2.3 Nonlinear centerline model*

The inelastic model is useful in assessing the behavior of existing buildings. To conduct nonlinear analysis, most commercial programs, as shown in Fig. 3, connect springs with nonlinear features with section properties of beams and columns.

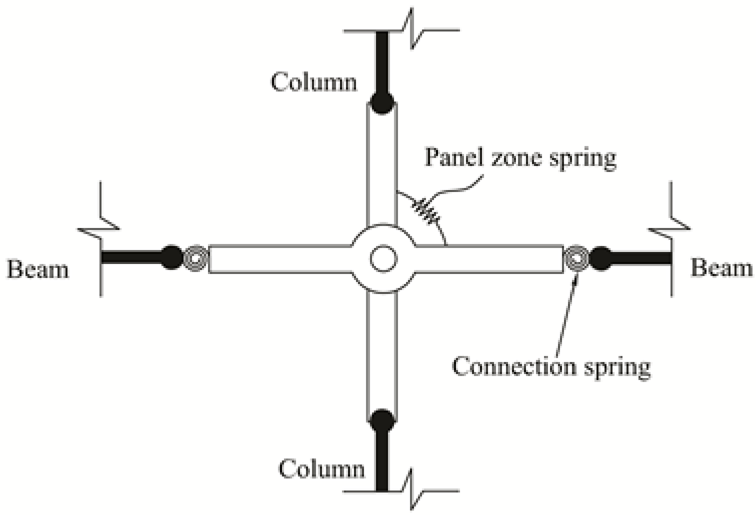



Figure 2. Joint panel of an interior beam-to-column connection

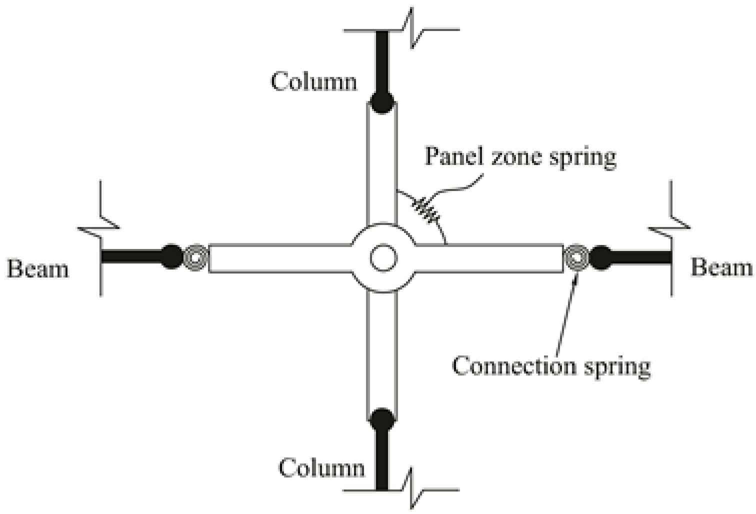



Figure 3. Nonlinear Centerline Model

*2.4 Nonlinear model with panel zone*

The nonlinear analytical models, which include PZs, are categorized into three techniques. The first model is the scissors model in Fig.2 containing nonlinear property for a spring element. The second model [23], shown in Fig. 4, uses two springs having a PZ stiffness and the average strength of the PZ and the beams.

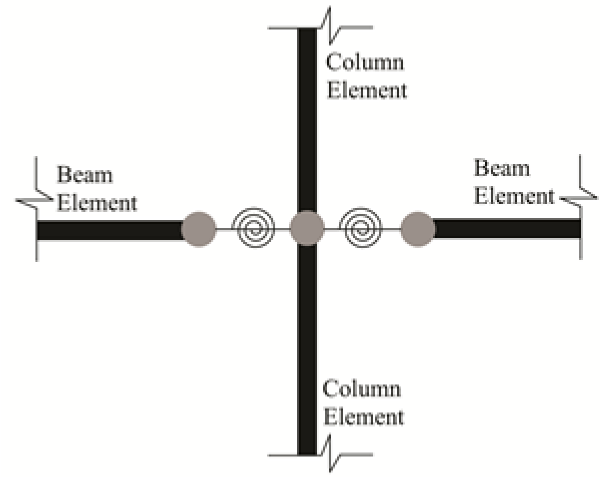



Figure 4. Shi's model [23]

Although this model can express stiffness and strength of a PZ, it cannot accurately express its shear deformation without expression of the accurate dimension of the PZ. The third model, developed by Krawinkler [24], models a PZ into 8 rigid bodies (Fig. 5). Actually, this model shows the least difference between the actual behavior of a structure and the behavior of the analytical model.

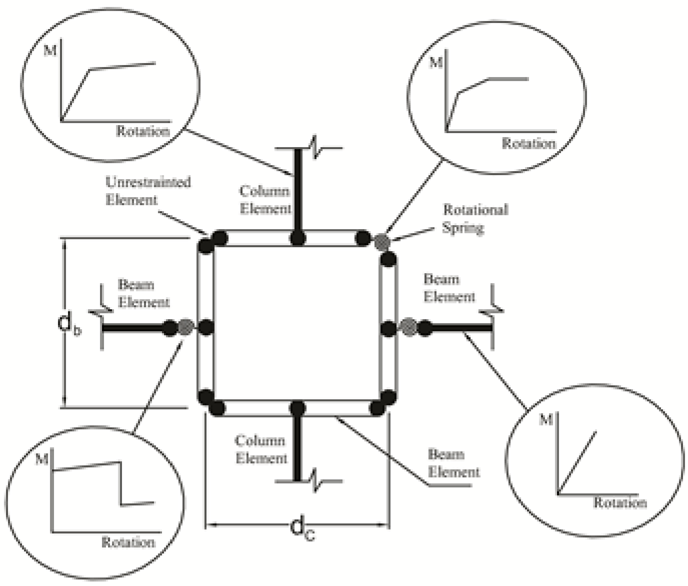



Figure 5. Krawinkler model [24]

*2.5 Characteristics of original Krawinkler model*

The Krawinkler joint model [18] is a simple and useful model to describe the shear force-shear distortion $(V-\gamma)$ behavior of a joint panel. Physically, this is the only model that makes sense. The others seem either much more complicated (for no reason) or physically, they don't make sense. AISC and FEMA 355D specifications also include this model in the PZ design.

The model is simple and gives generally conservative results. Krawinkler's model is the most simple model (requires the fewest number of parameters) that completely describes PZ behavior in steel moment frames. One could develop a more complicated model with increased number of parameters, and calibrate that model to better fit experimental response. However, complicated models are, by nature, more cumbersome to use. In many cases, even if extra effort is spent calibrating a model to a particular test, the same model will not work as well for another test result. For these reasons, Krawinkler's model is the model of choice.

The control values for the model are given as follows:

$$V_y = \frac{F_y}{\sqrt{3}} A_{eff} = \frac{F_y}{\sqrt{3}} (0.95 d_c t_p) \approx 0.55 F_y d_c t_p \tag{1}$$

where $V_y$ is the PZ shear yield strength, $F_y$ is the yielding strength of the PZ, $A_{eff}$ is the effective shear area, $d_c$ is the depth of the column, and $t_p$ is the panel thickness (thickness of the column web, plus the doubler plate thickness, if present). The corresponding yield distortion, $(V-\gamma)$, is given as:

$$\gamma_y = \frac{F_y}{G\sqrt{3}} \tag{2}$$

The elastic stiffness, $K_e$, of the PZ can then be written as:

$$K_e = \frac{V_y}{\gamma_y} = 0.95 d_c t_p G \tag{3}$$

where $G$ is the shear modulus of the column material.

When the panel shear exceeds $V_y$, the elastic stiffness contribution from the panel web is assumed to be zero. The stiffness contribution when $V > V_y$ can only come from the resistance of the elements surrounding the panel. This post-yield panel stiffness, $K_p$, is given as [18]:

$$K_p = \frac{1.095 b_c {t_{cf}}^2 G}{d_b} = \frac{1.04 b_c {t_{cf}}^2 G}{0.95 d_b} \tag{4}$$

where $b_c$ and $t_{cf}$ are the width and thickness of the column flange, respectively, and $d_b$ is the beam depth.

If it is assumed that the post-yield stiffness of the joint panel as given by Eq. (4) is valid for the range $\gamma_y < \gamma < 4\gamma_y$, the ultimate strength, $V_p$, of a joint panel is given by:

$$V_p = V_y\left(1+\frac{3K_p}{K_e}\right) \approx 0.55F_y d_c t_p\left(1+\frac{3b_c t_{cf}^{\ 2}}{d_b d_c t_p}\right) \tag{5}$$

This strength is assumed to be reached at a value of $\gamma_p = 4\gamma_y$. Beyond $\gamma_p$ an appropriate value of the strain-hardening can be assumed to fully define the trilinear shear force-shear deformation relationship of the PZs (α is the ratio between post-yield tangent and initial elastic tangent). A schematic plot of $V$ versus $\gamma$ for the Krawinkler panel model is shown in Fig. 6.

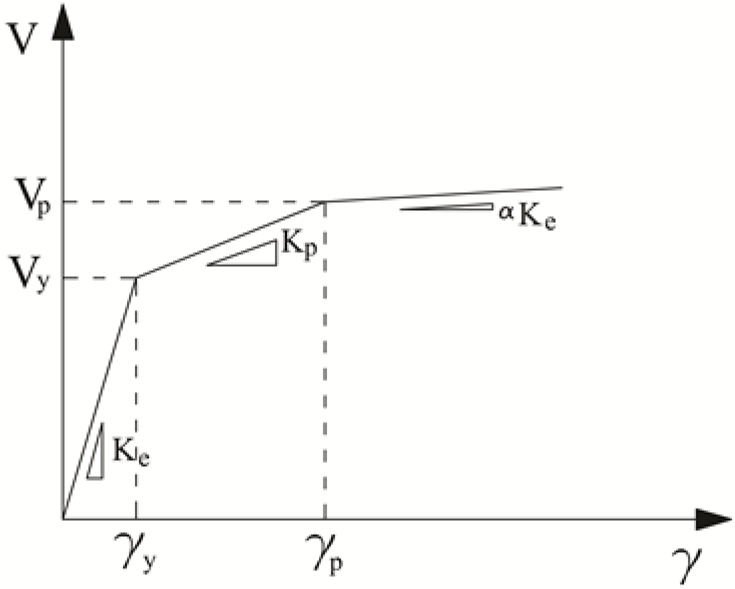



Figure 6. Shear force-distortion response of the Krawinkler joint panel model

## 3. FINITE ELEMENT MODEL

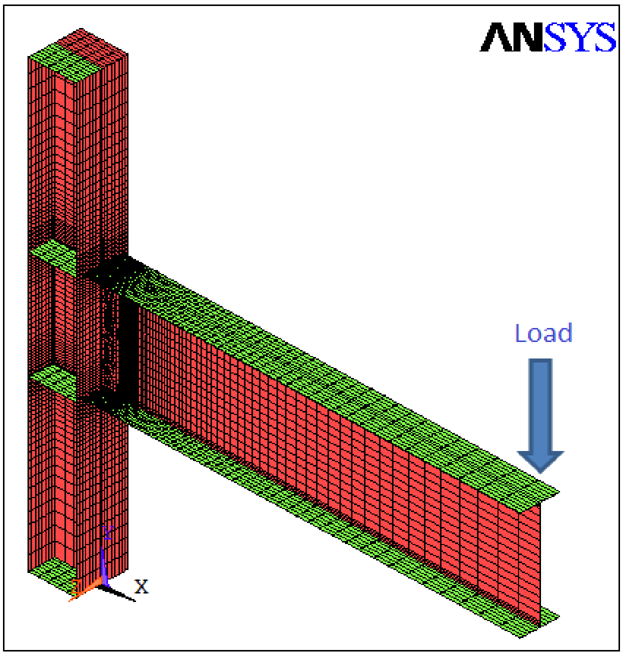
To determine the PZ model behavior, the effects of PZ strength and stiffness on connection response are parametrically studied using finite element models. Version 14.0 of the general purpose nonlinear finite element program ANSYS [25] was used to model 432 fully restrained bolted web-welded flange beam-to-column moment connections. The whole computational volume for the present parametric study is estimated to be of the order of (4 specimens) × (6 column flange thicknesses) × (6 column web thicknesses) × (3 beam flange thicknesses) = 432 finite element analysis. Shell-element models were prepared to study local and global instabilities in the connections because such models are computationally more efficient than solid-element models for this purpose [26]. A four-node shell element (Shell 181 element with six degrees of freedom at each node) has been used to model the specimens. Such elements were successfully employed by El-Tawil et al. [27] for a related study funded by the SAC Joint Venture. The size of the finite element mesh varied over the length and height of the specimen. A fine mesh was used near the connection of the beam to the column. A coarser mesh was used elsewhere in order to reduce the computational efforts. Beam flanges were modeled using 5 layers of elements through the flange depth and 10 elements across the flange half-width. The distribution of geometric imperfections matched the first eigenvector of the loaded connection configuration. The maximum imperfection was chosen as one percent of the beam flange thickness.

Two lines of nodes at each end of the column were restrained against translation only (i.e., a pinned connection) to approximately replicate the support conditions used for the laboratory tests. A vertical displacement history was imposed at the free end of the beam using the displacement control feature in ANSYS.

Since verification is necessary for numerical models, before performing the parametric study some well-known experimental programs were considered to verify the finite element modeling methodology and general assumptions on the nonlinear analysis.

*3.1 Verification study*

To verify the accuracy of finite element modeling, specimens SAC3, SAC5 and SAC7 (Fig. 7) of reference [28] and specimen SPE1 of reference [29] was remodeled using finite element method. Shown in Fig. 8 is a comparison between analytical and experimental results. As this figure shows, the analytical result is in good agreement with experimental result.



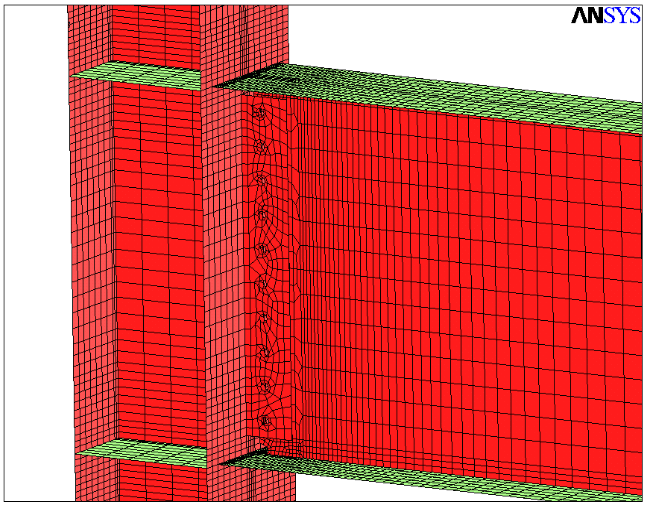



Figure 7. Finite element model of SAC specimens

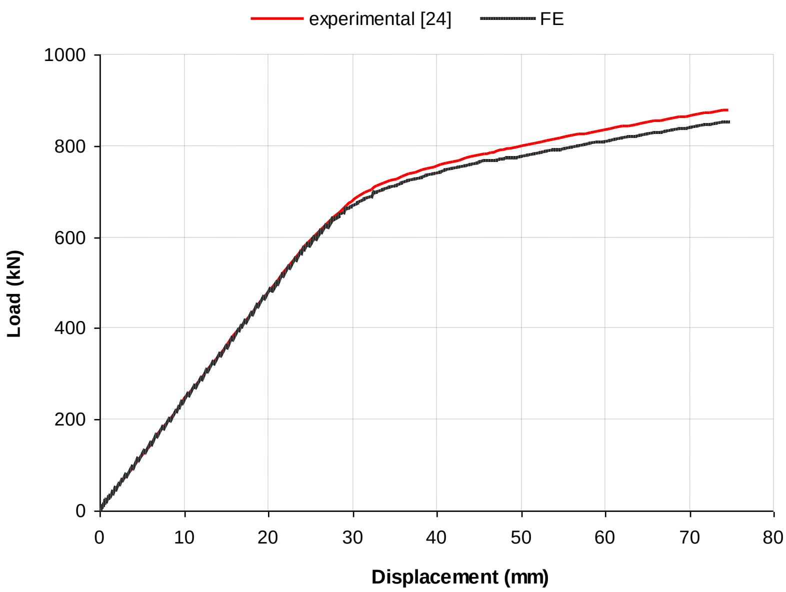
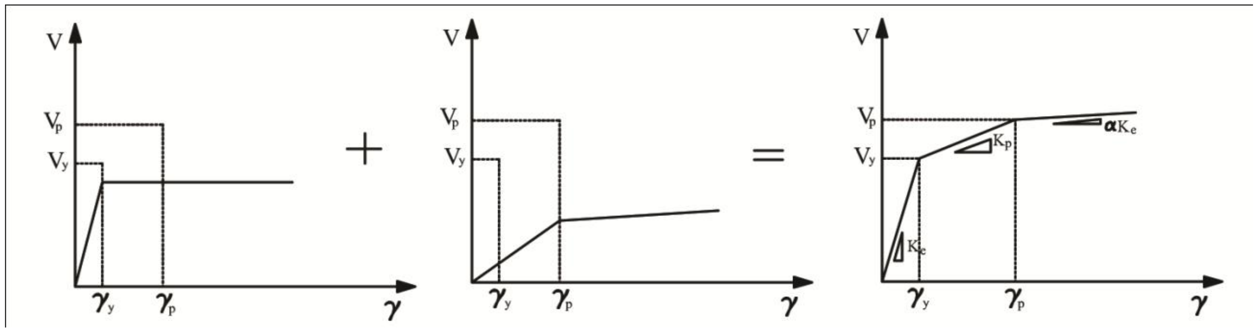



Figure 8. Comparison between the predicted response in FE verification study and experimental results of SAC7 specimen

The typical load response from the panel was characterized by three phases. First, elastic shear response followed by yielding, according to the von Mises criterion. Second, reserve in strength corresponded to the surrounding elements of the panel. Finally, a post yield strength characterized by strain hardening of the steel. The elements that determine the stiffness and strength of a PZ are the web and the flange of a column. The sum of these two elements determines the shear-force shear-distortion $(V-\gamma)$ curve of a PZ, and shows the trilinear behavior (Fig. 9).

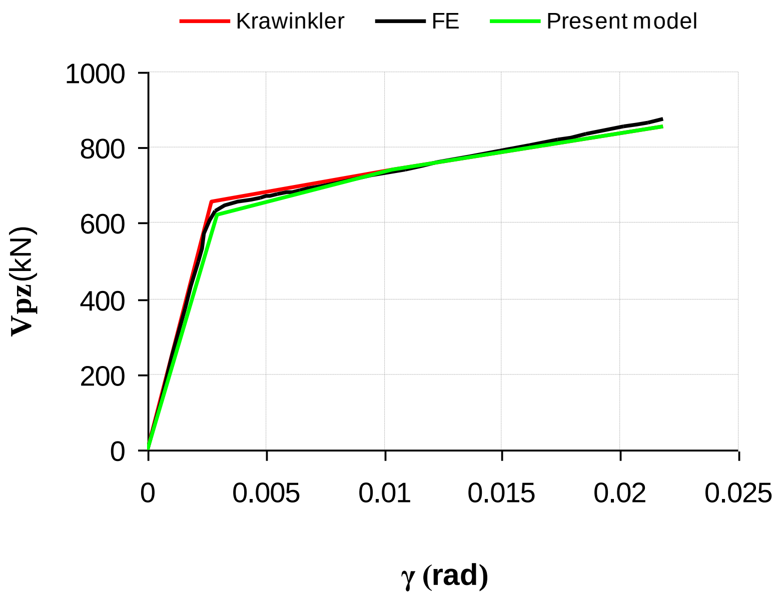



(a) column web (b) column flange (c) trilinear PZ

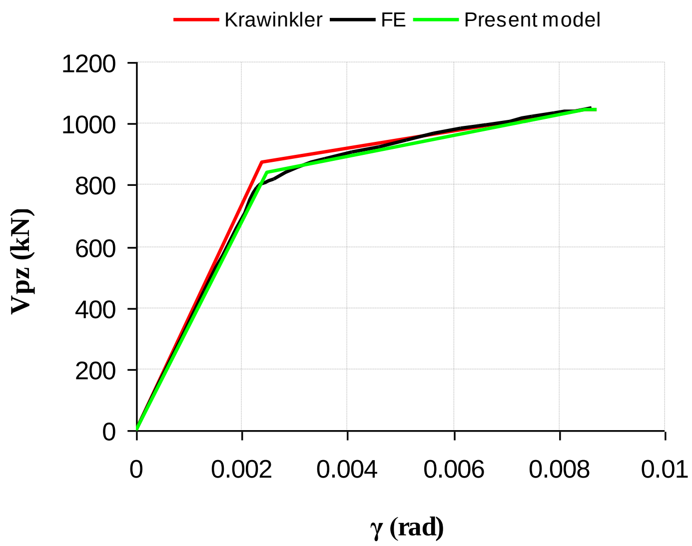
Figure 9. Moment-rotation Behavior of PZ

The Krawinkler PZ model proposes relationships between PZ shear force and deformation for monotonic loading. These relationships have been used as a basis of mathematical models for nonlinear rotational springs representing the PZ. As it can be seen in Figs. 10a and 10b, Krawinkler's model gives good results for joints with thin to medium thickness column flanges.

However, it is pointed out by Krawinkler that a new model might be needed for joints with thick column flanges [18]. This issue can be observed in Fig. 11. This paper proposes a model that considers both bending and shear deformation, covering the range of thin to thick column flanges.




(a)




(b)

Figure 10. Comparison of the FE model and Krawinkler's model (a) with $t_{cf}$ 16.3 mm (b) with $t_{cf}$ 21.7 mm

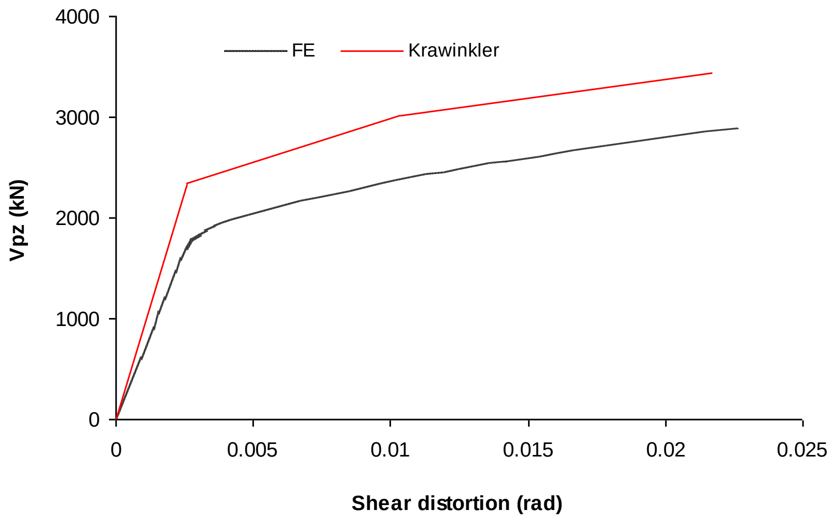



Figure 11. Comparison of the FE model and Krawinkler's model with $t_{cf}$ 48 mm

## 4. PROPOSED ANALYTİCAL MODEL

Column flange thickness effects PZ yield shear and elastic stiffness [30]. In this study, to find the mathematical model of PZ behavior both bending and shear deformations of the PZ are considered. For this purpose, it is assumed that the equivalent of column web and column flanges (Fig. 12a) is similar to an element shown in Fig. 12b.

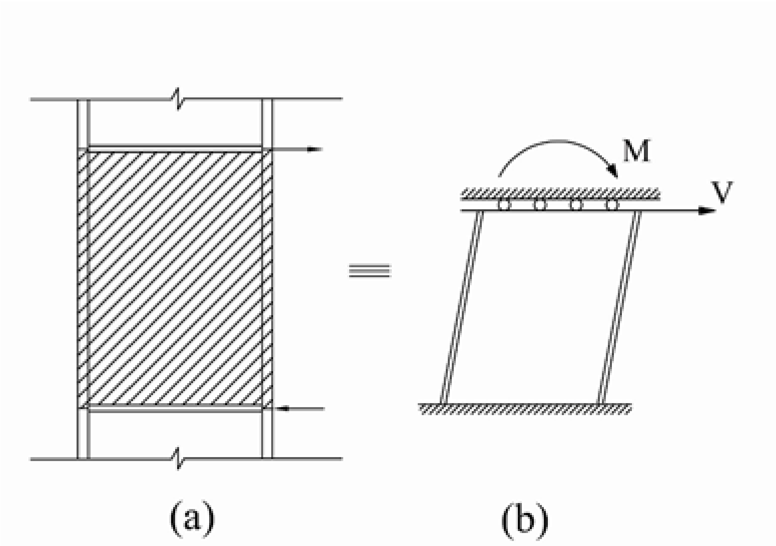



Figure 12. Panel zone (a) complex of column web and column flanges (b) equivalent model

In order to achieve the stiffness of PZ the energy method and a simplified model are used (Fig. 12b). Based on stiffness concept and using the method of least work the external displacement, Δ, is given by:

$$\Delta = \frac{\partial}{\partial K}\left(\int_0^{d_b}\frac{m^2\,dx}{2EI} + \int_0^{d_b}\frac{V^2\,dx}{2\eta GA}\right) = \frac{\partial}{\partial K}\left(\int_0^{d_b}\frac{(M+Kx)^2\,dx}{2EI} + \int_0^{d_b}\frac{K^2dx}{2\eta GA}\right) = 1 \tag{6}$$

where $m$ = internal moment in the member, expressed as a function of $x$; $E$ = modulus of elasticity of the steel; $I$ = moment of inertia of cross-sectional area; $V$ = internal shear in the member, expressed as a function of $x$; $A$ = cross-sectional area of the member, and $\eta$ = form factor for the cross-sectional area.

After simplifying:

$$\frac{Md_b^2}{2EI} + \frac{Kd_b^3}{3EI} + \frac{Kd_b}{\eta GA} = 1 \tag{7}$$

and it can be written that:

$$\theta = \frac{\partial U}{\partial M} = \int_0^{d_b}\frac{(M+Kx)\,dx}{EI} = 0 \tag{8}$$

therefore from Eq. (8):

$$\frac{Md_b}{EI} + \frac{Kd_b^2}{2EI} = 0 \Rightarrow M = \frac{-Kd_b}{2} \tag{9}$$

Substituting Eq. (9) into Eq. (7), the stiffness, $K$, is given as follows:

$$K = \frac{1}{\frac{d_b}{\eta GA} + \frac{d_b^3}{12EI}} \tag{10}$$

Often, the shear force-shear distortion $(V-\gamma)$ relation is used to investigate the behavior of PZ. Thus, the Eq. (10) is multiplied by $d_b$ and the related stiffness can be written as:

$$K = \frac{1}{\frac{1}{\eta GA} + \frac{d_b^2}{12EI}} = \frac{\eta GA}{1 + \frac{d_b^2}{12EI}(\eta GA)} \tag{11}$$

Replacing $E = 2.6G$, and $I \approx A_{cf} d_c^2 / 2$ ($A_{cf} = b_c t_{cf}$) in Eq. (11), the modified initial stiffness, $K_{e,\text{mod}}$, of the panel component is expressed as follows:

$$K_{e,\text{mod}} = \frac{\eta GA}{1 + 0.064\eta \frac{A}{A_{cf}} (\frac{d_b}{d_c})^2} \tag{12}$$

if $A = A_w$ (area of the web), then the parameter $\eta$ is equal to 1 [31]. A comparison between results of 432 finite element models and above formulation showed that if $A_w$ is considered as $d_c t_{cw}$, the following equation is better matched with results obtained from FE, therefore:

$$K_{e,\text{mod}} = \frac{GA_w}{1 + 0.064 \frac{A_w}{A_{cf}} (\frac{d_b}{d_c})^2} \tag{13}$$

Consequently, the following relationships can be used to make the mathematical model:

$$V_{y,\text{mod}} = K_{e,\text{mod}} \times \gamma_y \tag{14}$$

$$V_{p,\text{mod}} = V_{y,\text{mod}} \left(1 + \frac{3K_p}{K_{e,\text{mod}}}\right) \tag{15}$$

it is assumed that strain hardening begins at $\gamma = 4\gamma_y$. The results show that the modified relations also depend on the ratios of web area to flange area ($A_w/A_{cf}$) and beam depth to column depth ($d_b/d_c$).

A comparison between this mathematical model and all of 432 FE models shows the average and maximum error among are equal to 1.15% and 8.8%, respectively. As an example Fig. 13 shows a sample that using these corrections (Eqs. 13 through 15), the proposed trilinear model is compatible with FE results, especially in the case of thick column flanges.

Also, the obtained errors of the proposed mathematical model and other mathematical models in comparison with 432 FE models are listed in Table 1 and Table 2 to show the accuracy of the present model.

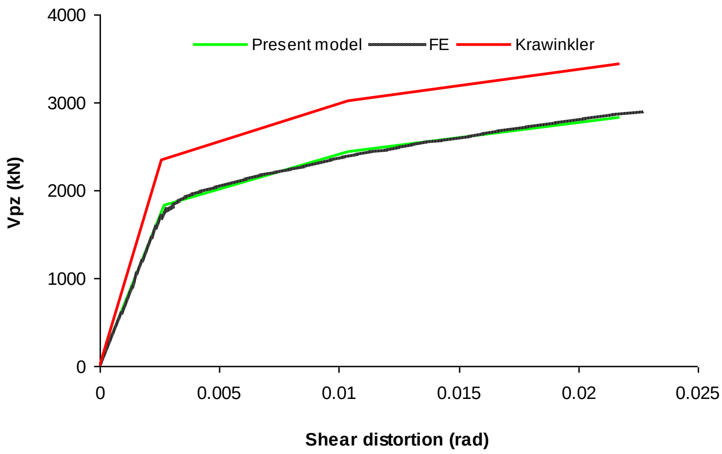



Figure 13. Comparison of the FE model, Krawinkler's model and proposed model with $t_{cf}$ 48 mm

Tables 1 and 2 show that the model introduced in the present work has better performance as compared to the other models; especially in comparison with initial model (Krawinkler's model) the proposed model has significantly reduced the error.

Table 1: Errors in PZ elastic stiffness of different mathematical models in comparison with 432 FE models

| elastic stiffness | $K_e$ | | | | |
|---|---|---|---|---|---|
| model | Krawinkler | scissors | Shi | Lu | proposed model |
| average error (%) | 18.05 | 18.05 | 16.94 | 12.77 | 1.15 |
| max error (%) | 31.95 | 31.95 | 24.55 | 22.72 | 8.8 |

Table 2. Errors in PZ shear ultimate strength of different mathematical models in comparison with 432 FE models

| ultimate shear strength | $V_p$ | | | | |
|---|---|---|---|---|---|
| model | Krawinkler | scissors | Shi | Lu | proposed model |
| average error (%) | 16.61 | 16.61 | 15.04 | 14.73 | 1.08 |
| max error (%) | 25.66 | - | 18.64 | 17.11 | 6.83 |

## 5. CONCLUSIONS

The research performed and presented in this paper aimed at inquiring the development of a panel zone model which eliminates the limitations of other mathematical models. More than four-hundred finite element modeling have been carried out in order to present the new model. In this model, both bending and shear deformation modes are considered which leads to more accurate results. The proposed model is simple and can be used for both thin and thick column flanges. The results show that the modified relations also depend on the ratios of web area to flange area and beam depth to column depth. The developed PZ model permits a realistic representation of the behavior of the PZ, especially in the case of thick column flanges.

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